

**2023
VCE
Mathematical
Methods
Year 12
Trial Examination 1
Detailed Answers**



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Quality educational content

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Question 1

a. $f(x) = \frac{\tan(2x)}{12x}$ quotient rule

$$f'(x) = \frac{\frac{2 \times 12x}{\cos^2(2x)} - 12 \tan(2x)}{(12x)^2} = \frac{\frac{2x}{\cos^2(2x)} - \tan(2x)}{12x^2} \quad \text{M1}$$

$$f\left(\frac{\pi}{6}\right) = \frac{\frac{\frac{\pi}{3}}{\cos^2\left(\frac{\pi}{3}\right)} - \tan\left(\frac{\pi}{3}\right)}{12\left(\frac{\pi}{6}\right)^2} = \frac{\frac{\pi}{3} \times 4 - \sqrt{3}}{\frac{\pi^2}{3}} \quad \text{A1}$$

$$f\left(\frac{\pi}{6}\right) = \frac{4\pi - 3\sqrt{3}}{\pi^2}, \quad p = 4, \quad q = -3\sqrt{3} \quad \text{A1}$$

b. $\int_{b-}^{b+} \frac{1}{x} dx = \log_e\left(\frac{5}{3}\right)$

$$\left[\log_e(x)\right]_{b-}^{b+} = \log_e\left(\frac{5}{3}\right)$$

$$\log_e(b+1) - \log_e(b-1) = \log_e\left(\frac{5}{3}\right) \quad \text{M1}$$

$$\log_e\left(\frac{b+1}{b-1}\right) = \log_e\left(\frac{5}{3}\right)$$

$$\frac{b+1}{b-1} = \frac{5}{3}$$

$$3(b+1) = 5(b-1)$$

$$3b+3 = 5b-5$$

$$2b = 8$$

$$b = 4 \quad \text{A1}$$

Question 2

Method 1

$$2^{x+3} = 3^{x+2}$$

$$\log_e(2^{x+3}) = \log_e(3^{x+2})$$

$$(x+3)\log_e(2) = (x+2)\log_e(3)$$

$$x\log_e(2) + 3\log_e(2) = x\log_e(3) + 2\log_e(3)$$

$$x(\log_e(3) - \log_e(2)) = 3\log_e(2) - 2\log_e(3)$$

$$x\log_e\left(\frac{3}{2}\right) = \log_e(2^3) - \log_e(3^2)$$

$$x\log_e\left(\frac{3}{2}\right) = \log_e(8) - \log_e(9)$$

$$x\log_e\left(\frac{3}{2}\right) = \log_e\left(\frac{8}{9}\right)$$

$$x = \frac{\log_e\left(\frac{8}{9}\right)}{\log_e\left(\frac{3}{2}\right)}, \quad c = \frac{8}{9}, \quad d = \frac{3}{2}$$

Method 2

$$2^{x+3} = 3^{x+2}$$

$$2^x \times 2^3 = 3^x \times 3^2$$

$$8 \times 2^x = 9 \times 3^x \quad \text{M1}$$

$$\frac{8}{9} = \frac{3^x}{2^x}$$

$$\frac{8}{9} = \left(\frac{3}{2}\right)^x$$

$$\log_e\left(\frac{8}{9}\right) = \log_e\left(\left(\frac{3}{2}\right)^x\right) \quad \text{M1}$$

$$\log_e\left(\frac{8}{9}\right) = x\log_e\left(\frac{3}{2}\right)$$

$$x = \frac{\log_e\left(\frac{8}{9}\right)}{\log_e\left(\frac{3}{2}\right)}, \quad c = \frac{8}{9}, \quad d = \frac{3}{2} \quad \text{A1}$$

Question 3

a. Let $f(x) = \cos\left(\frac{2\pi x}{3}\right) - x^2$

$$f(0) = \cos(0) - 0 = 1, \quad f(1) = \cos\left(\frac{2\pi}{3}\right) - 1 = -\frac{1}{2} - 1 = -\frac{3}{2}$$

Since $f(0) > 0$ and $f(1) < 0$ there is a sign change

between $x = 0$ and $x = 1$ then there is a root between $x = 0$ and $x = 1$ A1

b. $x_0 = \frac{3}{2}$ $f(x) = \cos\left(\frac{2\pi x}{3}\right) - x^2$, $f\left(\frac{3}{2}\right) = \cos(\pi) - \frac{9}{4} = -1 - \frac{9}{4} = -\frac{13}{4}$

$$f'(x) = -\frac{2\pi}{3}\sin\left(\frac{2\pi x}{3}\right) - 2x, \quad f'\left(\frac{3}{2}\right) = -\frac{2\pi}{3}\sin(\pi) - 3 = -3 \quad \text{A1}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = \frac{3}{2} - \left(\frac{-\frac{13}{4}}{-3}\right) = \frac{3}{2} - \frac{13}{12} = \frac{18-13}{12} \quad \text{M1}$$

$$x = \frac{5}{12} \quad \text{A1}$$

Question 4

$$\left(\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right) = (0.544, 0.736)$$

$$2\hat{p} = 0.736 + 0.544 = 1.28$$

A1

$$\hat{p} = \frac{1.28}{2}$$

$$\hat{p} = 0.64$$

$$2z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.736 - 0.544 = 0.192$$

$$z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \frac{0.192}{2} = 0.096, \quad 95\%, \quad z \approx 2$$

$$\sqrt{\frac{0.64 \times 0.36}{n}} = \frac{0.096}{2}$$

M1

$$\frac{0.48}{\sqrt{n}} = 0.048$$

$$\sqrt{n} = \frac{0.48}{0.048} = 10$$

$$n = 100$$

A1

Question 5

a. $\sin(x) + \sqrt{3}\cos(x) = 0$

$$\sin(x) = -\sqrt{3}\cos(x)$$

$$\frac{\sin(x)}{\cos(x)} = -\sqrt{3}$$

M1

$$\tan(x) = -\sqrt{3}$$

$$x = n\pi + \tan^{-1}(-\sqrt{3})$$

A1

$$x = n\pi - \frac{\pi}{3} = \frac{\pi}{3}(3n-1), \quad n \in \mathbb{Z}$$

b. $f : [0, 2\pi] \rightarrow \mathbb{R}, f(x) = \sin(x) + \sqrt{3}\cos(x)$

$f'(x) = \cos(x) - \sqrt{3}\sin(x) = 0$ for turning points

$\cos(x) = \sqrt{3}\sin(x)$

$\frac{\sin(x)}{\cos(x)} = \tan(x) = \frac{1}{\sqrt{3}}$

M1

$x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right), \pi + \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}, \pi + \frac{\pi}{6}$

$x = \frac{\pi}{6}, \frac{7\pi}{6}$

A1

$f\left(\frac{\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right) + \sqrt{3}\cos\left(\frac{\pi}{6}\right) = \frac{1}{2} + \sqrt{3} \times \frac{\sqrt{3}}{2} = 2, \left(\frac{\pi}{6}, 2\right)$

A1

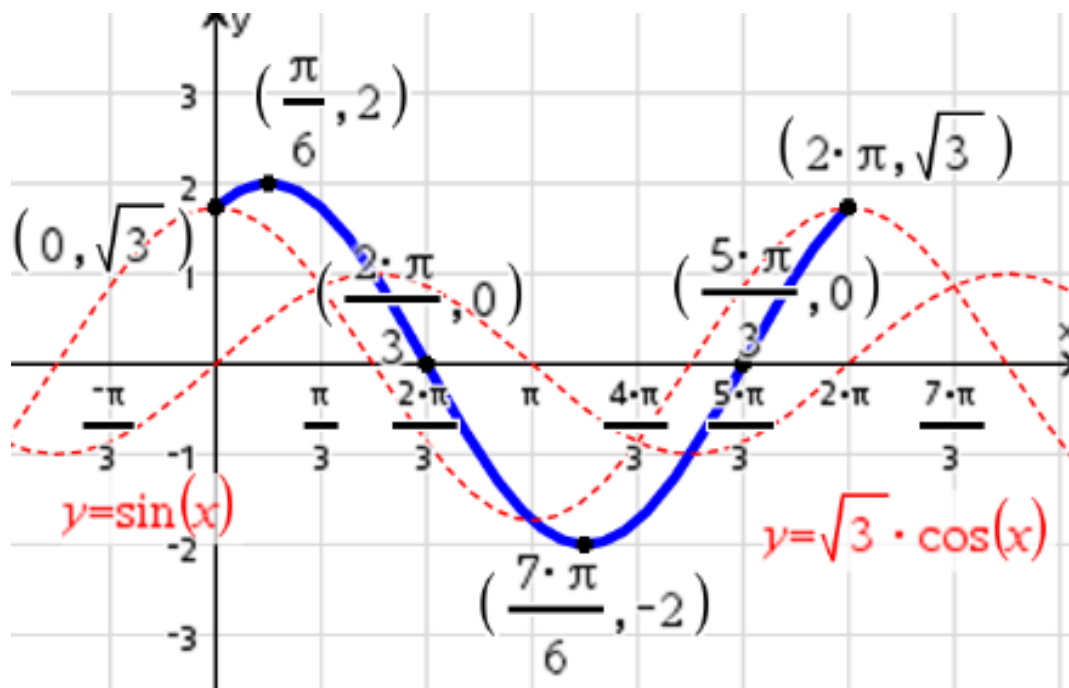
$f\left(\frac{7\pi}{6}\right) = \sin\left(\frac{7\pi}{6}\right) + \sqrt{3}\cos\left(\frac{7\pi}{6}\right) = -\frac{1}{2} - \sqrt{3} \times \frac{\sqrt{3}}{2} = -2, \left(\frac{7\pi}{6}, -2\right)$

c. Graph by addition of ordinates and from a. when $n=1$ $x = \frac{2\pi}{3}$ and when

$n=2$ $x = \frac{5\pi}{3}$ these are where the graph crosses the x-axis $\left(\frac{2\pi}{3}, 0\right), \left(\frac{5\pi}{3}, 0\right)$

endpoints $(0, \sqrt{3}), (2\pi, \sqrt{3})$

G3



Question 6 $f(x) = \frac{1}{x-3}$ and $g(x) = 1 + 2\sin(x)$

a. Complete a function domain and range table

function	domain	range
$f(x)$	$\mathbb{R} \setminus \{3\}$	$\mathbb{R} \setminus \{0\}$
$g(x)$	$\mathbb{R}$	$[-1, 3]$

$f \circ g(x)$ exists if $\text{range } g \subseteq \text{domain } f$, but $[-1, 3] \not\subseteq \mathbb{R} \setminus \{3\}$

so $f \circ g(x)$ does not exist

A1

$g \circ f(x)$ exist, if $\text{range } f \subseteq \text{domain } g$,

This is true, so $g \circ f(x) = g\left(f(x)\right) = g\left(\frac{1}{x-3}\right) = 1 + 2\sin\left(\frac{1}{x-3}\right)$

A1

and domain $g \circ f(x)$ is equal to the domain $f = \mathbb{R} \setminus \{3\}$

b.

$f \quad y = \frac{1}{x-3}$

swap x and y

$f^{-1} : \quad x = \frac{1}{y-3}, \quad y-3 = \frac{1}{x},$

$f^{-1}(x) = \frac{1}{x} + 3$

A1

solving

$f(x) = f^{-1}(x)$

$\frac{1}{x-3} = \frac{1}{x} + 3$

$x = x - 3 + 3x(x - 3)$

$3x^2 - 9x - 3 = 0$

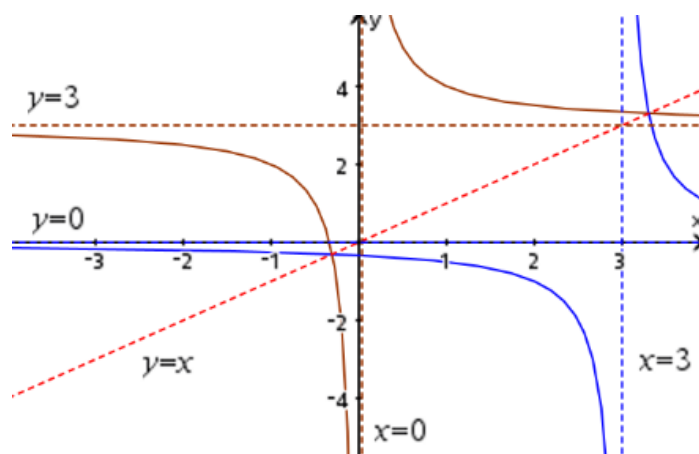
$x^2 - 3x - 1 = 0$

$x = \frac{3 \pm \sqrt{13}}{2}$

$f(x) < f^{-1}(x)$

$\Rightarrow x \in \left(-\infty, \frac{3 - \sqrt{13}}{2}\right) \cup (0, 3) \cup \left(\frac{3 + \sqrt{13}}{2}, \infty\right)$

A1



M1

Question 7

a. $F(t) = 50e^{-0.5t} - 0.4F(t)$

$$\frac{d}{dt}(F(t)e^{0.4t}) = \frac{d}{dt}(F(t))e^{0.4t} + F(t)\frac{d}{dt}(e^{0.4t}) \text{ using the product rule}$$

$$= F(t)e^{0.4t} + 0.4F(t)e^{0.4t} \quad \text{A1}$$

$$= (50e^{-0.5t} - 0.4F(t))e^{0.4t} + 0.4F(t)e^{0.4t} \quad \text{M1}$$

$$= 50e^{-0.1t} - 0.4F(t)e^{0.4t} + 0.4F(t)e^{0.4t} = 50e^{-0.1t}$$

b. $\frac{d}{dt}(F(t)e^{0.4t}) = 50e^{-0.1t}$

$$F(t)e^{0.4t} = \int 50e^{-0.1t} dt = \frac{50}{-0.1}e^{-0.1t} + c \quad \text{M1}$$

$$F(t)e^{0.4t} = -500e^{-0.1t} + c$$

when $t = 0$ $F(0) = 0$, $0 = -500 + c$, $c = 500$ A1

$$F(t)e^{0.4t} = 500 - 500e^{-0.1t} = 500(1 - e^{-0.1t})$$

$$F(t) = \frac{500(1 - e^{-0.1t})}{e^{0.4t}} = 500(e^{-0.4t} - e^{-0.5t})$$

c. $F(t) = 500(e^{-0.4t} - e^{-0.5t})$

$$F(t) = 500(-0.4e^{-0.4t} + 0.5e^{-0.5t}) = 0$$

$$\frac{2}{5}e^{-\frac{2}{5}} = \frac{1}{2}e^{-\frac{1}{2}} \quad \text{M1}$$

$$\frac{5}{4} = \frac{e^{-\frac{2}{5}}}{e^{-\frac{1}{2}}} = e^{\frac{t}{2} - \frac{2t}{5}} = e^{\frac{t}{10}}$$

$$\frac{t}{10} = \log_e\left(\frac{5}{4}\right) \quad \text{A1}$$

$$t = 10\log_e\left(\frac{5}{4}\right)$$

Question 8

a. $\frac{d}{dx} \left[x^2 \log_e(x) - \frac{x^2}{2} \right] = 2x \log_e(x)$ using the product rule

$$x^2 \frac{d}{dx} [\log_e(x)] + \log_e(x) \frac{d}{dx} (x^2) - \frac{d}{dx} \left(\frac{x^2}{2} \right)$$

$$= x^2 \times \frac{1}{x} + 2x \log_e(x) - x$$

A1

$$= x + 2x \log_e(x) - x$$

$$= 2x \log_e(x)$$

b. $f(x) = \begin{cases} \log_e(x) & \text{for } 1 \leq x \leq e \\ 0 & \text{otherwise} \end{cases}$

$$E(X) = \int_1^e xf(x) dx = \int_1^e x \log_e(x) dx$$

$$E(X) = \frac{1}{2} \left[x^2 \log_e(x) - \frac{x^2}{2} \right]_1^e \quad \text{from a.}$$

M1

$$E(X) = \frac{1}{2} \left[\left(e^2 \log_e(e) - \frac{e^2}{2} \right) - \left(\log_e(1) - \frac{1}{2} \right) \right] = \frac{1}{2} \left(e^2 - \frac{e^2}{2} + \frac{1}{2} \right)$$

$$E(X) = \frac{1}{4} (e^2 + 1)$$

A1

Question 9

a. $f : [0, \infty) \rightarrow \mathbb{R}$, $f(x) = e^{\sin\left(\frac{\pi x}{2}\right)}$ using the chain rule

$$f'(x) = \frac{\pi}{2} \cos\left(\frac{\pi x}{2}\right) e^{\sin\left(\frac{\pi x}{2}\right)} = 0 \text{ for turning points}$$

$$\cos\left(\frac{\pi x}{2}\right) = 0, \quad \frac{\pi x}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \quad \text{A1}$$

$$x = 1, 3, 5, 7, \quad f(1) = f(5) = e, \quad f(3) = f(7) = \frac{1}{e}$$

$(1, e), (5, e)$ maximum turning points

$$\left(3, \frac{1}{e}\right), \left(7, \frac{1}{e}\right) \text{ minimum turning points} \quad \text{A1}$$

b. since $f(x) = f(x+4)$, $\sin\left(\frac{\pi x}{2}\right) = \sin\left(\frac{\pi(x+4)}{2}\right) = \sin\left(2\pi + \frac{\pi x}{2}\right) = \sin\left(\frac{\pi x}{2}\right)$

the period is 4 A1

**End of detailed answers for the
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