

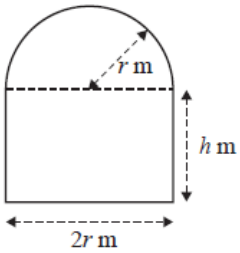
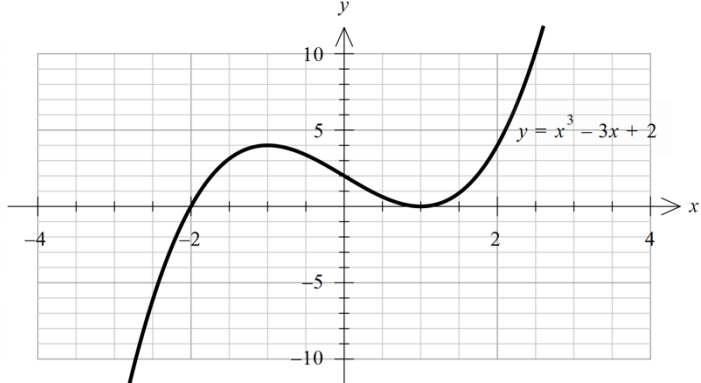
2022 VCE Mathematical Methods 2 (NHT) external assessment report

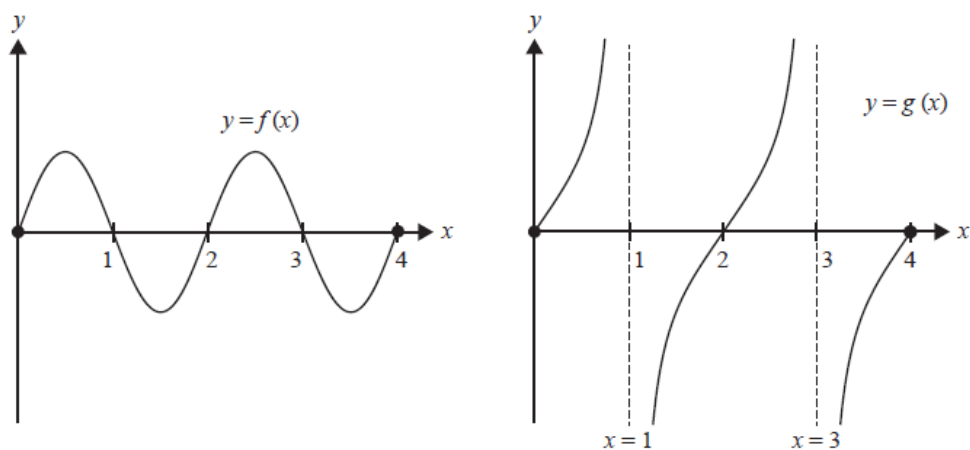
Specific information

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

Section A – Multiple-choice questions

Question	Correct answer	Comments
1	B	
2	E	
3	E	
4	B	
5	C	
6	D	
7	C	
8	D	The range is $(-\infty, 2.866\dots]$. So it is contained within the interval $(-\infty, 2.9)$.
9	A	
10	D	

Question	Correct answer	Comments
11	E	 $A = 2rh + \frac{\pi r^2}{2}, P = \pi r + 2h + 2r = 8, h = 4 - \frac{\pi r}{2} - r, A = 8r - 2r^2 - \frac{\pi r^2}{2}$
12	B	
13	D	
14	D	
15	E	
16	A	
17	C	
18	C	$x^2 + (y-1)^2 = 1, y = -\sqrt{1-x^2} + 1, \tan(135^\circ) = -1, \text{ solve } \frac{d}{dx}(-\sqrt{1-x^2} + 1) = -1 \text{ at } x = k, k = -\frac{\sqrt{2}}{2}$
19	A	 $x^3 - px + 2 = 0$ has three distinct real solutions for $p \in (3, \infty)$. When $x = 3$, there are two distinct real solutions as shown. For values of p greater than three, the y-coordinate of the local maximum turning point is positive and the y-coordinate of the local minimum turning point is negative, which means there will be three distinct real solutions.

Question	Correct answer	Comments
20	A	$h = f \times g$ has three x-intercepts on the interval $x \in [0, 4]$. The x-intercepts are at $x = 0$, $x = 2$ and $x = 4$. 

Section B

Question 1a.

$$f'(x) = -\frac{6}{5}(x-2)^2 \text{ or } f'(x) = -\frac{6}{5}x^2 + \frac{24x}{5} - \frac{24}{5}$$

Question 1b.

$$\left(2, \frac{3}{5}\right)$$

Question 1ci.

$$f'(x) = -\frac{6}{5}(x-2)^2 = -\frac{6}{5},$$

$$(x-2)^2 = 1,$$

$$x-2 = \pm 1,$$

$x = 1$ or $x = 3$, since $x = 1$ is given, the other point D is at $x = 3$.

Question 1cii.

$$y = \frac{19}{5} - \frac{6x}{5}$$

Question 1ciii.

Solving $f(x) = \frac{19}{5} - \frac{6x}{5}$

$$\left(0, \frac{19}{5}\right) \text{ or } (0, 0.38)$$

Question 1civ.

$$129.8^\circ$$

Question 1cv.

$$A_{\text{Total}} = \int_0^3 \left(\frac{19}{5} - \frac{6x}{5} - f(x) \right) dx + \int_3^{\frac{19}{6}} \left(\frac{19}{5} - \frac{6x}{5} \right) dx - \int_3^{\frac{\frac{1}{3^3} \times 2^{\frac{2}{3}} + 4}{2}} f(x) dx = 2.7015,$$

or

$$A_{\text{Total}} = \int_0^{\frac{\frac{1}{3^3} \times 2^{\frac{2}{3}} + 4}{2}} \left(\frac{19}{5} - \frac{6x}{5} - f(x) \right) dx + \int_{\frac{\frac{1}{3^3} \times 2^{\frac{2}{3}} + 4}{2}}^{\frac{19}{6}} \left(\frac{19}{5} - \frac{6x}{5} \right) dx = 2.7015$$

Question 2ai.

$$x \in (-\infty, 2]$$

Question 2aai.

$$h_1(x) = \sqrt{2+x}$$

Question 2bi.

Dilation by a factor 2 from the x-axis, translation of 1 unit to the right, or

Dilation by a factor of $\frac{1}{4}$ from the y-axis, translation by $\frac{5}{2}$ units to the right.

Question 2bii.

$$a=1, b=2, c=1, d=0 \text{ or } a=\frac{1}{4}, b=1, c=\frac{5}{2}, d=0$$

Question 2biii.

$$x = \frac{5}{3}$$

Question 2c.

$$h_3(x) = a(x-4)^2 + 6,$$

$$h_3(x) = -(x-4)^2 + 6 \text{ or } h_3(x) = -x^2 + 8x - 10$$

Question 2d.

$$x \in [0, \infty), \quad h^{-1}(x) = -x^2 + 2$$

Question 2e.

Method 1:

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ performs the inverse transformation to get $y = 2 - x^2$ for $x \geq 0$, adding $\begin{bmatrix} 4 \\ 4 \end{bmatrix}$ translates the graph to $y = -(x-4)^2 + 6$ for $x \geq 4$, although the rule is correct, because the maximal domain of the image is $x \geq 4$, the transformation cannot give h_3 .

Method 2:

Let $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 4 \\ 4 \end{bmatrix}$, $x' = y + 4$, $y = x' - 4$, $y' = x + 4$, $x = y' - 4$, substitute into $y = \sqrt{2-x}$, $y = -(x-4)^2 + 6$ for $x \geq 4$, although the rule is correct, because the maximal domain of the image is $x \geq 4$, the transformation cannot give h_3 .

Question 3a.

$$A = \int_0^1 (q(x) - p(x)) dx \text{ or } 2 \int_0^1 (q(x) - 2) dx,$$

$$A = \frac{4(e-1)}{e} \text{ or } A = -4e^{-1} + 4$$

Question 3b.

p is strictly increasing and q is strictly decreasing.

Question 3c.

$$p^{-1}(x) = \log_e \left(\frac{2}{2-x} \right) \text{ or } p^{-1}(x) = \log_e \left(\frac{-2}{x-2} \right) \text{ or } p^{-1}(x) = \log_e \left(\frac{-1}{x-2} \right) + \log_e(2)$$

$$q^{-1}(x) = \log_e \left(\frac{2}{x-2} \right) \text{ or } q^{-1}(x) = -\log_e(x-2) + \log_e(2)$$

Question 3di.

(1.805, 2.329)

Question 3dii.

$$y = -x + 4.134$$

Question 3ei.

$$\begin{aligned} r(x) &= p(x)q(x) \\ &= 2(1 - e^{-x}) \times 2(1 + e^{-x}) \\ &= 4(1^2 - (e^{-x})^2) \\ &= 4(1 - e^{-2x}) \end{aligned}$$

Question 3eii.

Domain R , Range $(-\infty, 4)$

Question 3eiii.

$$\begin{aligned} r^{-1}(x) &= \frac{1}{2} \log_e \left(\frac{4}{4-x} \right) \\ r^{-1}(x) &= \frac{1}{2} \log_e \left(\frac{2^2}{2^2-x} \right) \end{aligned}$$

Question 3fi.

$$y = x$$

Question 3fii.

(0, 0), (3.999, 3.999)

Question 4ai.

$$\begin{aligned} \int_0^6 xf(x) dx \\ = \frac{18}{5} \end{aligned}$$

Question 4aii.

$$\frac{6}{5} \text{ or } 1.2$$

Question 4bi.

3.6856

Question 4bii.

$$\Pr(X > 2 \mid X < 3.685634\dots)$$

$$= \frac{\Pr(2 < X < 3.685634)}{\Pr(X < 3.685634\dots)}$$

$$= \frac{\int_2^{3.685634\dots} f(x) dx}{0.5}$$

$$= 0.78$$

Question 4c.

$$0.5319$$

Question 4d.

$$\Pr(J \geq 15.1) = 0.95, \quad -1.64485\dots = \frac{15.1 - \mu}{\sigma},$$

$$\Pr(J > 23.9) = 0.10, \quad 1.28155\dots = \frac{23.9 - \mu}{\sigma},$$

$$\mu = 20, \quad \sigma = 3$$

Question 4ei.

$$11$$

Question 4eii.

$$0.0773$$

Question 4f.

$$35$$

Question 4gi.

$$0.0624$$

Question 4gii.

$$(0.3465, 0.5910)$$

Question 4giii.

For approximately 95% of all randomly selected samples from the population, the confidence interval calculated in this manner will capture the population proportion of customers who said the doughnuts are delicious.

Question 5a.

$$b_2(t) = -20e^{-\frac{t}{5}} + 20$$

Question 5b.

$$\frac{1}{10-0} \int_0^{10} h(t) dt$$

$$= 20.01$$

Question 5ci.

$$h'(t) = -4e^{-\frac{t}{5}} (\cos(2\pi t) + 10\pi \sin(2\pi t))$$

Question 5cii.

Max of $h'(t)$, (0.7, 18.9)

Question 5d.

Method 1:

$$(40 - 1.894...) + (36.382... - 1.894...) + (36.382... - 5.176...) + (33.413... - 5.176...) + (33.413... - 33.406...) = 132$$

Method 2:

$$\int_0^2 |h'(t)| dt = 132$$