INSTITUTE OF MATHEMATICS VICTORIA

IMV

Mathematical Methods (CAS) U34
Written Examination 1

Accreditation Period ∼ 2023–2027

Name: ____________________

Section	Time Recommended	Time Given	Marks Allocated	Marks Awarded
A	35 minutes	60 minutes	40	

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Write your student number in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted access to mobile phones and/or any other unauthorised electronic devices.

Instructions

- Answer all questions in the spaces provided.
- In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working must be shown.
- Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Question 1 **(2+2 = 4 marks)**

a) If $y = \tan(\frac{1}{x})$, find $\frac{dy}{dx}$. (2 marks)

b) Evaluate $h'(\pi)$ where $h(x) = e^{\cos(x)}$ (2 marks)

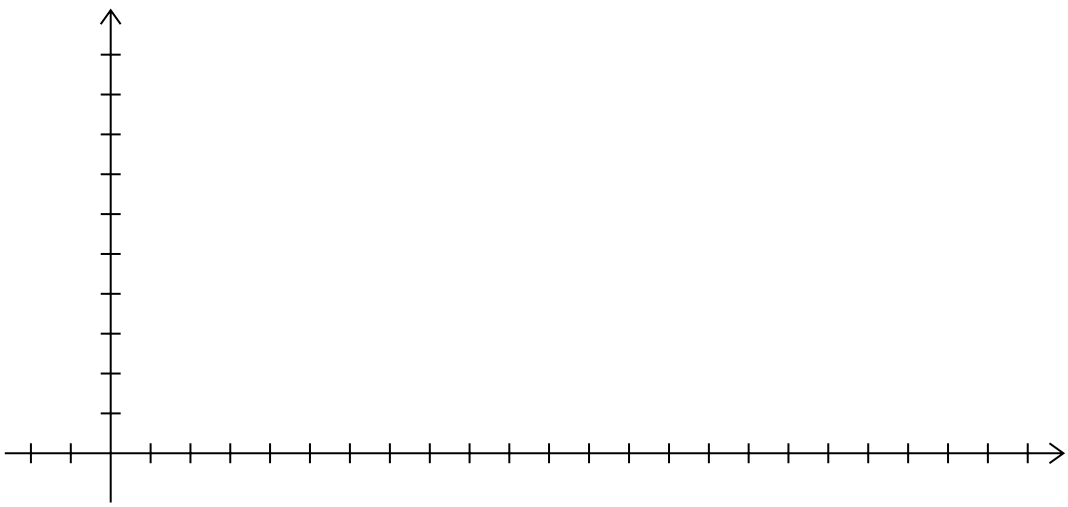
Question 2 **(1+1+2+2 = 6 marks)**

a) State an antiderivative of $\frac{1}{2}\cos(\frac{x}{4})$ (1 mark)

It is known that $\frac{dy}{dx} = \frac{1}{2}\cos(\frac{x}{4})$ and the graph of y touches the x axis once each period.

b) Find the possible expressions for y in terms of x (1 mark)

c) Sketch the graph of $y = 2\sin(\frac{x}{4}) + \sqrt{3}$ for $x \in [0, 8\pi]$ on the axes below. (2 mark)



d) Consider $y = 2\sin(\frac{x}{4}) + \sqrt{3}$. If $y = \frac{8}{5}$ when $x = \alpha$, state the value of $\cos(\alpha)$, where $0 \leq \alpha \leq \frac{\pi}{2}$. (2 marks)

Question 3 **(1+1+1+2+1+1 = 7 marks)**

Let $f : (a, b) \to \mathbb{R}, f(x) = \log_e(\sin(x)) - \log_e \cos(x))$, where $f(x)$ is defined over its maximal domain while satisfying $0 \leq a < b \leq 2\pi$.

a) Find the value of a and b (1 mark)

b) Hence, find the value(s) of c such that $f(c) = 0$. (1 mark)

c) Find $f'(x)$, the derivative of $f(x)$ (1 mark)

d) Find all possible value(s) of k such that $\sin(x+k) = \cos(x-k)$. (2 marks)

e) Show that $f(x+\frac{\pi}{4})$ is an odd function. (1 mark)

Hence, state the value of q in terms of p such that $\int_q^p f(x+\frac{\pi}{4})dx = 0$, where $0 < p < b - \frac{\pi}{4}$
(1 mark)

Question 3 **(1+1+2+2 = 6 marks)**

Consider the function $h(x) = e^x - e^{-x}$.

a) State the derivative of $h(x)$ (1 mark)

b) The graph of $y = h(x)$ and $y = a^x$ have no points of intersection. State the possible values of a, where $a > 0$ (1 mark)

c) Show that $\frac{h(x)}{h'(x)} = 1 - \frac{2}{(e^x)^2+1}$ (2 marks)

d) State the axial intercepts of $y = h(x) + c$ in terms of c, where $c < -2$. (2 marks)

Question 5 (1+2+2+1 = 6 marks)

Let X be a random variable with a density function of p(x)=$\begin{cases} \frac{k}{(x+1)^n} & x \geq 0 \\ 0 & x < 0 \end{cases}$ and $n \in \mathbb{Z}^+ \setminus \{1\}$. a) Show that n cannot equal 1 (1 mark)

b) Find the value of k in terms of n, where $n \in \mathbb{Z}^+ \setminus \{1\}$ (2 marks)

c) Show that $(m+1)(2k-(2k-n+1)(m+1)^{n-1})=0$, where m represents the median of X. (2 marks)

d) Hence, find the possible value(s) of m, in terms of n and k. (1 mark)

Question 6 **(1+1+3 = 5 marks)**

Matilda is making a biased coin in her product design class, for a game in which the coin is to be flipped four times. The probability that the coin lands on heads on any individual throw is z. Let $\hat{P}$ be the random variable that represents the proportion of throws that result in heads.

a) Find $\Pr(\hat{P} = 0.5)$ in terms of z (1 mark)

b) Find the value of z that corresponds with the highest possible value of $\Pr(\hat{P} = 0.5)$ (1 mark)

c) Find the value(s) of z such that $\hat{P}$ is the most likely outcome. (3 marks)

Question 7 **(1+2+1 = 4 marks)**

a) By using two rectangles of equal width and the left endpoint estimate, show that the approximate area bounded by the graph of $y = x^2 - \sqrt{x}$ and the x axis is equal to $\frac{2\sqrt{2}-1}{8}$. (1 mark)

b) Find the exact area bounded by the graph of $y = x^2$ and $y = \sqrt{x}$ (2 marks)

c) Hence, if the approximate area is A_A and the exact area is A_T, find the proportion of error, $\frac{A_T - A_A}{A_T}$. (1 mark)

Question 8 **(2 marks)**

A triangle is formed on the unit circle with vertices $(0,0)$, $(\cos(\theta),0)$ and $(\cos(\theta),\sin(\theta))$. $0 \leq \theta \leq \frac{\pi}{2}$. The area of the triangle can be determined by the expression $A = \frac{\sin(\theta)\cos(\theta)}{2}$. Find the maximum area of this triangle.

End of Written Examination 1