



Mathematical Methods Units 1 and 2

Written examination 1

September 2023

Reading time: 5 minutes

Writing time: 30 minutes

Name : _____ Teacher : _____

Question and Answer Booklet

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
1	7	7	22

- Students are permitted to bring into the room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are **NOT** permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book.
- A formula sheet

Instructions

- Write your **name** and your **teacher's** name in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorized electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, **an exact value** must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (3 marks)

The line with the equation $(a + 2)x - y = 5$ is perpendicular to the line $2y - x = 3$.

- a. Find the gradient of $2y - x = 3$ 1 mark

$\frac{1}{2}$ (1A)

- b. Find the value of a 2 marks

$a + 2 = -2$ (1M) or $(a + 2) \times \frac{1}{2} = -1$

$a = -4$ (1A)

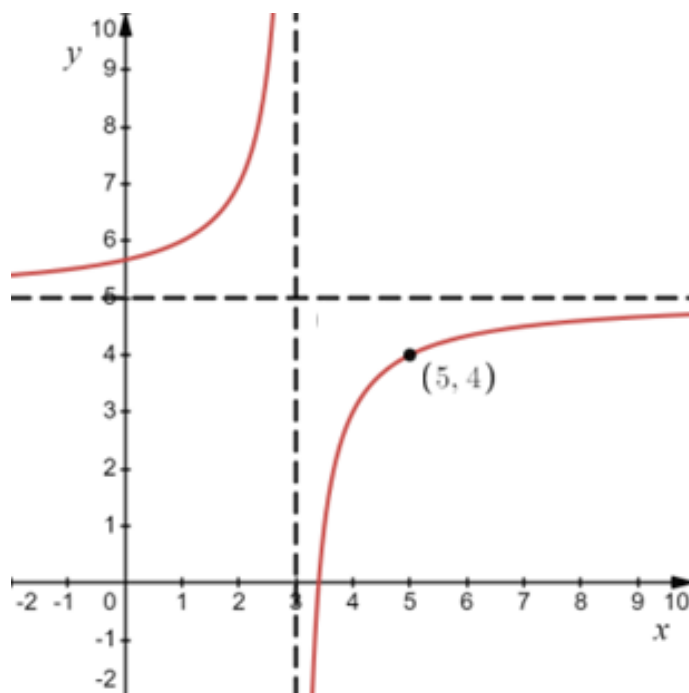
CM to be awarded according to a.

Question 2 (3 marks)

Determine the rule for the following graph in the form of $y = \frac{a}{x-h} + k$,

where $a, h, k \in \mathbb{R}$

3 marks



$$y = \frac{a}{x-3} + 5 \text{ (1 mark for } h = 3 \text{ and 1 mark for } k = 5 \text{)}$$

$$\text{sub } (5,4): a = -2$$

$$y = \frac{-2}{x-3} + 5 \text{ (1A)}$$

Question 3 (3 marks)

Two events A and B are such that: $\Pr(A)=0.6$, $\Pr(B)=0.3$ and $\Pr(A' \cap B)=0.1$.

- a. Find $\Pr(A \cap B)$ (1 mark)

$\Pr(A \cap B) = 0.2$ (1 mark)

- b. Find $\Pr(A' \mid B)$ (1 mark)

$\Pr(A|B') = \frac{1}{3}$ (1 mark)

- c. Are the events A and B independent? Justify your answer (1 mark)

No. $\Pr(A|B') = \frac{1}{3} \neq \Pr(A) = 0.6$ (1 mark) any other equivalent relationship

accepted as long as justified

Question 4 (5 marks)

a. Solve the equation $2 \sin(2x) - 1 = 0$ for $x \in [-\pi, \pi]$.

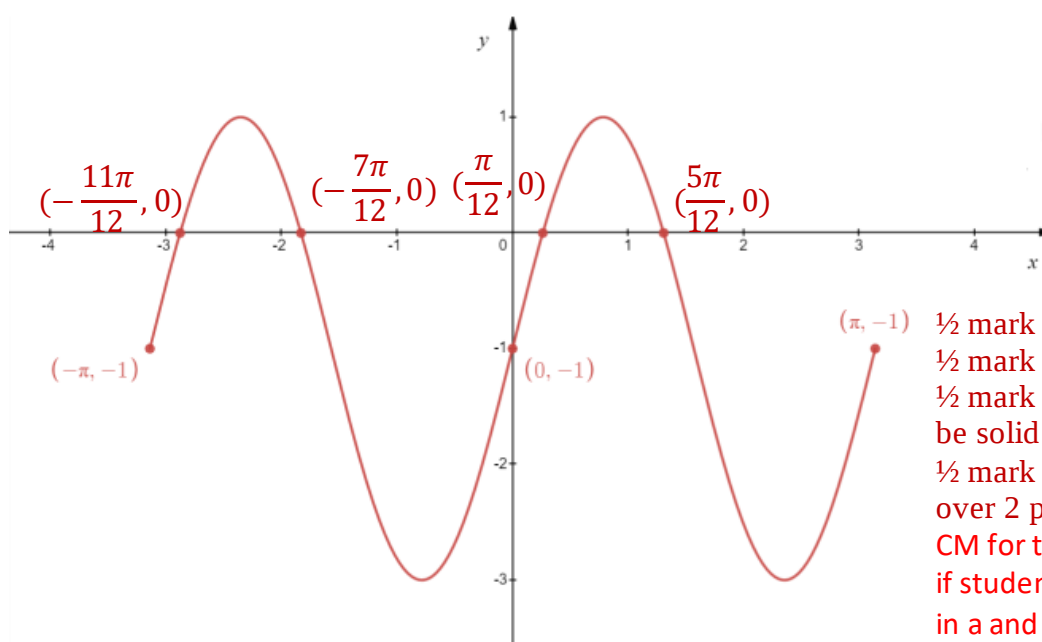
(3 marks)

$$2 \sin(2x) - 1 = 0 \therefore \sin(2x) = \frac{1}{2} \therefore$$

$$2x = \frac{\pi}{6} + 2n\pi \text{ or } 2x = \pi - \frac{\pi}{6} + 2n\pi \text{ (1 mark for each RA highlighted)}$$

$$x \in \left\{ -\frac{11\pi}{12}, -\frac{7\pi}{12}, \frac{\pi}{12}, \frac{5\pi}{12} \right\} \text{ (1 mark)}$$

b. Sketch the graph of $f: [-\pi, \pi] \rightarrow \mathbb{R}, f(x) = 2 \sin(2x) - 1$. Label the axis intercepts and end points with their coordinates. (2 marks)



½ mark for all x-intercepts
 ½ mark for all y-intercept
 ½ mark for endpoints ; must be solid dots
 ½ mark for shape; must be over 2 periods
 CM for the x intercepts only if students have 4 solutions in a and they are two positive and two negative solutions.

Question 6 (3 marks)

Evaluate the following:

a. $\log_2(16) =$

1 mark

$$\log_2(16) = 4 \text{ (1 mark)}$$

b. $2\log_2 12 + 3\log_2 5 - \log_2 15 - \log_2 150 =$

2 marks

$$2\log_2 12 + 3\log_2 5 - \log_2 15 - \log_2 150 = \log_2 \left(\frac{12^2 \times 5^3}{15 \times 150} \right) \text{ (1 mark)}$$

$$= \log_2 \left(\frac{2^4 \times 3^2 \times 5^3}{3^2 \times 5^3 \times 2} \right) = \log_2 2^3 = 3 \text{ (1 mark)}$$

Question 6 (5 marks)

Solve the following equations for x :

a. $5^{3x} = 25$ 1 mark

$3x = 2, x = \frac{2}{3}$ (1 mark)

b. $3^{2x} = 27 - 6 \times 3^x$ 3 marks

$3^{2x} = 27 - 6 \times 3^x \therefore t^2 = 27 - 6t, t = 3^x \therefore t^2 + 6t - 27 = 0$ (1 mark)

$\therefore (t + 9)(t - 3) = 0 \therefore t = -9, t = 3 \therefore$

$3^x = -9$ (no sol) (1 mark), $3^x = 3 \therefore x = 1$ (1 mark)

c. $\log_5(2x - 3) = 2$ 1 mark

$2x - 3 = 5^2 \therefore 2x = 28 \therefore x = 14$ (1 mark)

END OF EXAM 1