

The Mathematical Association of Victoria

Trial Exam 2023

MATHEMATICAL METHODS

WRITTEN EXAMINATION 1

STUDENT NAME _____

Reading time: 15 minutes

Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room any technology (calculators or software) or notes of any kind. blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 11 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions

- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale .

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

THIS PAGE IS BLANK

Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (3 marks)

a. Find $\frac{d}{dx}(x^3 \sin(2x))$.

1 mark

b. If $f(x) = \frac{e^{2x}}{2x+1}$ find $f'(2)$.
marks

2

TURN OVER

Question 2 (5 marks)

Let $f : (-2, \infty) \rightarrow \mathbb{R}, f(x) = \frac{1}{x+2}$ and $g : (3, \infty) \rightarrow \mathbb{R}, g(x) = \frac{1}{x-3}$.

- a. Explain why $f \circ g$ exists.

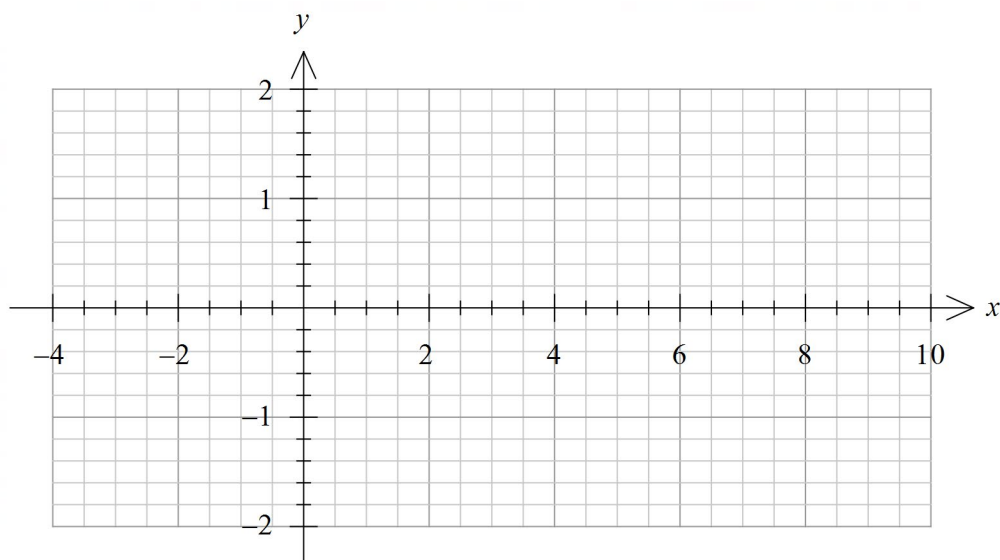
1 mark

- b. Show that $(f \circ g)(x) = \frac{1}{2} - \frac{1}{4x-10}$.

2 marks

- c. Sketch the graph of $f \circ g$ on the set of axes below.
Label any asymptotes with their equations.

2 marks



Question 3 (4 marks)

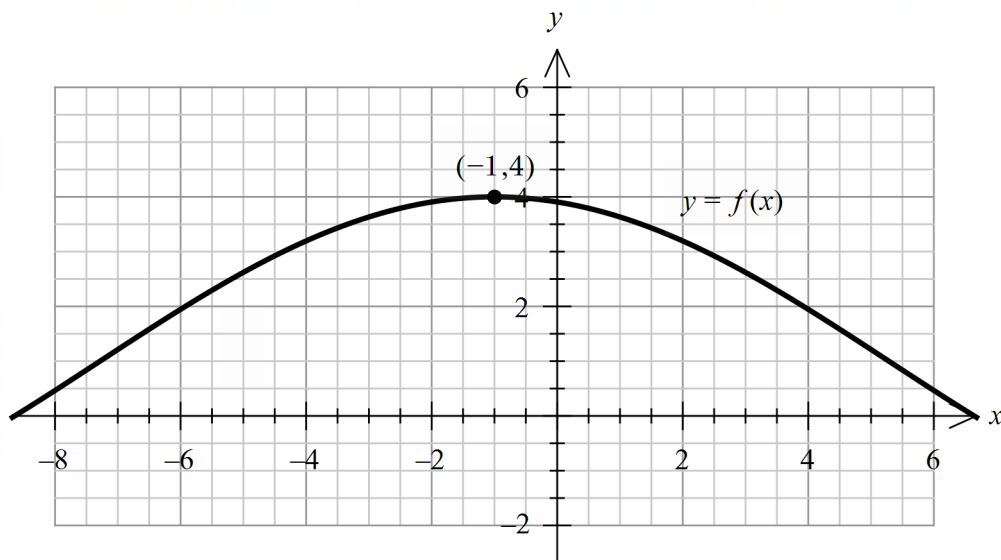
- a. Given that $g'(x) = \frac{1}{(3x+1)^2}$, $x \in \mathbb{R} \setminus \left\{-\frac{1}{3}\right\}$ find an expression for $g(x)$ if $g(0) = 3$. 2 marks

- b. Evaluate $\int_{-\frac{\pi}{18}}^{\frac{\pi}{4}} \left(-\frac{1}{2}\sin(6x)\right) dx$. 2 marks

TURN OVER

Question 4 (4 marks)

Part of the graph of $y = f(x)$ is shown below. It has a turning point at $(-1, 4)$ and the y -coordinates of the points of inflection are 1. The rule for the graph is $f(x) = A \cos\left(nx + \frac{1}{4}\right) + c$, where $n \in \mathbb{Q}$ and $A, c \in \mathbb{Z}^+$.



- a. i. Find the values of A and c .

1 mark

- ii. Find the value of n and give the general solution to the x -coordinates of the points of inflection.

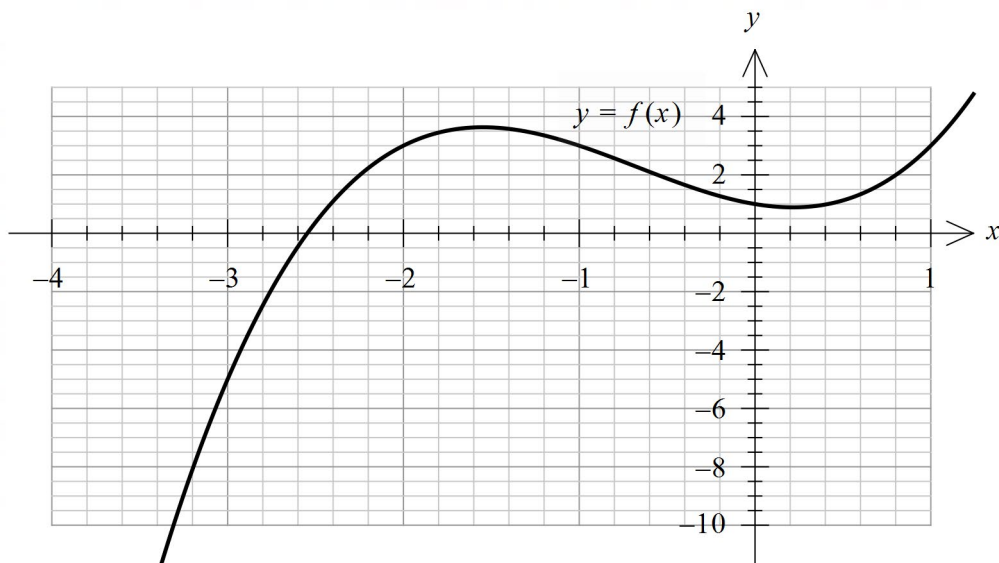
2 marks

- b. State a sequence of transformations that map the graph of $f(x) = A \cos\left(nx + \frac{1}{4}\right) + c$ onto the graph of $g(x) = \cos\left(nx + \frac{1}{4}\right) + c$.

1 mark

Question 5 (5 marks)

Part of the graph of f where $f(x) = x^3 + 2x^2 - x + 1$ is shown below.



- a. Find the x -coordinates of the turning points.

2 marks

The solution to $f(x) = 0$ occurs between $x = -2$ and $x = -3$.

- b. Use one iteration of Newton's Method, with $x = -3$ as the initial value, to find a better approximation.

2 marks

- c. Draw the tangent line to f at $x = -3$ on the above graph and label the x -axis intercept with its coordinates.

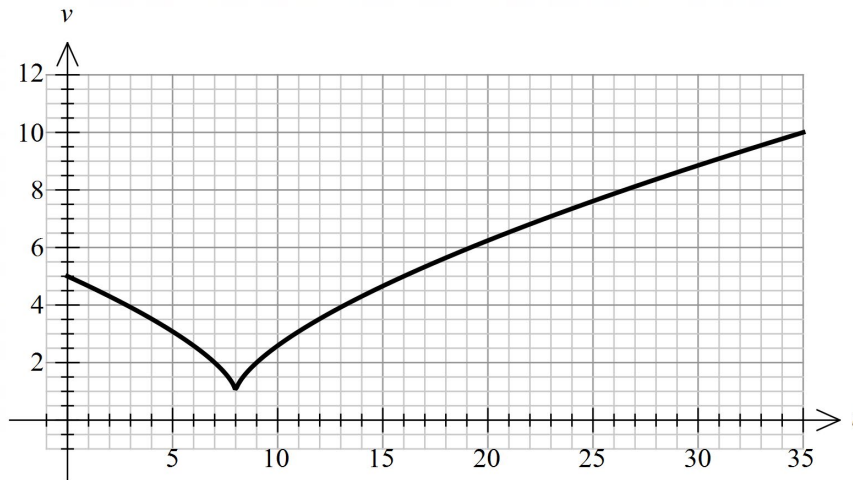
1 mark

TURN OVER

Question 6 (5 marks)

The velocity, $v \text{ ms}^{-1}$ of a particle at time t seconds has rule $v(t) = (t-8)^{\frac{2}{3}} + 1$ where $t \geq 0$.

Part of the graph of v is shown below.



- a. Find the time in seconds, when the acceleration, $a \text{ ms}^{-2}$, of the particle is equal to 1 ms^{-2} , given $a = v'(t)$.

2 marks

- b. Find the average rate of change of v from $t = 0$ to $t = 35$.

2 marks

A second particle is travelling at $v_2 \text{ ms}^{-1}$, where $v_2(t) = -v(t) + 10$. The bounded area between the graphs of v and v_2 represents how much further the second particle has travelled than the first particle over that time period.

- c. Write down a definite integral which when evaluated will give this distance. Give your answer in terms of t . Do not evaluate the integral.

1 mark

Question 7 (6 marks)

Hadiza either walks or takes the bus to work, depending on the weather. The probability it is going to rain on a particular morning is 0.2. If it rains on a particular morning the probability Hadiza will walk to work is 0.1. If it does not rain, the probability she will walk to work is 0.8.

- a. Given that Hadiza walks to work on a particular morning what is the probability that it rained? 2 marks

Hadiza works for a large accounting company. The duration of her telephone calls to her clients is a random variable T minutes with probability density function

$$f(t) = \begin{cases} \frac{3}{5}\sqrt{t} & 0 \leq t \leq 1 \\ \frac{3e}{5}e^{-t} & 1 < t < \infty \\ 0 & t < 0 \end{cases}$$

- b. Show that $\Pr(T \leq 1) = \frac{2}{5}$.

1 mark

- c. Find the 50th percentile of T .

3 marks

TURN OVER

Question 8 (4 marks)

Let $g(x) = a \log_2(x + b)$ where $a, b \in \mathbb{R}$. If $g(2) = 6$ and $g(6) = 9$ find a and b .
