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Student Name.....

MATHEMATICAL METHODS UNITS 3 & 4

TRIAL EXAMINATION 1

2024

Reading Time: 15 minutes

Writing time: 1 hour

Instructions to students

This exam consists of 9 questions.
All questions should be answered in the spaces provided.
There is a total of 40 marks available.
The marks allocated to each of the questions are indicated throughout.
Students may **not** bring any calculators or notes into the exam.
Where a numerical answer is required, an exact value must be given unless otherwise directed.
Where more than one mark is allocated to a question, appropriate working must be shown.
Diagrams in this trial exam are not drawn to scale.
A formula sheet can be found on the last page of this exam.

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Question 1 (3 marks)

a. Let $y = \cos(1 - x^2)$.

Find $\frac{dy}{dx}$.

1 mark

b. If $f(x) = \frac{\sin(2x)}{1 + e^{2x}}$, find $f'(0)$.

2 marks

Question 2 (3 marks)

Let $f: (-1, \infty) \rightarrow \mathbb{R}$, $f(x) = \log_e(x+1)$ and $g: \mathbb{R} \rightarrow \mathbb{R}$, $g(x) = x^2$.

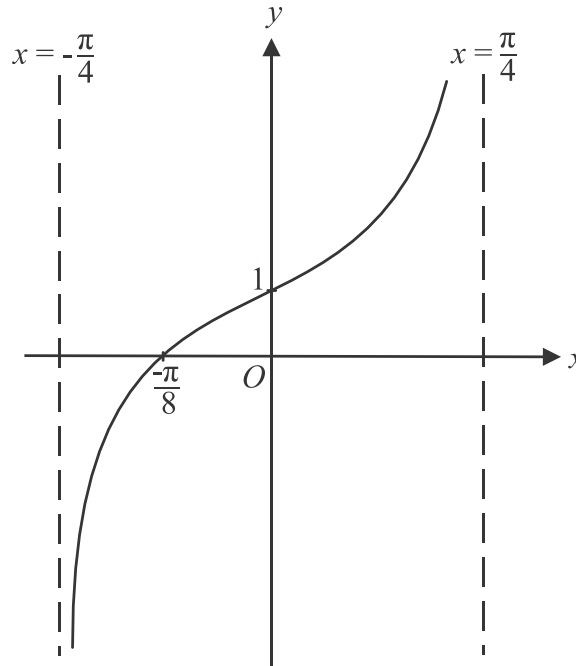
- a.** Find $(f \circ g)(x)$. 1 mark

- b.** State the domain and range of $(f \circ g)(x)$. 2 marks

Question 3 (5 marks)

Let $f: \left(-\frac{\pi}{4}, \frac{\pi}{4}\right) \rightarrow \mathbb{R}$, $f(x) = \tan(2x) + 1$.

Part of the graph of f is shown below.



- a.** Find the average rate of change of f between $x=0$ and $x=\frac{\pi}{8}$. 1 mark

- b.** Solve $f(x) = 1 + \frac{1}{\sqrt{3}}$ for x . 2 marks

Let $g: \left(-\frac{\pi}{4}, \frac{\pi}{4}\right) \rightarrow \mathbb{R}$, $g(x) = f(-x) - 2$.

- c.** Sketch the graph of g on the axes shown on page 4. Label any axis intercepts with their coordinates. 2 marks

Question 4 (4 marks)

- a.** Evaluate $\int_0^{e^{-1}} \frac{3}{x+1} dx$. 2 marks

- b.** Find $f(x)$ given that $f\left(\frac{1}{3}\right) = 0$ and $f'(x) = 2\sin(\pi x)$. 2 marks

Question 5 (4 marks)

Let $y = x^2 \log_e(2x)$.

- a.** Find $\frac{dy}{dx}$. 1 mark

- b.** Hence find the average value of the function $f(x) = x \log_e(2x)$ over the interval $x \in \left[\frac{1}{2}, 1\right]$.

Express your answer in the form $\log_e(a) - \frac{b}{a^3}$, where $a, b \in \mathbb{Z}^+$. 3 marks

Question 6 (2 marks)

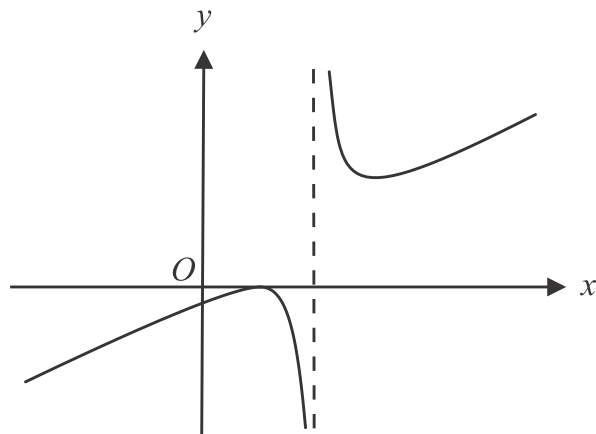
For random samples of four check-outs at a certain brand of supermarkets, $\hat{P}$ is the random variable that represents the proportion of check-outs that are occupied by customers.

It is known that $\Pr(\hat{P} = 0) = 0.25$.

Find the expected value of the proportion $E(\hat{P})$.

Question 7 (4 marks)

Consider the function f with rule $f(x) = x + \frac{1}{x-2}$. Part of the graph of f is shown below.



- a.** Find the coordinates of the stationary points of f . 2 marks

- b. i.** Find the values of c , where $c \in \mathbb{R}$, for which $f(x) + c = 0$ has no solutions. 1 mark

- ii.** Find the value of a , where $a \in \mathbb{R}$, for which the graph of $y = 1 + f(a - x)$ has no y -intercepts. 1 mark

Question 8 (7 marks)

A random variable X has the probability density function f given by

$$f(x) = \begin{cases} e^x - 1 & 0 \leq x \leq a \\ 6e^{-x} & a < x \leq b \\ 0 & \text{elsewhere} \end{cases}$$

where a and b are real constants. The function is continuous at $x = a$.

a. Show that $a = \log_e(3)$.

2 marks

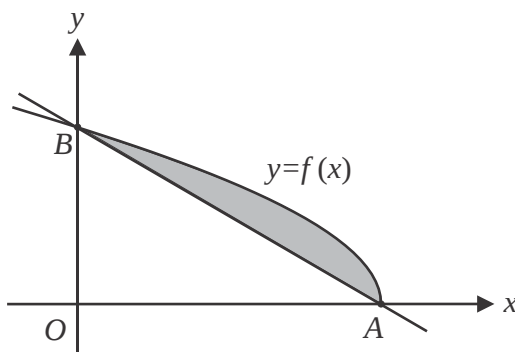
3 marks

$$b = \log_e \left(\frac{m}{n - \log_e(n)} \right) \quad \text{where } m, n \in N$$

[illegible]

Question 9 (8 marks)

Consider the function $f: (-\infty, 3] \rightarrow \mathbb{R}$, $f(x) = \sqrt{3-x}$. Part of the graph of f is shown below.



The points A and B represent the x and y intercepts of f respectively.
The shaded region between $y=f(x)$ and the straight line that passes through points A and B is also shown.

- a. Show that the equation of the line through A and B is given by $y = -\frac{\sqrt{3}}{3}x + \sqrt{3}$. 2 marks

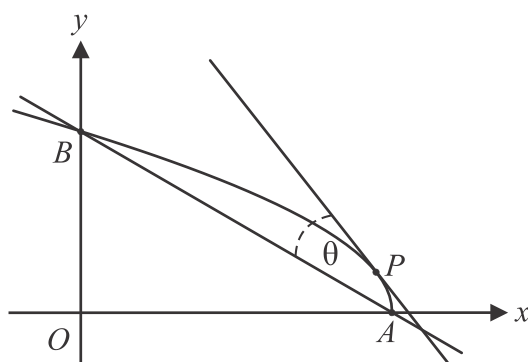
- b. Find the area of the shaded region. 2 marks

- c.** Find the rule for the derivative of f .

1 mark

The point $P(x, f(x))$ lies on the graph of f .

Let θ be the angle between the line AB and the tangent to f at P such that $0^\circ < \theta < 60^\circ$ as shown in the diagram below.



- d.** Find the coordinates of P when $\theta = 30^\circ$.

3 marks

Mathematical Methods formula sheet

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc \sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$
$\frac{d}{dx}((ax+b)^n) = an(ax+b)^{n-1}$	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e(x) + c, x > 0$
$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$	$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$	
product rule	quotient rule
$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
chain rule	Newton's method
$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
trapezium rule approximation	
	$Area \approx \frac{x_n - x_0}{2n} [f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-2}) + 2f(x_{n-1}) + f(x_n)]$

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A B) = \frac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = E(X)$	variance	$\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$
binomial coefficient	$\binom{n}{x} = \frac{n!}{x!(n-x)!}$		

Probability distribution		Mean	Variance
discrete	$\Pr(X=x) = p(x)$	$\mu = \sum x p(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
binomial	$\Pr(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$	$\mu = np$	$\sigma^2 = np(1-p)$
continuous	$\Pr(a < X < b) = \int_a^b f(x) dx$	$\mu = \int_{-\infty}^{\infty} x f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$

Sample proportions

$\hat{P} = \frac{X}{n}$	mean	$E(\hat{P}) = p$
standard deviation	$\text{sd}(\hat{P}) = \sqrt{\frac{p(1-p)}{n}}$	approximate confidence interval $\left(\hat{p} - z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right)$

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End of formula sheet