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YEAR 12 *Trial Exam Paper*

2023

MATHEMATICAL METHODS

Written examination 1

Reading time: 15 minutes
Writing time: 1 hour

STUDENT NAME:

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
8	8	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 11 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions

- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination

- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (3 marks)

a. Let $y = \frac{2x}{3x+5}$.

Find $\frac{dy}{dx}$.

1 mark

b. Let $f(x) = 3 - e^{1-2x}$.

Evaluate $f'\left(\frac{1}{2}\right)$.

2 marks

Question 2 (3 marks)

Determine the values of a for which the simultaneous equations $ax + y = 3$ and $y = x - 7$ have a solution (x, y), where $x > 0$ and $y > 0$.

3 marks

Question 3 (6 marks)

Talula enjoys bushwalking and sometimes sees kangaroos on her walks. Let X be a random variable describing the number of kangaroos that Talula sees on any given walk. The probability distribution of X is given in the table below.

x	0	1	2	3
$\Pr(X = x)$	0.6	0.1	0.2	0.1

a. Find the mean, or expected value $E(X)$, of X.

1 mark

b. In a particular week, Talula goes on three bushwalks. Find the probability that Talula sees one kangaroo on exactly two of those walks.

2 marks

c. During another week, Talula went on two bush walks and she saw at least one kangaroo on each of those walks. Find the probability that Talula saw a total of three kangaroos on her walks during that week.

3 marks

Question 4 (6 marks)

Let $f: R \to R, f(x) = 3 + 2\sin\left(\frac{x}{2}\right)$.

a. The graph of f is transformed to produce the graph of the function g, where $g(x) = f(x - c)$.

i. If $c > 0$, find the smallest value of c such that the graph of g has a y-intercept at $(0, 3)$.

1 mark

ii. If $c \in R$, state all values of c such that the graph of g has a y-intercept at $(0, 3)$.

1 mark

b. Calculate the average value of f between $x = 0$ and $x = \pi$.

2 marks

c. The average value of f between $x = \frac{5\pi}{4}$ and $x = b$ is 3.

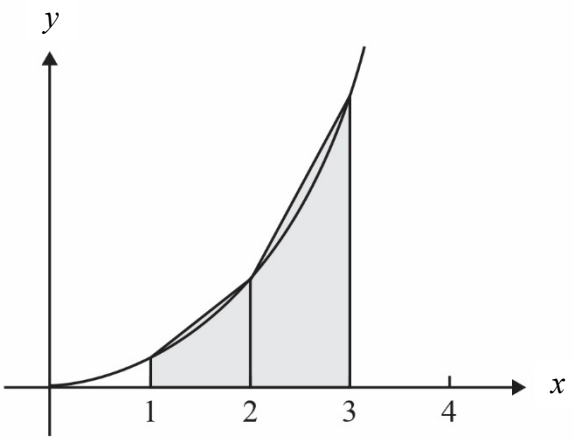
If $b \in \left(\frac{5\pi}{4}, 8\pi\right]$, state the possible values of b.

2 marks

Question 5 (6 marks)

Let $f : [0, \infty) \rightarrow R, f(x) = e^x - 1$

a. Two trapeziums of equal width are used to approximate the area between the graph with equation $y = f(x)$ and the x-axis between the lines with equations $x = 1$ and $x = 3$. The heights of the left and right edges of each trapezium are the values of $y = f(x)$, as shown on the graph below.



Find the total area of the trapeziums.

2 marks

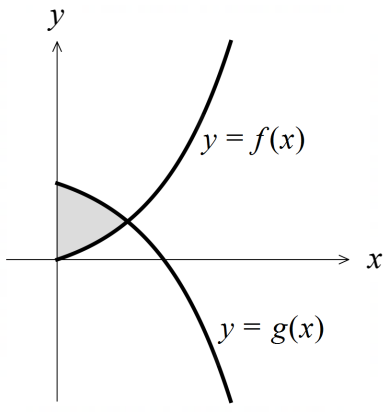
b. Let $g : [0, \infty) \rightarrow R$, $g(x) = m - e^x$, where $m \geq 1$.

The graphs of $y = f(x)$ and $y = g(x)$ intersect when $x = k$.

Show that $k = \log_e\left(\frac{m+1}{2}\right)$.

1 mark

c. The graphs of the functions $f(x) = e^x - 1$ and $g(x) = m - e^x$ are shown in the diagram below.



The area of the shaded region, which is bounded by the graphs of f and g and the y-axis, is $6\log_e(3) - 4$ square units.

Find the values of m and k.

3 marks

Question 6 (7 marks)

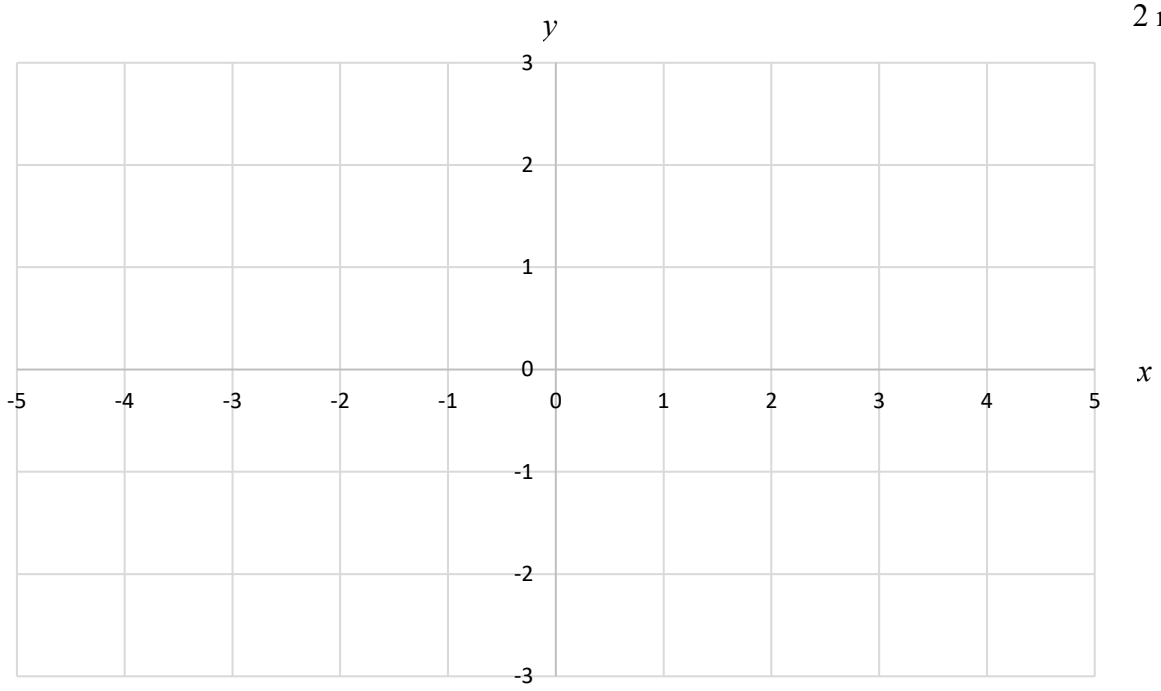
Consider the function f where $f: [-1, \infty) \rightarrow R, f(x) = \log_e(x + 2)$.

a. Find $f(-1)$.

1 mark

b. On the axes below, sketch the graph of f. Label any axial intercepts with their exact coordinates.

2 marks



c. Let $g : [d, \infty) \to R$, $g(x) = x^2 - 17$.

i. Find the smallest possible value of d such that $f \circ g$ exists.

2 marks

ii. For the value of d found in **part c.i.** state the range of $f \circ g$.

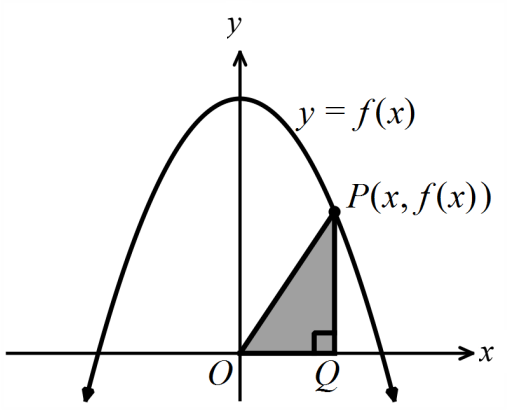
1 mark

d. Let $h : R \to R$, $h(x) = x^2$. State the range of $f \circ h$.

1 mark

Question 7 (4 marks)

The graph of $f(x) = 9 - x^2$ is shown below. The point P, which has coordinates $(x, f(x))$, where $1 \leq x \leq 2$, lies on the graph of f.



Triangle OQP has vertex O at the origin and vertex Q lies on the x-axis. Angle PQO is a right angle.

a. The area of triangle OQP is A square metres. Show that $A = \frac{9}{2}x - \frac{1}{2}x^3$.

1 mark

b. Find the minimum and maximum areas of triangle OQP.

3 marks

Question 8 (5 marks)

Let $f: \left[-\frac{v}{6}, v\right] \rightarrow R, f(x) = \sqrt{6x + v} - 3$ where v is a positive real number.

a. Find the value of $f\left(-\frac{v}{6}\right)$.

1 mark

b. The graphs of f and its inverse f^{-1} may have up to two points of intersection.

Find the x-coordinates of any possible points of intersection of the graphs of f and f^{-1} in terms of v.

2 marks

c. Determine the set of values of v for which the graphs of f and f^{-1} have two points of intersection.

2 marks

END OF QUESTION AND ANSWER BOOK