MATHEMATICAL METHODS

Written examination 2

TSSM™
Creating VCE Success

2023 Trial Examination

SOLUTIONS

SECTION 1: Multiple-choice questions (1 mark each)

1.	A	11.	A
2.	C	12.	C
3.	E	13.	E
4.	B	14.	A
5.	C	15.	B
6.	B	16.	C
7.	C	17.	D
8.	E	18.	E
9.	D	19.	E
10.	A	20.	A

Question 1

Answer: A

Explanation:

$-1 = 1 - 2x, \quad x = 1$
$4 = 1 - 2x, \quad x = -\frac{3}{2}$

Question 2

Answer: C

Explanation:

$\left(\frac{\pi}{2}, 0\right)$ and $(\pi, -2)$

$(0 - -2)^2 + \left(\frac{\pi}{2} - \pi\right)^2$
$= 4 + \frac{\pi^2}{4}$

Question 3

Answer: E

Explanation:

Range: $1 \pm 4 \quad T = \frac{2\pi}{2} = \pi$

Question 4

Answer: B

Explanation:

$\text{dom } h^{-1} = \text{ran } h = [4, \infty)$

$x = 2(x-3)^2 + 4$
$x - 4 = 2(x-3)^2$
$\frac{x-4}{2} = (x-3)^2$
$\pm\sqrt{\frac{x-4}{2}} = x - 3$

$so\ h^{-1}(x) = \pm\sqrt{\frac{x-4}{2}} + 3$

Discard negative as we want the upper section.

Question 5

Answer: C

Explanation:

$0.2^2 + 0.35^2 + 0.35^2 + 0^2 + 0.1^2 = 0.295$

Question 6

Answer: B

Explanation:

$\frac{y}{a} = \log_b(c - 2x)$
$b^{\frac{y}{a}} = c - 2x$
$2x = c - b^{\frac{y}{a}}$
$x = \frac{c - b^{\frac{y}{a}}}{2}$

Question 7

Answer: C

Explanation:

$$14 = \frac{2\pi}{b}, b = \frac{\pi}{7}$$

Negative cosine graph with mean value 2.5 and amplitude 2.5
Hence $y = 2.5 - 2.5\cos\left(\frac{\pi t}{7}\right)$

Question 8

Answer: E

Explanation:
Sample proportion is midpoint so $\frac{0.428+0.612}{2} = 0.52$

Question 9

Answer: D

Explanation:

$$y''(x) = 12x - 4, so\ y''(x) = 0\ at\ x = \frac{1}{3}, with\ y''(x) > 0\ and\ y''(x) < 0\ on\ either\ side.$$

Question 10

Answer: A

Explanation:

$64 + 16m + n = 100$
$f'(x) = 3x^2 + 2mx$
$f'(4) = 0$
$48 + 8m = 0$
$m = -6, n = 132$

Question 11

Answer: A

Explanation:

$(-5 \times a) + (0 \times 4a) + 5(1 - 5a)$
$= 5 - 30a$

Question 12

Answer: C

Explanation:

Need same gradient but different y intercepts.

$$\frac{a}{5} = \frac{2}{a} \neq \frac{b}{5}$$
$$a^2 = 10$$
$$a = \sqrt{10}$$

$$\frac{5}{\sqrt{10}} \neq \frac{b}{2}, \ b \neq \sqrt{10}$$

Question 13

Answer: E

Explanation:

$$Area: A(x) = x\sqrt{16 - x^2}$$
$$A'(x) = \sqrt{16 - x^2} - \frac{x^2}{\sqrt{16 - x^2}}$$
$$A'(x) = 0 \ at \ x = 2\sqrt{2}$$
$$A\left(2\sqrt{2}\right) = 8$$

Question 14

Answer: A

Explanation:

$\Pr(\hat{P}=0)=(1-p)^4=\frac{16}{625}=\left(\frac{2}{5}\right)^4$
Hence $p=\frac{3}{5}$
$\Pr(\hat{P}=1)=p^4=\frac{81}{625}$

Question 15

Answer: B

Explanation:

$Pr(X>265)=0.10565$
$So\ Pr\left(Z<\frac{265-250}{\sigma}\right)=0.89435$
$\frac{15}{\sigma}=1.25$
$\sigma=12$

Question 16

Answer: C

Explanation:

$f(x)=x^3-3x,\quad f'(x)=3x^2-3$
So $x_{n+1}=x_n-\frac{x_n^3-3x_n}{3x_n^2-3}$
$x_{n+1}=\frac{4x_n^3-6x_n}{3x_n^2-3}$

Question 17

Answer: D

Explanation:

Area$=\frac{1}{2}\times\frac{1}{2}\left(f(1)+2f(1.5)+2f(2)+2f(2.5)+2f(3)+2f(3.5)+f(4)\right)$
$=\frac{245}{16}=15.3125$

Question 18

Answer: E

Explanation:

$\int_0^{\frac{\pi}{4}} f(x)\,dx = 1$ so $a = 2$

$$E(X) = \int_0^{\frac{\pi}{4}} xf(x)\,dx = \frac{\pi - 2}{4}$$

Question 19

Answer: E

Explanation:
Look for first time abs f(x)<0.001

	N	x	f(x)	g(x)
3	0	2	-10.5858	-12.7071
4	1	1.16694	-3.00499	-7.92735
5	2	0.787873	-0.97461	-5.85384
6	3	0.621382	-0.37007	-4.99688
7	4	0.547322	-0.15887	-4.63563
8	5	0.51305	-0.07339	-4.47441
9	6	0.496649	-0.03525	-4.39887
10	7	0.488636	-0.01727	-4.36238
11	8	0.484677	-0.00855	-4.34446
12	9	0.48271	-0.00425	-4.33558
13	10	0.481729	-0.00212	-4.33116
14	11	0.481239	-0.00106	-4.32895
15	12	0.480994	-0.00053	-4.32785
16	13	0.480872	-0.00026	-4.3273
17	14	0.480811	-0.00013	-4.32702
18	15	0.48078	-6.6E-05	-4.32689
19	16	0.480765	-3.3E-05	-4.32682
20	17	0.480757	-1.7E-05	-4.32678

Question 20

Answer: A

Explanation:

$$\Pr(both\ same) = \Pr(blue, blue) + \Pr(black, black) = \frac{u}{u+v} \times \frac{u-1}{u+v-1} + \frac{v}{u+v} \times \frac{v-1}{u+v-1}$$

$$= \frac{u^2 + v^2 - u - v}{(u+v)(u+v-1)}$$

SECTION 2

Question 1

a. $dom\ f = (2, \infty) \quad ran\ f = R$

2 marks

b. $f'(x) = \frac{4}{(x-2)\log_e(2)}$

1 mark

c. $f'(6) = \frac{1}{\log_e(2)}$

$y - 9 = \frac{1}{\log_e(2)}(x - 6)$

$y = \frac{1}{\log_e(2)}(x - 6) + 9$

2 marks

d. $f'(x) = \frac{1}{2\log_e 2}\ at\ x = 10$

1 mark

$f(10) = 13$

1 mark

Distance $= \sqrt{(10-0)^2 + (13-0)^2} = \sqrt{269}$

1mark

e. Translation of 2 units left and one unit down followed by a dilation of factor $\frac{1}{4}$ from the x axis

3 marks

Total 11 marks

Question 2

a. Max temp occurs at $T = 24.2°C$ occurring at $t = 6\ ad\ t = 18$

2 marks

b. $T = \frac{2\pi}{\frac{\pi}{6}} = 12\ hours$

1 mark

c. $T(t) = 22\ at\ t = 3.607, 8.393, 15.607\ and\ 20.393$

$2 \times (8.393 - 3.607) = 9.572$

$\frac{9.572}{24} \times 100 = 40\%$ of the time greater than $22°C$

2 marks

d.

$T(t) = 23$ at $t = 4.289, 7.711, 16.289\ and\ 19.711$

$2 \times (7.711 - 4.289) = 6.842$ so 17.158 hours

$\frac{17.158}{24} \times 100 = 71\%$ of the time less than $23°C$

2 marks

e. $4 \times (4.289 - 3.607) = 2.728 hours = 2h\ 44min$

2 marks

Total 9 marks

Question 3

a. $\Pr(X = 4) = \binom{10}{4}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^6 = 0.2051$

1 mark

b. $\Pr(X \leq 7) = \sum_{x=0}^{7}\binom{10}{x}\left(\frac{1}{2}\right)^x\left(\frac{1}{2}\right)^{10-x} = 0.9453$

1 mark

c. $\Pr(X \leq 4) = \sum_{x=4}^{10}\binom{10}{x}\left(\frac{1}{2}\right)^x\left(\frac{1}{2}\right)^{10-x} = 0.9453$

$$\Pr(4 \leq X \leq 7) = \sum_{x=4}^{7}\binom{10}{x}\left(\frac{1}{2}\right)^x\left(\frac{1}{2}\right)^{10-x} = 0.7734$$

1 mark

So $\Pr(X \geq 4 | X \leq 7) = \frac{0.7734}{0.9453} = 0.818$

1 mark

d. $E(X) = np = 10 \times \frac{1}{2} = 5$

1 mark

$$SD(X) = \sqrt{np(1-p)} = \sqrt{10 \times \frac{1}{2} \times \frac{1}{2}} = \frac{\sqrt{10}}{2}$$

2 marks

e. $\Pr(X = 10) = \left(\frac{1}{2}\right)^{10} = \frac{1}{1024}$

1 mark

Total 8 marks

Question 4

a. $t = 0\ when\ A = 8.1$ so $c = 8.1$

1 mark

$A(2) = 11.043$ and $A(10) = 8.370$

$a = 4$, $b = 2$, c= 8

2 marks

b. From graph see that max occurs at $t = 2$ where $A = 11.043$mmol/L

1 mark

c. $A(t) = 10$ at $t = 0.66106$ and $t = 4.49464$

1 mark

$4.49464 - 0.66106 = 3.8336min = 3min50sec$

1 mark

d. $A'(t) = 4e^{-\frac{t}{2}} - 2te^{-\frac{t}{2}}$

1 mark

e. From graph of $A'(t)$ can see that max increase occurs at $t = 0$ and max decrease occurs at $t = 4$

2 marks

f. 7.9mmol/L

1 mark

g. Max is 10.659mmol/L and this occurs at $t = 2.5$

2 marks

h. $A(t) = B(t)$ at $t = 3.702$ and 11.508

1 mark

Hannah's blood sugar is greater than Freddie's during these times
Hence $11.508 - 3.702 = 7.806$ min= 7min 48sec

2 marks
Total 15 marks

Question 5

a. $\int_0^4 a(4x^3 - x^4)\,dx = 1$

1 mark

$a = \frac{5}{256}$

1 mark

b. $E(X) = \int_0^4 x\,f(x)\,dx$

1 mark

$x = \frac{8}{3}$ days

1 mark

c. $\int_3^4 f(x)\,dx = \frac{47}{128} = 37\%$

2 marks

d. $\int_2^3 f(x)\,dx = \frac{57}{128}$

$\int_2^4 f(x)\,dx = \frac{13}{16}$

1 mark

$\frac{\frac{57}{128}}{\frac{13}{16}} = \frac{57}{104}$

1 mark

e. $\Pr(30 < X < 40) = 0.5249$

1 mark

f. $\Pr(X < a) = 0.1$
$a = 26.02$mm is the largest small worm.

$\Pr(X > b) = 0.15$
$b = 42.26$mm is the shortest large worm.

2 marks

g. 75% of all worms are standard length.
$0.75^{20} = 0.0032$

1 mark

h. $E(\hat{P}) = p = 0.75$

1 mark

$$SD(\hat{P}) = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.75 \times 0.25}{20}} = 0.0968$$

2 marks

i. $\mu - 2\sigma = 0.75 - 2(0.0968) = 0.5564$
$\mu + 2\sigma = 0.75 + 2(0.0968) = 0.9436$

1 mark

In a random sample of 20 worms, the sample proportion will lie between 0.5564 and 0.9436 95% of the time.

1 mark

Total 17 marks