

Trial Examination 2022

MATHEMATICAL METHODS

Trial Written Examination 2 - SOLUTIONS

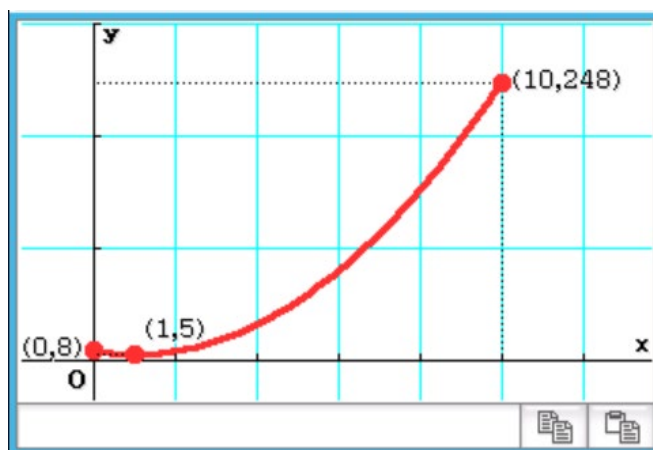
SECTION A: Multiple Choice

Question	Answer	Question	Answer
1	B	11	C
2	A	12	D
3	E	13	C
4	E	14	D
5	B	15	E
6	C	16	A
7	A	17	A
8	D	18	C
9	B	19	E
10	D	20	B

Question 1 Answer B

$f : [0, 10] \rightarrow R, f(x) = 3(x - 1)^2 + 5$ has restricted domain $[0, 10]$.

Minimum turning point is at $(1, 5)$ and the y -intercept is at $(0, 8)$



range $[5, 248]$

Question 2 **Answer A**

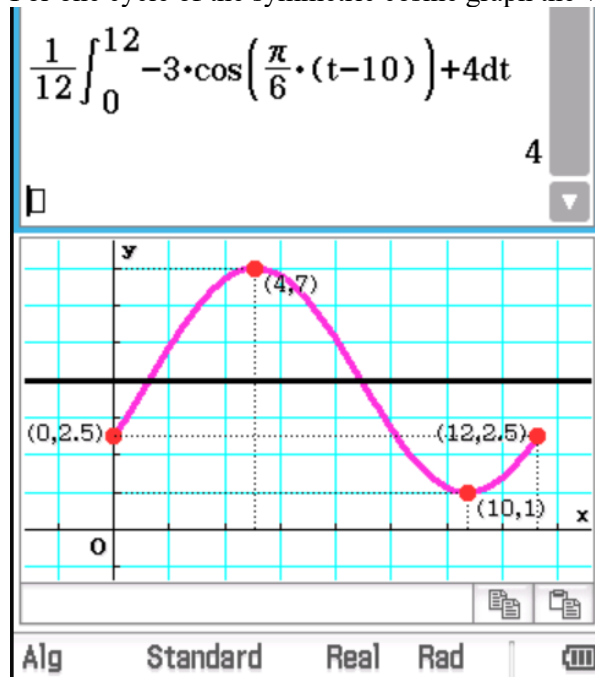
$$h(t) = -3 \cos\left(\frac{\pi}{6}(t-10)\right) + 4$$

The average height of water from 9 am ($t=0$) to 9 pm ($t=12$) is found by

$$\frac{1}{12-0} \int_0^{12} h(t) dt = 4$$

Alternatively

For one cycle of the symmetric cosine graph the vertical translation of 4 becomes the average value.

**Question 3** **Answer E**

For $y = -\pi \sin(2\pi x) - \pi$, the amplitude is π

Question 4 **Answer E**

$$f(x) = 2^{x-1}$$

$$f(x) \times f(y)$$

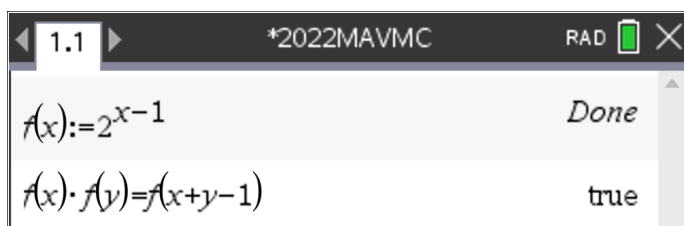
$$= 2^{x-1} \times 2^{y-1}$$

$$= 2^{x+y-2}$$

$$= 2^{x+y-1-1}$$

$$= f(x+y-1)$$

Question 4 (continued)



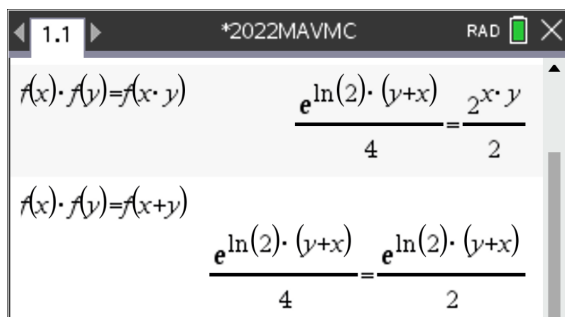
1.1 *2022MAVMC RAD

$$f(x) := 2^{x-1}$$

Done

$$f(x) \cdot f(y) = f(x+y-1)$$

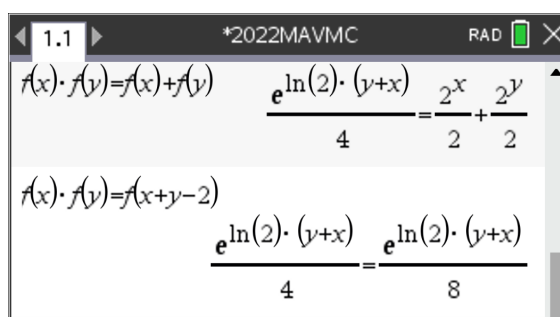
true



1.1 *2022MAVMC RAD

$$f(x) \cdot f(y) = f(x \cdot y) \quad \frac{e^{\ln(2) \cdot (y+x)}}{4} = \frac{2^{x \cdot y}}{2}$$

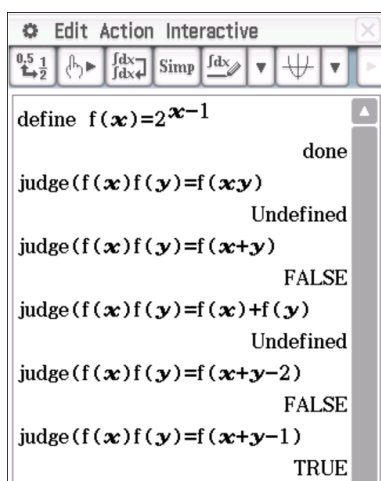
$$f(x) \cdot f(y) = f(x+y) \quad \frac{e^{\ln(2) \cdot (y+x)}}{4} = \frac{e^{\ln(2) \cdot (y+x)}}{2}$$



1.1 *2022MAVMC RAD

$$f(x) \cdot f(y) = f(x) + f(y) \quad \frac{e^{\ln(2) \cdot (y+x)}}{4} = \frac{2^x}{2} + \frac{2^y}{2}$$

$$f(x) \cdot f(y) = f(x+y-2) \quad \frac{e^{\ln(2) \cdot (y+x)}}{4} = \frac{e^{\ln(2) \cdot (y+x)}}{8}$$



Edit Action Interactive

0.5 1 2

```

define f(x)=2^{x-1}
done
judge(f(x)f(y)=f(x*y))
Undefined
judge(f(x)f(y)=f(x+y))
FALSE
judge(f(x)f(y)=f(x)+f(y))
Undefined
judge(f(x)f(y)=f(x+y-2))
FALSE
judge(f(x)f(y)=f(x+y-1))
TRUE
  
```

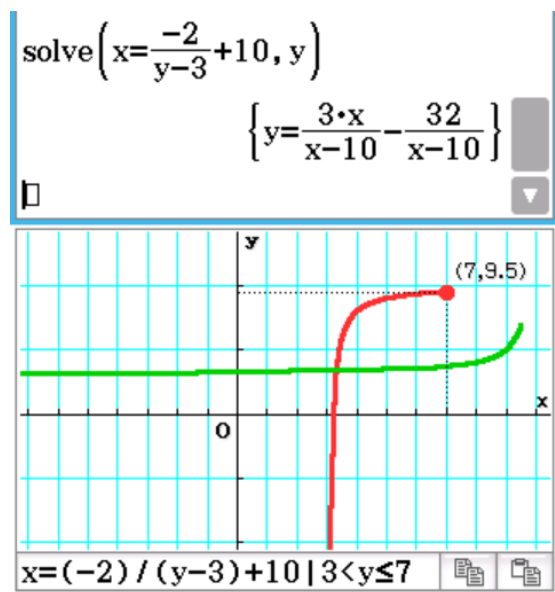
Question 5 **Answer B**

$$g: (3, 7] \rightarrow \mathbb{R}, g(x) = \frac{-2}{x-3} + 10 \text{ with domain } (3, 7] \text{ and range } (-\infty, 9.5]$$

Using CAS to swap x and y to get the rule for inverse

$$g^{-1}(x) = \frac{3x}{x-10} - \frac{32}{x-10}$$

$$\text{Dom } g^{-1}(x) = \text{Ran } g(x) = (-\infty, 9.5]$$

Question 5 (continued)**Question 6****Answer C**

$T: \mathbb{R}^2 \rightarrow \mathbb{R}^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ has been applied to get the image $y = e^{2x}$.

We need to find the original exponential rule.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x \\ 3y \end{bmatrix}$$

Giving $x' = -x$ and $y' = 3y$

Substitute into image equation $y' = e^{2x'}$

$$3y = e^{-2x}$$

$$y = \frac{1}{3}e^{-2x}$$

Question 7**Answer A**

The gradient of both equations is 1

$$2x - 2y = m \text{ and } 6x - 6y = m$$

$$y = x - \frac{m}{2} \text{ and } y = x - \frac{m}{6}$$

Hence parallel lines

Question 8**Answer D**

$$y = a \log_e (bx + c)$$

$$(3, -6 \log_e(2)) \text{ and } \left(\frac{2}{3}, 0\right)$$

From the graph the vertical asymptote is at $x = \frac{1}{3}$ and the graph is dilated by $\frac{1}{3}$ from the y-axis to get

$$\text{the } x\text{-intercept of } \left(\frac{2}{3}, 0\right)$$

So the rule is $y = a \log_e (3x - 1)$

Question 8 (continued)

Substitute $(3, -6\log_e(2))$

$$-6\log_e(2) = a\log_e(8)$$

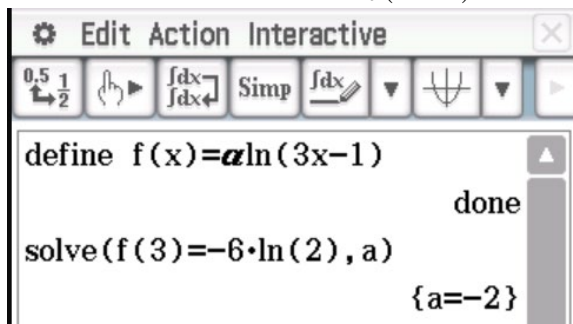
$$a = \frac{-6\log_e(2)}{\log_e(8)}$$

$$a = \frac{-6\log_e(2)}{\log_e(2)^3}$$

$$a = \frac{-6\log_e(2)}{3\log_e(2)}$$

Giving $a = -2$

Equation of graph is $y = -2\log_e(3x-1)$

**Question 9****Answer B**

A and B are two independent events.

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

$$= \frac{\Pr(A) \times \Pr(B)}{\Pr(B)} \text{ for independent events}$$

$$= \Pr(A) = \frac{1}{2}$$

$$\Pr(A') = \frac{1}{2}$$

$$\Pr(A' \cap B') = \Pr(A') \times \Pr(B') = \frac{1}{5}$$

$$\Pr(B') = \frac{2}{5}$$

$$\Pr(A \cap B') = \Pr(A) \times \Pr(B') = \frac{1}{2} \times \frac{2}{5} = \frac{1}{5}$$

OR

$$\Pr(A' \cap B') = \frac{1}{5} \text{ given}$$

$$\Pr(A' \cap B) = 1 - \left(\frac{1}{2} + \frac{1}{5} \right) = \frac{3}{10}$$

Question 9 (continued)

$$\Pr(A \cap B) = \Pr(A) \times \Pr(B) = \frac{1}{2} \times \left(\frac{3}{10} + \Pr(A \cap B) \right)$$

$$\Pr(A \cap B) = \frac{3}{10}$$

$$\Pr(A \cap B') = \frac{1}{2} - \frac{3}{10} = \frac{1}{5}$$

	A	A'	
B	$\frac{3}{10}$	$\frac{3}{10}$	$\frac{3}{5}$
B'	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{2}{5}$
	$\frac{1}{2}$	$\frac{1}{2}$	1

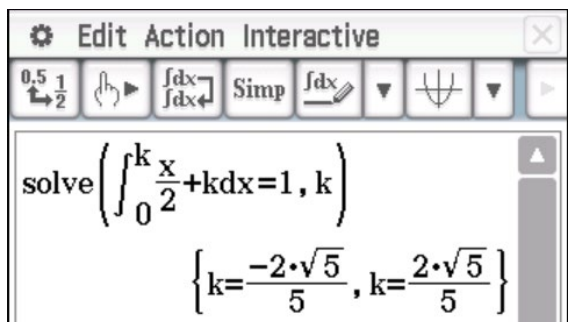
Question 10**Answer D**

$$f(x) = \begin{cases} \frac{x}{2} + k, & 0 \leq x \leq k \\ 0, & \text{otherwise} \end{cases} \quad \text{where } k \in \mathbb{R}^+.$$

Firstly, find constant k

For PDF let $\int_0^k \left(\frac{x}{2} + k \right) dx = 1$

For $k \in \mathbb{R}^+$, $k = \frac{2\sqrt{5}}{5}$

To find the median of $f(x)$ solve

$$\int_0^m \left(\frac{x}{2} + \frac{2\sqrt{5}}{5} \right) dx = 0.5$$

$$m = \frac{\sqrt{130} - 4\sqrt{5}}{5}$$

Question 10 (continued)

$$\text{solve}\left(\int_0^m \frac{x}{2} + \frac{2\sqrt{5}}{5} dx = 0.5, m\right)$$

$$\left\{m = \frac{\sqrt{130} - 4\sqrt{5}}{5}, m = \frac{-(\sqrt{130} + 4\sqrt{5})}{5}\right\}$$

Median is $\frac{\sqrt{130} - 4\sqrt{5}}{5}$

Question 11 **Answer C**

To find the mean of the sample, find the midpoint of the confidence interval (0.3346, 0.4654)

$$\frac{0.3346 + 0.4654}{2} = 0.4$$

Using the left edge of the CI solve for n where

$$0.4 - 1.96 \sqrt{\frac{0.4(1-0.4)}{n}} = 0.3346$$

Giving $n = 215.5598\dots$

Number of students in the random sample is closest to 216

$$\text{solve}\left(0.4 - 1.96 \sqrt{\frac{0.4 \cdot (1 - 0.4)}{n}} = 0.3346, n\right)$$

$$\{n = 215.5598575\}$$

Question 12 **Answer D**

To find k solve $0.2 + \frac{1+k^2}{10} + \frac{4-k}{10} + \frac{k}{20} = 1$

gives $k = 2, k = -\frac{3}{2}$

As $k \in \mathbb{R}^+$, $k = 2$

To find the mean evaluate $0 \times 0.2 + 1 \times \frac{1+k^2}{10} + 2 \times \frac{4-k}{10} + 3 \times \frac{k}{20}$ where $k = 2$

Mean = 1.2

Question 12 (continued)

The screenshot shows a TI-Nspire calculator window with the following content:

- Top menu: Edit Action Interactive
- Toolbar: $\frac{0.5}{1.2}$, $\frac{1}{2}$, $\frac{f(x)}{g(x)}$, $\frac{f(x)}{g(x)}$, $\frac{f(x)}{g(x)}$, $\frac{f(x)}{g(x)}$, $\frac{f(x)}{g(x)}$, $\frac{f(x)}{g(x)}$
- Equation: $\text{solve}\left(0.2 + \frac{1+k^2}{10} + \frac{4-k}{10} + \frac{k}{20} = 1, k\right)$
- Solution: $\{k=2, k=-\frac{3}{2}\}$
- Assignment: $2 \rightarrow k$
- Expression: $\frac{1+k^2}{10} + 2\left(\frac{4-k}{10}\right) + 3\left(\frac{k}{20}\right)$
- Bottom menu: Alg Standard Real Rad

Question 13 **Answer C**

$$f(x) = x^4 + bx^3 + x^2 - 2$$

$$\text{Solve } \frac{d}{dx}(x^4 + bx^3 + x^2 - 2) = 0 \text{ for } x$$

$$x = \frac{\pm\sqrt{9b^2 - 32} + 3b}{8} \text{ or } x = 0$$

There will be three solutions when $9b^2 - 32 > 0$.

$$b < -\frac{4\sqrt{2}}{3} \text{ or } b > \frac{4\sqrt{2}}{3}$$

TI-Nspire calculator screen showing the derivative equation and its solution:

$$\text{solve}\left(\frac{d}{dx}(x^4 + b \cdot x^3 + x^2 - 2) = 0, x\right)$$

$$x = \frac{-\left(\sqrt{9 \cdot b^2 - 32} + 3 \cdot b\right)}{8} \text{ or } x = \frac{\sqrt{9 \cdot b^2 - 32} - 3 \cdot b}{8}$$

TI-Nspire calculator screen showing the derivative equation and its solution:

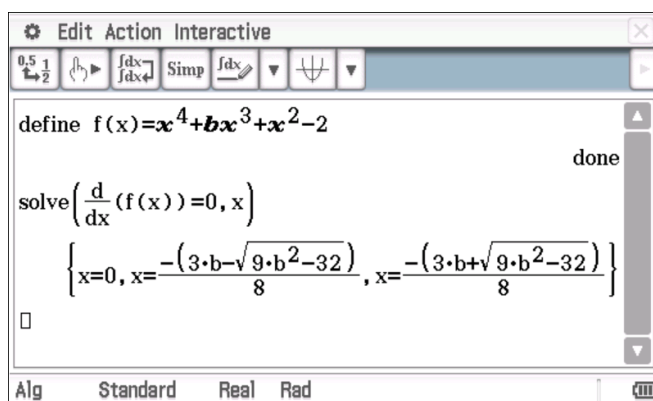
$$\text{solve}\left(\frac{d}{dx}(x^4 + b \cdot x^3 + x^2 - 2) = 0, x\right)$$

$$\frac{\sqrt{b^2 - 32} + 3 \cdot b}{8} \text{ or } x = \frac{\sqrt{9 \cdot b^2 - 32} - 3 \cdot b}{8} \text{ or } x = 0$$

TI-Nspire calculator screen showing the inequality for three solutions:

$$\text{solve}(9 \cdot b^2 - 32 > 0, x)$$

$$b < \frac{-4 \cdot \sqrt{2}}{3} \text{ or } b > \frac{4 \cdot \sqrt{2}}{3}$$

Question 13 (continued)**Question 14****Answer D**

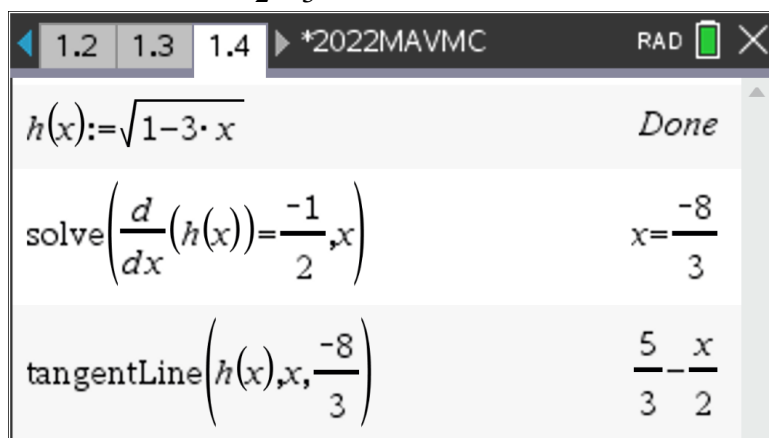
$$h(x) = \sqrt{1-3x}$$

The gradient of the line perpendicular to the line $y = 2x$ is $-\frac{1}{2}$.

Solve $h'(x) = -\frac{1}{2}$ for x

$$x = -\frac{8}{3}$$

The tangent is $y = -\frac{x}{2} + \frac{5}{3}$.



Question 14

Edit Action Interactive
 define $h(x) = \sqrt{1-3x}$
 done
 solve $\left(\frac{d}{dx}(h(x)) = -\frac{1}{2}, x\right)$
 $\left\{x = -\frac{8}{3}\right\}$
 tanLine $\left(h(x), x, -\frac{8}{3}\right)$
 $-\frac{x}{2} + \frac{5}{3}$

Question 15**Answer E**

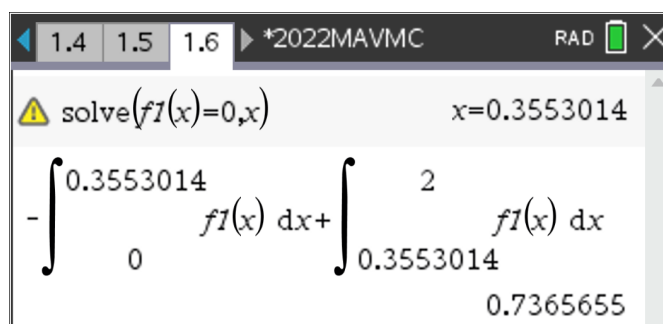
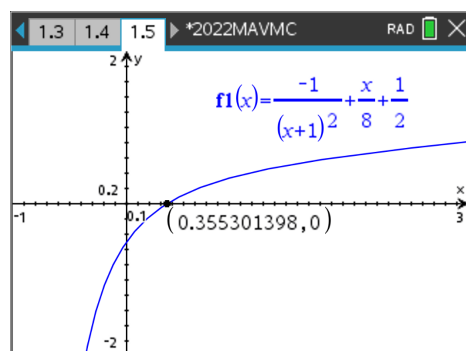
$$v = \frac{-1}{(t+1)^2} + \frac{t}{8} + \frac{1}{2}$$

Solve $v(t) = 0$ for t

$$t = 0.3553\dots$$

$$- \int_0^{0.3553\dots} (v(t)) dt + \int_{0.3553\dots}^2 (v(t)) dt$$

$$= 0.737 \text{ correct to three decimal places}$$



Question 15 (continued)

define $v(t) = \frac{-1}{(t+1)^2} + \frac{t}{8} + \frac{1}{2}$

done

solve($v(t)=0, t$)

$\{t=0.3553013976\}$

$-\int_0^{0.35530139} v(t) dt + \int_{0.35530139}^2 v(t) dt$

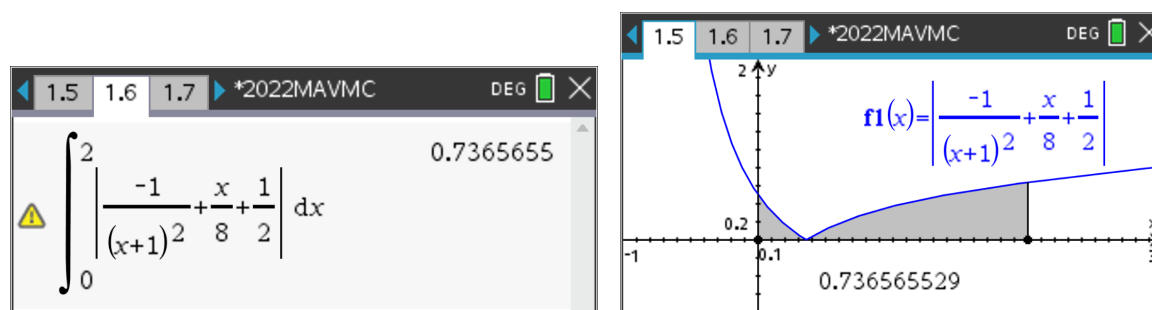
0.7365655325

Alg Standard Real Rad

OR

Please note that absolute values are not on the course but can be used for these types of questions.

$$\int_0^2 (|v(t)|) dt = 0.737$$

**Question 16****Answer A**

$$f(x) = e^{\sin(x)} \text{ and } g(x) = e^{\cos(x)}$$

$$\text{Area} = \int_{\frac{3\pi}{4}}^{\frac{\pi}{4}} (g(x) - f(x)) dx + \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (f(x) - g(x)) dx$$

$$= 2 \int_{\frac{3\pi}{4}}^{\frac{\pi}{4}} (g(x) - f(x)) dx$$

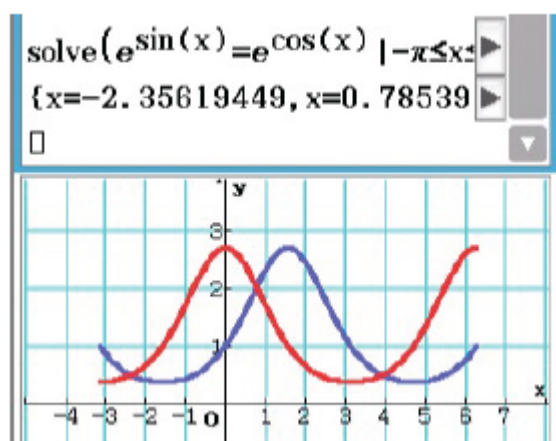
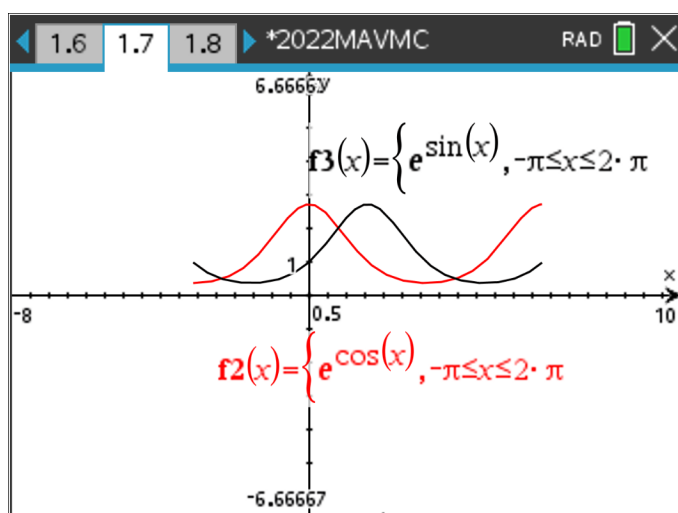
solve($e^{\sin(x)} = e^{\cos(x)}, x$)

$x = \frac{(4 \cdot n1 - 3) \cdot \pi}{4}$

solve($e^{\sin(x)} = e^{\cos(x)}, x$) | $-\pi < x < 2 \cdot \pi$

$x = \frac{-3 \cdot \pi}{4}$ or $x = \frac{\pi}{4}$ or $x = \frac{5 \cdot \pi}{4}$

Question 16



Question 17

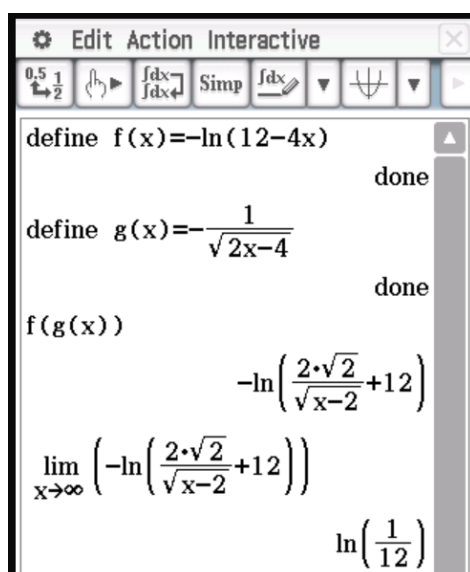
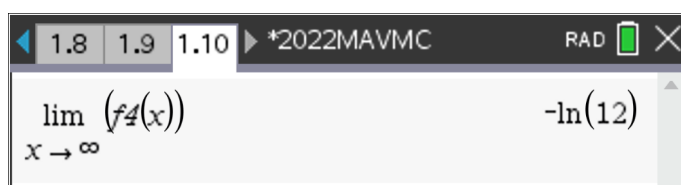
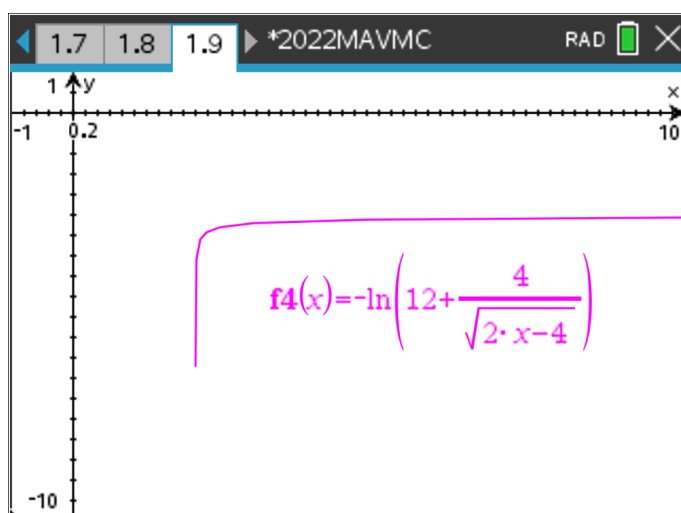
Answer A

$$f(x) = -\log_e(12 - 4x) \text{ and } g(x) = -\frac{1}{\sqrt{2x-4}}$$

$$f(g(x)) = -\log_e\left(12 + \frac{4}{\sqrt{2x-4}}\right)$$

Domain of $f(g(x))$ is $x > 2$

Range is $(-\infty, -\log_e(12))$

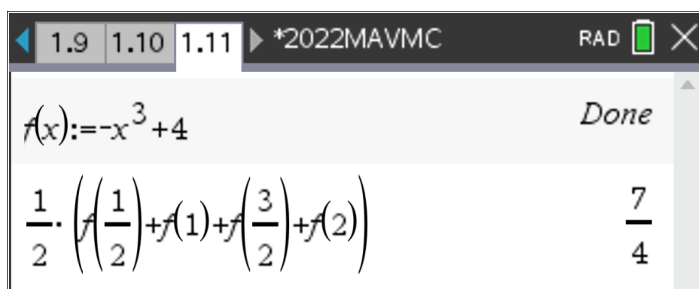
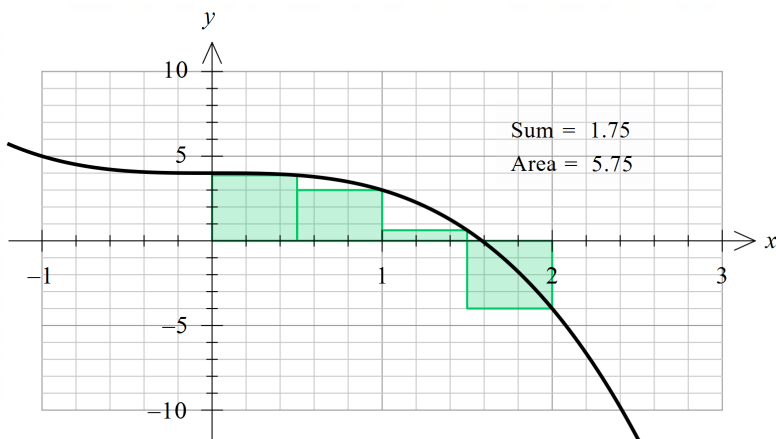
Question 17 (continued)**Question 18****Answer C**

$$y = f(x) = -x^3 + 4$$

$$\int_0^2 (-x^3 + 4) dx$$

$$\approx \frac{1}{2} \left(f\left(\frac{1}{2}\right) + f(1) + f\left(\frac{3}{2}\right) + f(2) \right)$$

$$= \frac{7}{4}$$

Question 18 (continued)**Question 19** **Answer E**

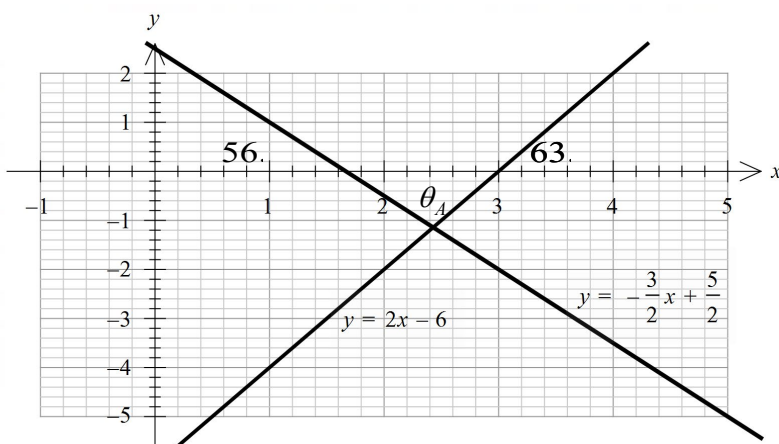
$$2x - y = 6 \text{ and } 3x + 2y = 5$$

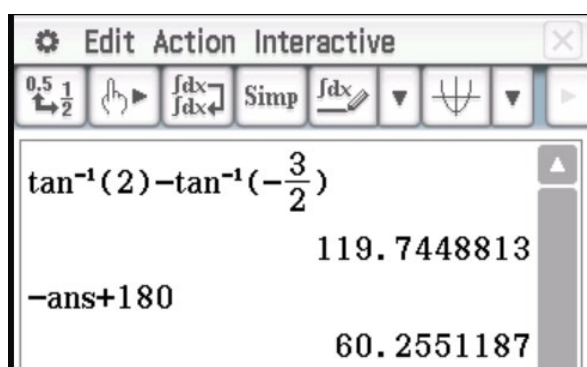
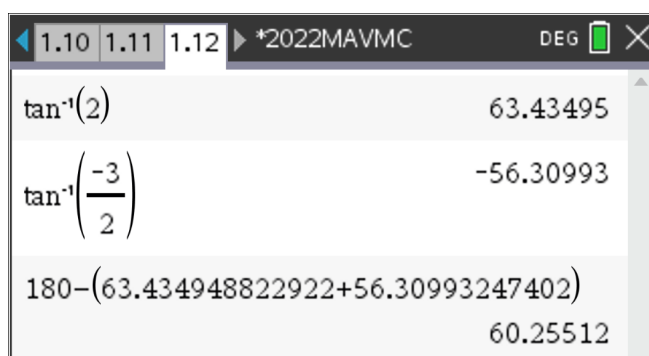
$$y = 2x - 6 \text{ and } y = -\frac{3}{2}x + \frac{5}{2}$$

$$\tan(\theta_1) = 2 \text{ and } \tan(\theta_2) = -\frac{3}{2}$$

$$\theta_1 = \tan^{-1}(2) = 63.43\dots, \theta_2 = \tan^{-1}\left(-\frac{3}{2}\right) = -56.30\dots$$

$$\text{Acute angle} = \theta_A = 180 - (63.43\dots + 56.30\dots) = 60.26 \text{ correct to two decimal places}$$



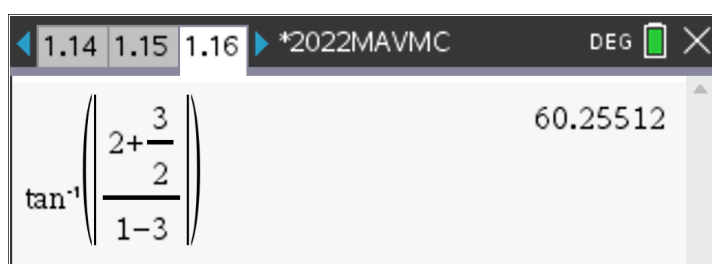
Question 19 (continued)

OR

Please note absolute values are not on the course but can be used for these types of questions.

$$\tan(\theta) = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| = \left| \frac{2 + \frac{3}{2}}{1 - 3} \right|$$

Acute angle = 60.26 correct to two decimal places



Question 20**Answer B**

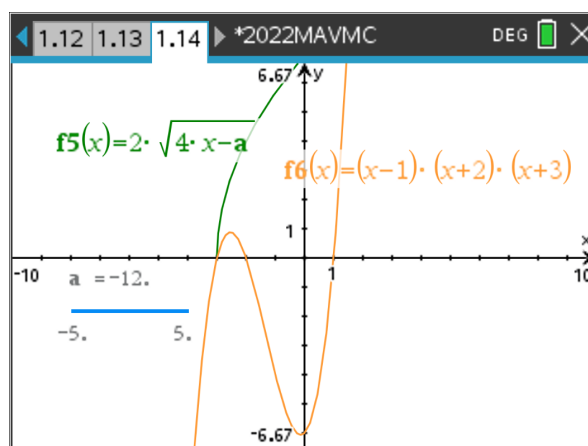
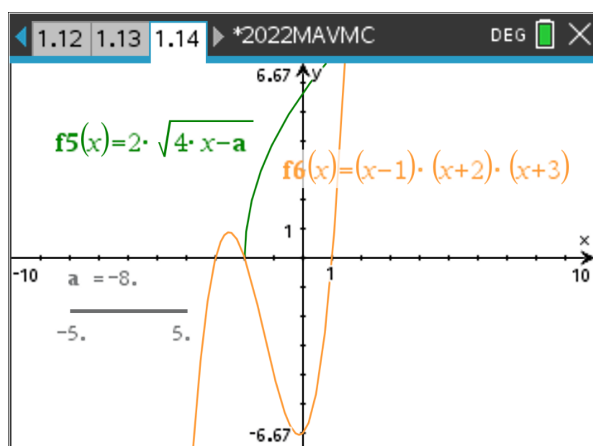
$$f(x) = 2\sqrt{4x-a} \text{ and } g(x) = (x-1)(x+2)(x+3)$$

The maximum number of points of intersection is two.

When $x = -2$, $2\sqrt{4 \times -2 - a} = 0$, $a = -8$

When $x = -3$, $2\sqrt{4 \times -3 - a} = 0$, $a = -12$

So two points of intersection occur when $-12 \leq a \leq -8$.



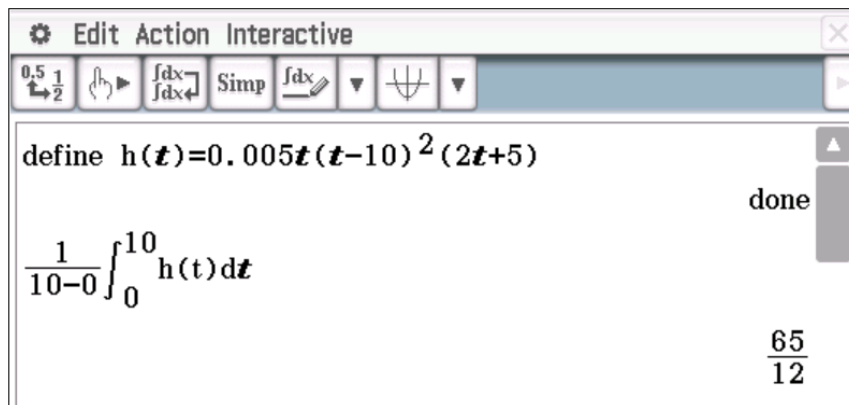
END OF SECTION A SOLUTIONS

SECTION B**Question 1**

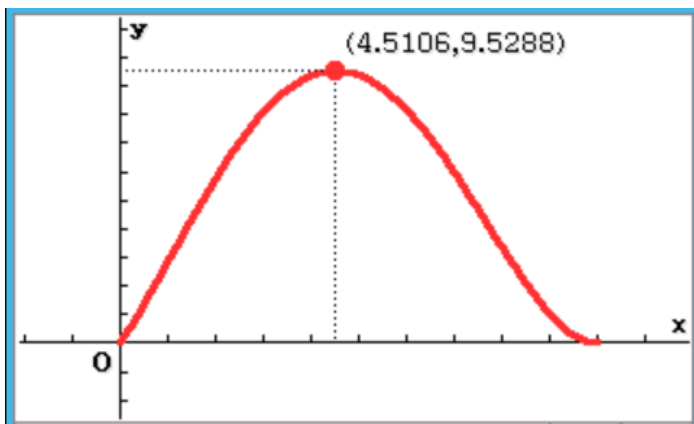
$$h: [0, 10] \rightarrow \mathbb{R}, h(t) = 0.005t(t-10)^2(2t+5)$$

a. Average height = $\frac{1}{10-0} \int_0^{10} h(t) dt$

$$= \frac{65}{12} \text{ cm} \quad \mathbf{1A}$$



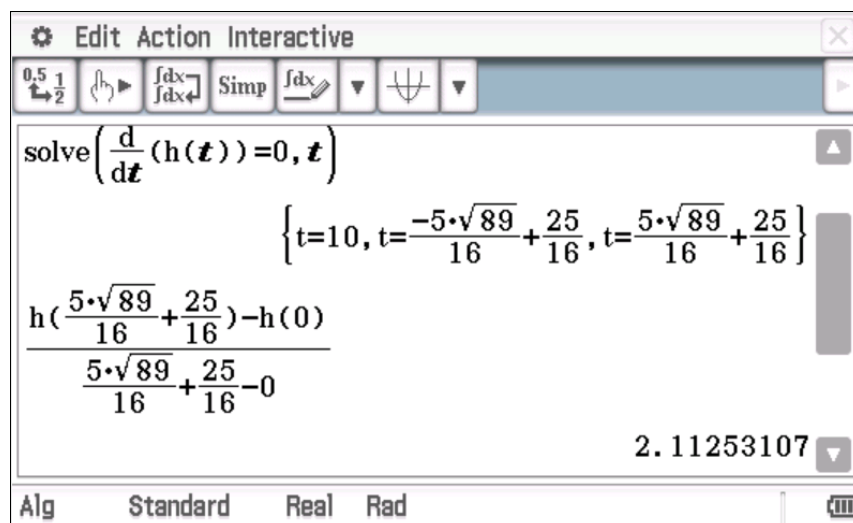
b. Maximum height = 9.5 cm correct to one decimal place $\mathbf{1A}$



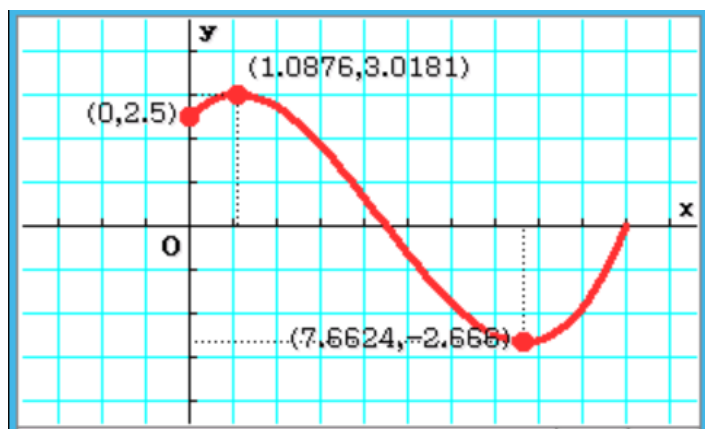
c. Average rate of change of h for $t \in [0, 4.5106\dots]$ or $t \in \left[0, \frac{25+5\sqrt{89}}{16}\right]$

$$= \frac{h\left(\frac{25+5\sqrt{89}}{16}\right) - h(0)}{\frac{25+5\sqrt{89}}{16} - 0} \quad \mathbf{1M}$$

$$= 2.11 \text{ cm/min} \quad \mathbf{1A}$$

Question 1 (continued)

- d. From the gradient graph the height of liquid is increasing at its maximum rate at $t = 1.09$ min correct to two decimal places. **1A**



- e. Volume of cone $= \frac{1}{3}\pi r^2 h$ where $r = \frac{1}{4}h$ **1M**

$$V = \frac{1}{3}\pi \left(\frac{h}{4}\right)^2 h = \frac{1}{3}\pi \frac{h^3}{16}$$

$$V = \frac{\pi}{48}h^3 \quad \text{as required} \quad \mathbf{1M \textit{ Show that}}$$

- f. Let glass have half volume.

$$\text{When full, volume of glass} = \frac{1}{3}\pi \times 3^2 \times 12 = 36\pi \text{ cm}^3$$

$$\text{Solve for } t, \frac{\pi}{48}(h(t))^3 = 18\pi \quad \mathbf{1M}$$

Glass is half full at $t = 4.43, 4.59$ mins correct to two decimal places **1A**

Question 1 (continued)

define $h(t)=0.005t(t-10)^2(2t+5)$

done

solve $\left(\frac{\pi}{48} \cdot (h(t))^3 = 18 \cdot \pi \mid 0 \leq t \leq 10, t\right)$

$\{t=4.428042507, t=4.593252896\}$

g. $h_1: [a, 10+a] \rightarrow \mathbb{R}, h_1(t) = 0.005(t-a)(t-a-10)^2(2(t-a)+5)$

Recognise $h_1(t) = h(t-a)$ for $t \in [a, 10+a]$

Equate $\frac{h_1(2+a) - h_1(a)}{2+a-a} = h_1'(4.428\dots)$ **1M**

$a = 2.73$ or $a = 3.92$ correct to 2 decimal places **1A**

define $h(t)=0.005t(t-10)^2(2t+5)$

done

define $H(t)=h(t-a)$

done

solve $\left(\frac{H(2+a) - H(a)}{2+a-a} = \text{diff}(H(t), t, 1, 4.4280425), a\right)$

$\{a=-6.486105018, a=2.729710444, a=3.915522073\}$

□

Alg Standard Real Rad

Question 2

$w: [-4, 4] \rightarrow \mathbb{R}, w(x) = \sqrt{r^2 - x^2} + c$ where $r > 0$ and $c \in \mathbb{R}^+ \cup \{0\}$.

a. Radius = 4 dm **1A**

b. $w(x) = \sqrt{16 - x^2} + 1$ **1A**

c. Area of the rectangle is $A_r = 2x_1(y_1 - 1)$.

$A_r = 2x_1(\sqrt{16 - x_1^2} + 1 - 1)$

$A_r = 2x_1\sqrt{16 - x_1^2}$ as required **1M Show that**

d. $A_r = 2x_1\sqrt{16 - x_1^2}$

Question 2 (continued)

Solve $\frac{d}{dx_1}(A_r) = 0$

Gives $x_1 = \pm 2\sqrt{2}$

for $x_1 > 0, x_1 = 2\sqrt{2}$

maximum area of rectangle = 16 dm².

1A

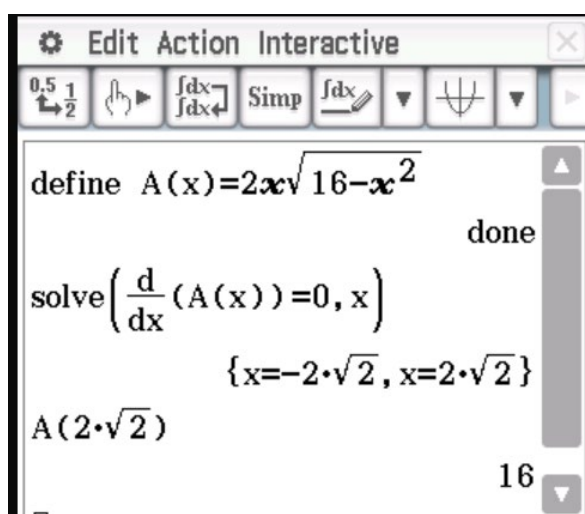
Justification

x	1	$2\sqrt{2}$	3
$A_r'(x)$	$\frac{28\sqrt{15}}{15}$	0	$-\frac{4\sqrt{7}}{7}$
	/	—	\

1M

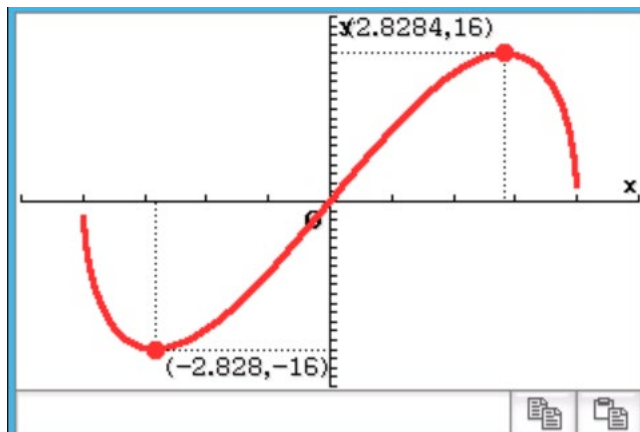
OR

$A_r''(2\sqrt{2}) = -8$, $A_r'' < 0$, hence a maximum

1M

Question 2 (continued)

$\frac{d^2}{dx^2}(A(x)) \mid x=2\sqrt{2}$	-8
$\frac{d}{dx}(A(x)) \mid x=1$	$\frac{28 \cdot \sqrt{15}}{15}$
$\frac{d}{dx}(A(x)) \mid x=3$	$\frac{-4 \cdot \sqrt{7}}{7}$
☐	v



e. Given radius = 4 dm

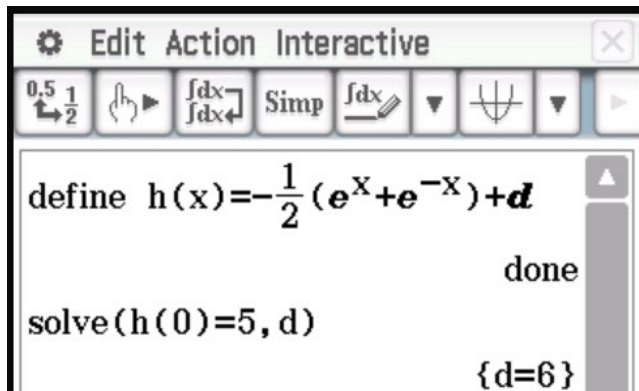
$$\text{Area of semicircle} = \frac{1}{2}\pi 4^2 = 8\pi$$

$$\text{Proportion stained glass} = \frac{16}{8\pi} = \frac{2}{\pi} \quad \mathbf{1A}$$

f. $h(x) = -\frac{1}{2}(e^x + e^{-x}) + d$ where $d \in \mathbb{R}^+ \cup \{0\}$.

Substitute $(0, 5)$ into $h(x) = -\frac{1}{2}(e^x + e^{-x}) + d$

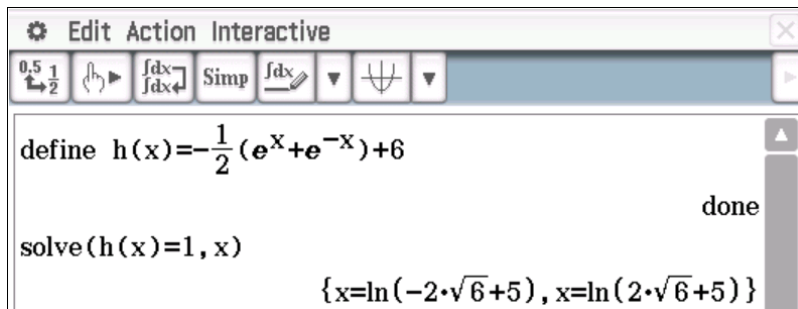
$$d = 6 \quad \mathbf{1A}$$

Question 2 (continued)

g. Solve $-\frac{1}{2}(e^x + e^{-x}) + 6 = 1$

Intersects with line $y = 1$ at

$(\log_e(5 - 2\sqrt{6}), 1)$ and $(\log_e(5 + 2\sqrt{6}), 1)$ **1A**



h. For the rectangle:

solve $16 = 2 \times 2\sqrt{2} \times \text{width}$

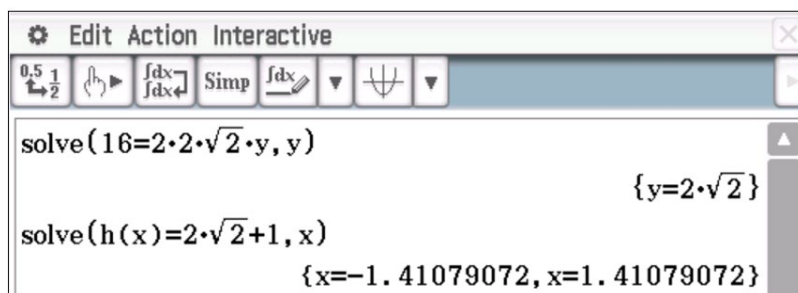
width $= 2\sqrt{2}$

Solve $h(x) = 2\sqrt{2} + 1$ for x

$x = -1.410\dots$ **1M**

$2 \left(\int_{\log_e(5-2\sqrt{6})}^{-1.410\dots} (h(x) - 1) dx + 1.410\dots \times 2\sqrt{2} \right)$ **1M**

$= 10.9 \text{ dm}^2$ **1A**



Question 2 (continued)

$$2 \left(\int_{\ln(5-2\sqrt{6})}^{-1.41079} (h(x)-1) dx + 1.41079(2\sqrt{2}) \right)$$

10.85433313

Alg Standard Real Rad

OR

$$\int_{\log_e(5-2\sqrt{6})}^{\log_e(5+2\sqrt{6})} (h(x)-1) dx - \int_{-1.41079}^{1.41079} (h(x) - (2\sqrt{2}+1)) dx \quad 2M$$

$$= 10.9 \text{ dm}^2 \quad 1A$$

$$\int_{\ln(5-2\sqrt{6})}^{\ln(5+2\sqrt{6})} (h(x)-1) dx - \int_{-1.41079}^{1.41079} (h(x) - (2\sqrt{2}+1)) dx$$

10.85433313

Alg Standard Real Rad

Question 3

a.i. $X \sim N(15,1)$

$$\Pr(X > 17.5) = 0.0062 \quad 1A$$

1.1 *2022MAVEA DEG

normCdf(17.5,∞,15,1) 0.0062097

a.ii. $\Pr(X > 17.5 | X \geq 14)$

$$= \frac{\Pr(X > 17.5 \cap X \geq 14)}{\Pr(X \geq 14)}$$

$$= \frac{\Pr(X > 17.5)}{\Pr(X \geq 14)} \quad 1M$$

$$= 0.0074 \text{ correct to four decimal places } 1A$$

1.1 1.2 *2022MAVEA DEG

normCdf(14,∞,15,1) 0.8413447

normCdf(17.5,∞,15,1) 0.0073807

0.0074

Question 3 (continued)

b. $M \sim N(\mu, \sigma)$

$$\frac{3.8 - \mu}{\sigma} = -1.666\dots$$

$$\frac{4.2 - \mu}{\sigma} = 1.042\dots$$

1M

$$\mu = 4.05 \text{ kg}, \sigma = 0.15 \text{ kg}$$

1A

Calculator screenshot showing the following calculations:

- $\text{invNorm}(0.0478, 0, 1) = -1.66657$
- $\text{invNorm}(1 - 0.1487, 0, 1) = 1.042025$
- Solve $\left(\frac{3.8 - a}{b} = -1.6665696892669 \text{ and } \frac{4.2 - a}{b} = 1.042025 \right)$
- Result: $a = 4.046116$ and $b = 0.147678$

c. $X \sim \text{Bi}(25, 0.8035)$

$$p = 1 - (0.0478 + 0.1487) = 0.8035$$

1M

$$\Pr(X > 20) = \Pr(21 \leq X \leq 25) = 0.4380 \text{ correct to four decimal places}$$

1A

Calculator screenshot showing the following calculations:

- $1 - (0.0478 + 0.1487) = 0.8035$
- $\text{binomCdf}(25, 0.8035, 21, 25) = 0.4379719$

d. $X_1 \sim \text{Bi}(n, 0.8035)$

$$\Pr(21 \leq X_1 \leq n) > 0.95$$

1M

Trial and error

$$n = 30 \quad \Pr(21 \leq X_1 \leq 30) = 0.944\dots$$

$$n = 31 \quad \Pr(21 \leq X_1 \leq 31) = 0.970\dots$$

$$n = 31$$

1A

Calculator screenshot showing the following calculations:

- $\text{binomCdf}(n, 0.8035, 21, n) | n=30 = 0.9449049$
- $\text{binomCdf}(n, 0.8035, 21, n) | n=31 = 0.9709725$
- $\text{invBinomN}(0.05, 0.8035, 20, 1)$
- Result: $\begin{bmatrix} 30 & 0.0550951 \\ 31 & 0.0290275 \end{bmatrix}$

Question 3 (continued)

$$\text{e. } s(x) = \begin{cases} \frac{x}{4} - 3 & 12 \leq x \leq 14 \\ a(x-16)^2 + b & 14 < x \leq 16 \\ 0 & \text{elsewhere} \end{cases}$$

$$\int_{12}^{14} \left(\frac{x}{4} - 3 \right) dx = \frac{1}{2}$$

$$\text{Solve } \int_{14}^{16} (a(x-16)^2 + b) dx = \frac{1}{2} \text{ and } a(14-16)^2 + b = \frac{14}{4} - 3 \text{ for } a \text{ and } b \quad \mathbf{1M}$$

$$a = \frac{3}{32}, b = \frac{1}{8} \quad \mathbf{1A}$$

The screenshot shows a TI-84 Plus calculator interface. At the top, the mode is set to RAD. The screen displays the equation: $\text{solve}\left(\int_{14}^{16} (a \cdot (x-16)^2 + b) dx = \frac{1}{2} \text{ and } a \cdot (14-16)^2 + b = \frac{14}{4} - 3\right)$. Below the equation, the solution is shown: $a = \frac{3}{32} \text{ and } b = \frac{1}{8}$.

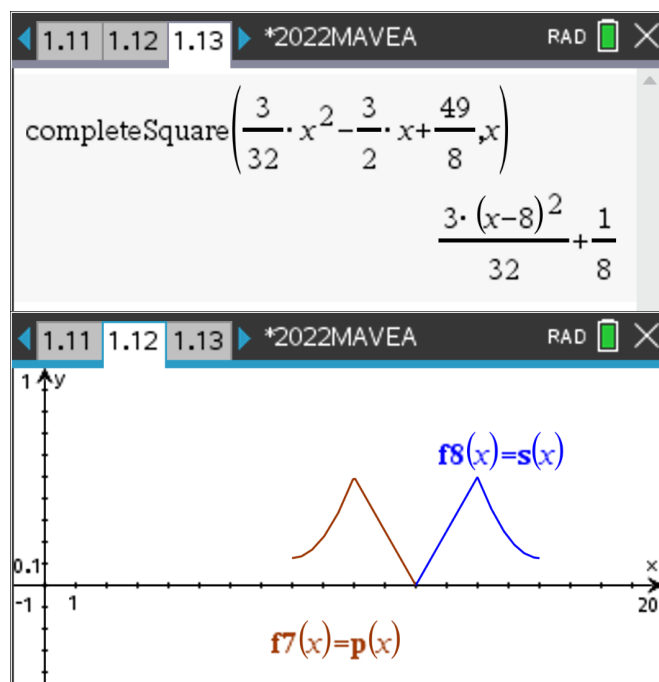
$$\text{f.i. } p(x) = \begin{cases} \frac{3x^2}{32} - \frac{3x}{2} + \frac{49}{8} & 8 \leq x \leq 10 \\ -\frac{x}{4} + 3 & 10 < x \leq 12 \\ 0 & \text{elsewhere} \end{cases}$$

Reflection in the line $x = 12$ $\mathbf{1A}$

OR

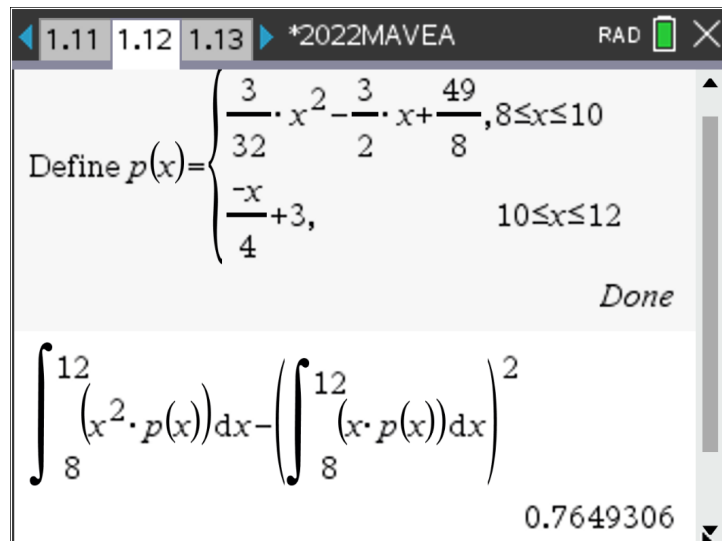
Reflection in the y -axis

Translation of 24 units to the right $\mathbf{1A}$

Question 3 (continued)

$$\text{f.ii. } \int_8^{12} (x^2 p(x)) dx - \left(\int_8^{12} (x p(x)) dx \right)^2 \quad \mathbf{1M}$$

$$= 0.765 \text{ correct to three decimal places} \quad \mathbf{1A}$$



$$\text{g. Solve } 0 \times \frac{\binom{8}{0} \binom{k}{3}}{\binom{8+k}{3}} + \frac{1}{3} \times \frac{\binom{8}{1} \binom{k}{2}}{\binom{8+k}{3}} + \frac{2}{3} \times \frac{\binom{8}{2} \binom{k}{1}}{\binom{8+k}{3}} + 1 \times \frac{\binom{8}{3} \binom{k}{0}}{\binom{8+k}{3}} = \frac{8}{17} \text{ for } k \quad \mathbf{1M}$$

OR

$$\text{Solve } \frac{3}{3} \times \frac{8}{8+k} = \frac{8}{17} \text{ for } k \quad \mathbf{1M}$$

$$k = 9 \quad \mathbf{1A}$$

Question 3 (continued)

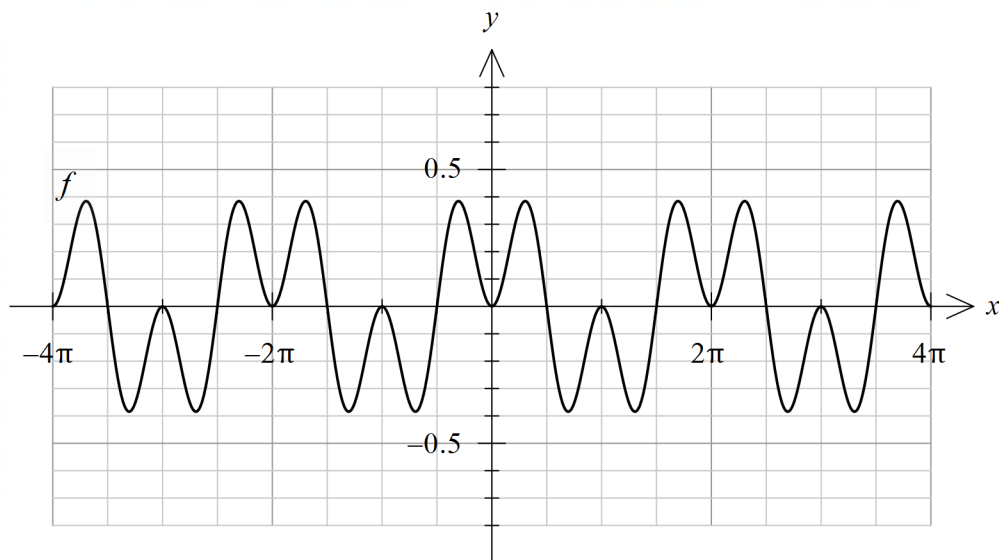
$$\text{solve}\left(\frac{1}{nCr(k+8,3)} \cdot \left(\frac{1}{3} \cdot nCr(8,1) \cdot nCr(k,2) - \frac{1}{3} \cdot nCr(8,2) \cdot nCr(k,1) + nCr(8,3)\right) = \frac{8}{17}, k\right)$$

$$k=9$$

$$k=9$$

Question 4

$$f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = \sin^2(x) \cos(x)$$



a. 2π **1A**

b. $\left[-\frac{2\sqrt{3}}{9}, \frac{2\sqrt{3}}{9}\right]$ **1A**

$$f(x) := (\sin(x))^2 \cdot \cos(x)$$

$$fMin(f(x), x) | 0 < x < 2 \cdot \pi$$

$$x = \sin^{-1}\left(\frac{\sqrt{3}}{3}\right) + \frac{\pi}{2} \text{ or } x = \frac{3 \cdot \pi}{2} - \sin^{-1}\left(\frac{\sqrt{3}}{3}\right)$$

$$f\left(\sin^{-1}\left(\frac{\sqrt{3}}{3}\right) + \frac{\pi}{2}\right) = \frac{-2 \cdot \sqrt{3}}{9}$$

$$f(x) := (\sin(x))^2 \cdot \cos(x)$$

$$fMax(f(x), x) | 0 < x < 2 \cdot \pi$$

$$x = \cos^{-1}\left(\frac{\sqrt{3}}{3}\right) \text{ or } x = \sin^{-1}\left(\frac{\sqrt{3}}{3}\right) + \frac{3 \cdot \pi}{2}$$

$$f\left(\cos^{-1}\left(\frac{\sqrt{3}}{3}\right)\right) = \frac{2 \cdot \sqrt{3}}{9}$$

c. $x = \frac{\pi k}{2}, k \in \mathbb{Z}$ **1A**

OR

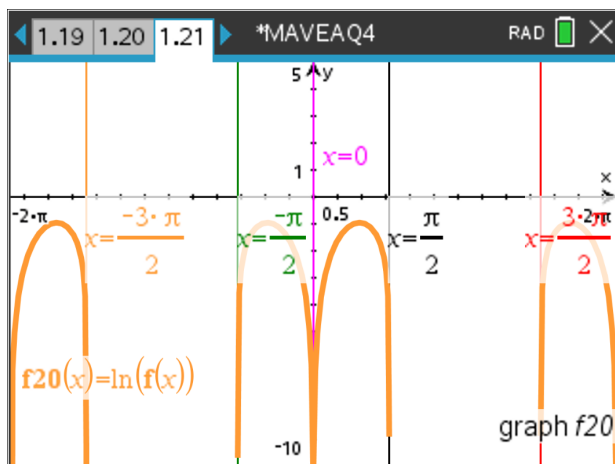
$x = \frac{(2k-1)\pi}{2}, x = k\pi, k \in \mathbb{Z}$ **1A**

Question 4 (continued)

$\text{solve}(f(x)=0,x)$
 $x = \frac{(2 \cdot n18 - 1) \cdot \pi}{2} \text{ or } x = n19 \cdot \pi$

d. $2\pi k < x < \frac{\pi}{2} + 2\pi k, k \in \mathbb{Z}$ or **1A**

$-\frac{\pi}{2} + 2\pi k < x < 2\pi k, k \in \mathbb{Z}$ **1A**



e. $A\left(\frac{\pi}{2}, -1\right), B\left(\frac{3\pi}{2}, 1\right)$ **1A**

$m = \frac{2}{\pi}$

$y + 1 = \frac{2}{\pi} \left(x - \frac{\pi}{2} \right)$

$y = \frac{2}{\pi} x - 2$ **1A**

Δ fMin $\left(\frac{d}{dx}(f(x)), x \right) | 0 < x < 2 \cdot \pi$ $x = \frac{\pi}{2}$
 Δ fMax $\left(\frac{d}{dx}(f(x)), x \right) | 0 < x < 2 \cdot \pi$ $x = \frac{3 \cdot \pi}{2}$
 $\frac{d}{dx}(f(x))$ $2 \cdot \sin(x) \cdot (\cos(x))^2 - (\sin(x))^3$
 $\frac{d}{dx}(f(x))$ $2 \cdot \sin(x) \cdot (\cos(x))^2 - (\sin(x))^3$ $x = \frac{\pi}{2}$ -1
 $\frac{d}{dx}(f(x))$ $2 \cdot \sin(x) \cdot (\cos(x))^2 - (\sin(x))^3$ $x = \frac{3 \cdot \pi}{2}$ 1

f.i. Midpoint $(\pi, 0)$, $m = -\frac{\pi}{2}$

Question 4 (continued)

$$y = -\frac{\pi}{2}(x - \pi) = -\frac{\pi}{2}x + \frac{\pi^2}{2} \quad \mathbf{1A}$$

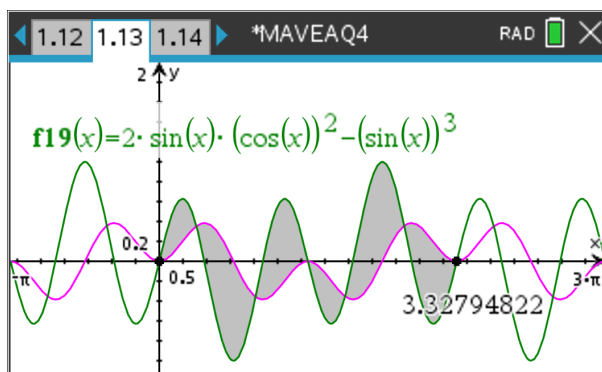
$$\mathbf{f.ii.} \ C \left(\pi - \sin^{-1} \left(\frac{\sqrt{2}}{3} \right), \frac{4\sqrt{2}}{9} \right) \quad \mathbf{1M}$$

$$y = -\frac{\pi}{2} \times \left(\pi - \sin^{-1} \left(\frac{\sqrt{2}}{3} \right) \right) + \frac{\pi^2}{2}$$

$$= \frac{\sin^{-1} \left(\frac{\sqrt{2}}{3} \right) \pi}{2}$$

$$\neq \frac{4\sqrt{2}}{9}$$

1M Show that

g. Area = 3.33 correct to two decimal places **1A**

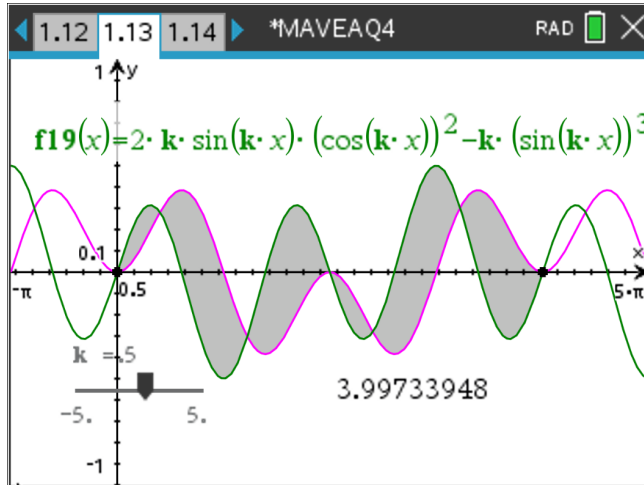
Question 4 (continued)

$$\text{h. } \frac{2\pi}{k} = 4\pi$$

$$k = \frac{1}{2}$$

$$g(x) = \sin^2(kx) \cos(kx), \quad g'(x) = 2k \sin(kx) \cos^2(kx) - k \sin^3(kx) \quad \mathbf{1M}$$

Area = 4.00

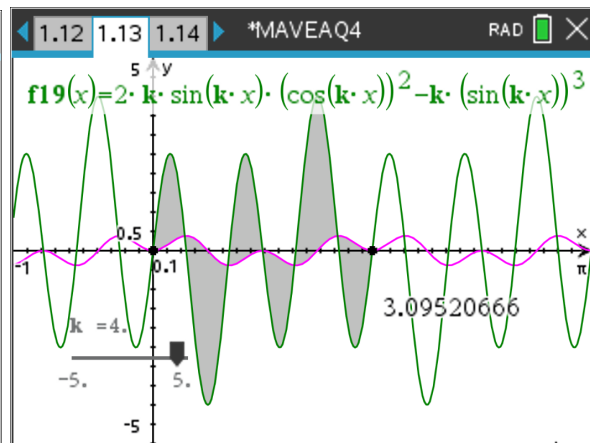
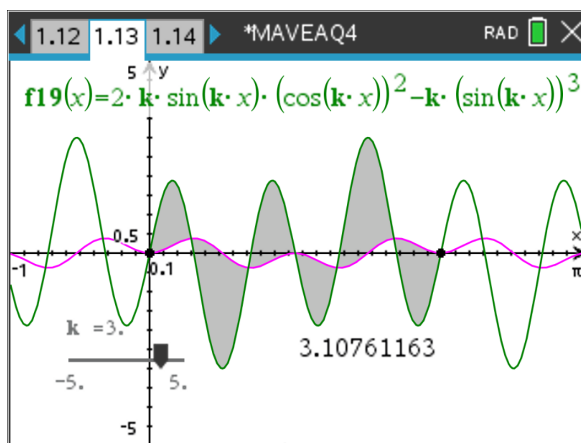
1A

i. Find the bounded area over the interval $\left[0, \frac{2\pi}{k}\right]$.

Use trial and error

When $k = 3$, the bounded area is 3.107...When $k = 4$, the bounded area is 3.095...

$$k = 4$$

1A**Question 5**

$$p: R \rightarrow R, p(x) = -(x-1)^3$$

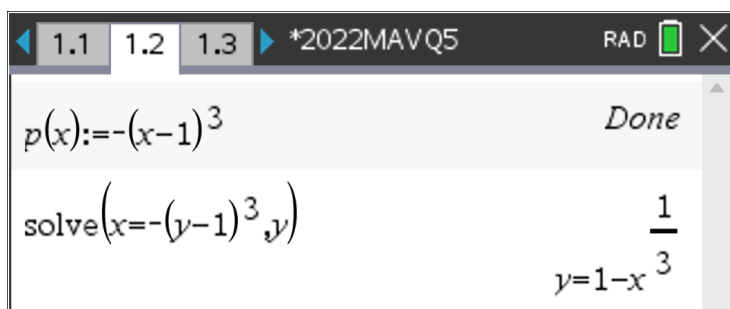
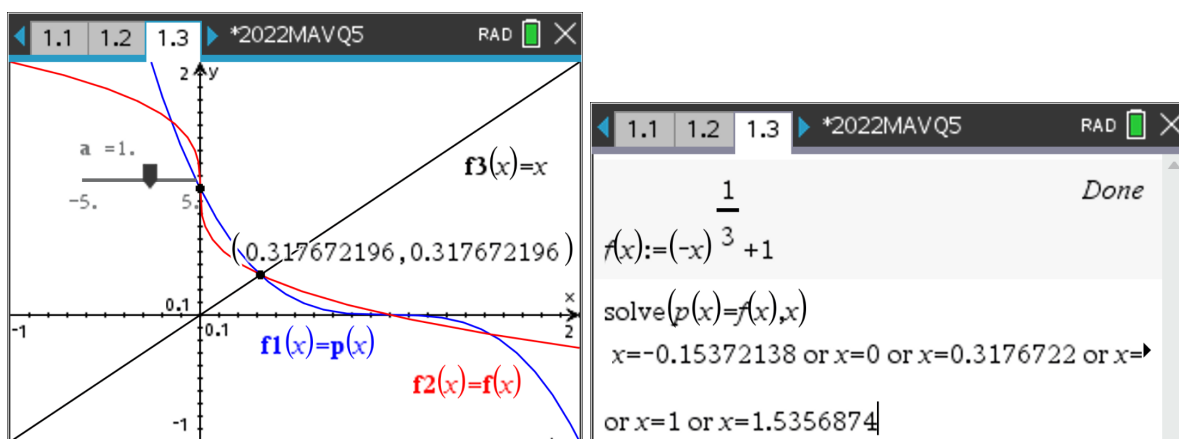
a. Let $y = -(x-1)^3$

Inverse swap x and y

$$x = -(y-1)^3$$

$$p^{-1}(x) = \sqrt[3]{-x} + 1 = 1 - \sqrt[3]{x}$$

1A (either form)

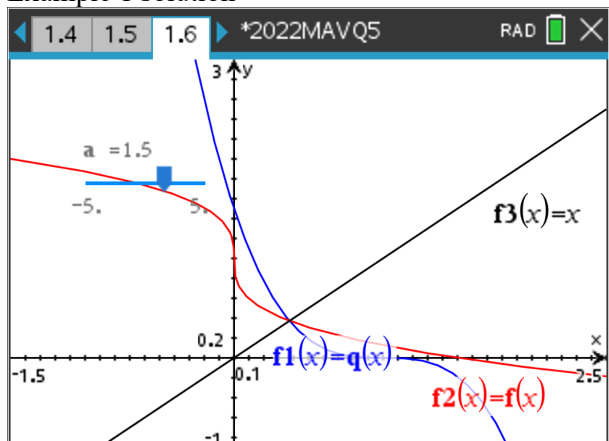
Question 5 (continued)**b. 5****1A**

c. $q: \mathbb{R} \rightarrow \mathbb{R}, q(x) = -a(x-1)^3$, $q^{-1}: \mathbb{R} \rightarrow \mathbb{R}, q^{-1}(x) = 1 - \sqrt[3]{\frac{x}{a}}$

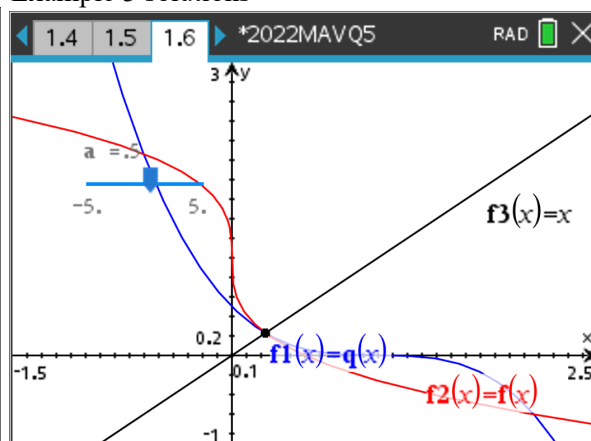
1, 3 or 5

1A

Example 1 solution



Example 3 solutions



d. Solve $q(x) = q^{-1}(x)$ and $\frac{d}{dx}(q(x)) = \frac{d}{dx}(q^{-1}(x))$ **1M**

$$a = \frac{16}{27}$$

1A

Question 5 (continued)

$\text{solve}\left(q(x)=f(x) \text{ and } \frac{d}{dx}(q(x))=\frac{d}{dx}(f(x)), x\right) | a > 0$
 $x = \frac{-(3 \cdot \sqrt{3} - 5)}{8} \text{ and } a = \frac{32}{27} \text{ or } x = \frac{1}{4} \text{ and } a = \frac{16}{27}$

Edit Action Interactive
 define $q(x) = -a(x-1)^3$
 solve($q(y)=x, y$)
 $\left\{ y = \left(\frac{-x}{a} \right)^{\frac{1}{3}} + 1 \right\}$
 Define $f(x) = \left(\frac{-x}{a} \right)^{\frac{1}{3}} + 1$

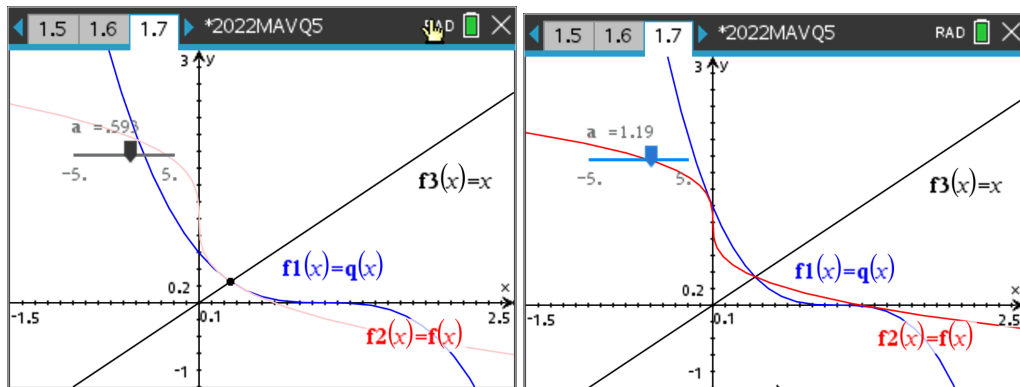
$\frac{d}{dx}(q(x)) = -3 \cdot a \cdot (x-1)^2$
 $\frac{d}{dx}(f(x)) = \frac{-1}{\left(\frac{-x}{a} \right)^{\frac{2}{3}}}$
 $\text{solve}(q(x)=x, a)$
 $\left\{ a = \frac{-x}{x^3 - 3 \cdot x^2 + 3 \cdot x - 1} \right\}$
 $\text{solve}\left(\frac{d}{dx}(q(x)) = \frac{d}{dx}(f(x)) \mid a\right)$
 $\{x = -0.5, x = 0.25\}$

$0.25 \rightarrow x$
 $\frac{1}{4}$
 $a = \frac{-x}{x^3 - 3 \cdot x^2 + 3 \cdot x - 1}$
 $a = \frac{16}{27}$

e. Three solutions occur when $0 < a \leq \frac{16}{27}$ and $a = \frac{32}{27}$ **1A**

$$a = \frac{16}{27}$$

$$a = \frac{32}{27}$$

Question 5 (continued)

f. $t: \mathbb{R} \rightarrow \mathbb{R}, t(x) = -(x-1)(x^2 + bx + c)$

One solution for $x^2 + bx + c = 0$

$$b^2 - 4c = 0$$

$$b = \pm 2\sqrt{c}, \quad b \neq -2 \quad \mathbf{1A}$$

Note if $b = -2$, $t(x) = -(x-1)(x^2 - 2x + 1) = -(x-1)^3$

Two solutions for $x^2 + bx + c = 0$ if one of the factors is $(x-1)$

$$x^2 + bx + c = (x-1)(x-c) = x^2 - (c+1)x + c$$

$$b = -c-1, \quad b \neq -2 \quad \mathbf{1A}$$

END OF SOLUTIONS