

The Mathematical Association of Victoria

Trial Exam 2022

MATHEMATICAL METHODS

WRITTEN EXAMINATION 1

STUDENT NAME _____

Reading time: 15 minutes

Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room : any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 11 pages,
- Formula sheet
- Working space is provided throughout the book.

Instructions

- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale .
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

- a. Find $\frac{d}{dx}(-3e^{x^2+1})$. 1 mark

- b. Let $f(x) = 2 \sin(2x) \cos\left(x + \frac{\pi}{4}\right)$.

Find $f'(\pi)$. 3 marks

TURN OVER

Question 2 (3 marks)

Evaluate $\int_{-1}^1 \left(\frac{1}{(3-2x)^2} + 5 \right) dx$.

Question 3 (6 marks)

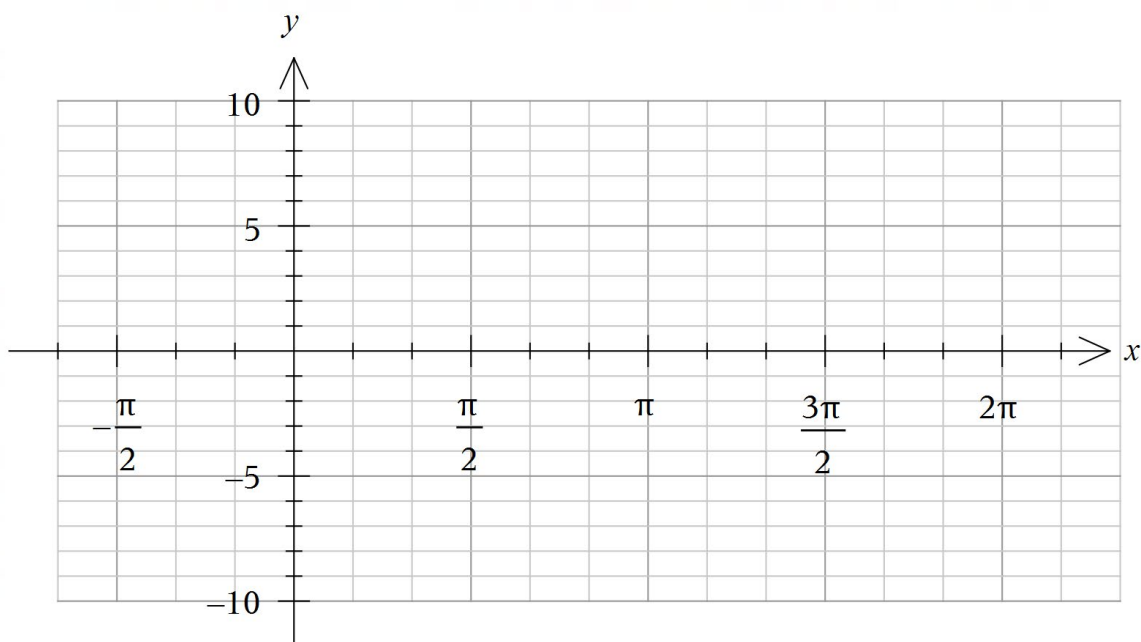
- a. Find the solutions to the equation $3 \tan\left(2x - \frac{\pi}{2}\right) + \sqrt{3} = 0$ for $x \in (0, 2\pi)$.

3 marks

- b. Sketch the graph of $y = 3 \tan\left(2x - \frac{\pi}{2}\right) + \sqrt{3}$ for $x \in \left[\frac{\pi}{6}, \frac{5\pi}{3}\right]$ on the set of axes below.

Label the axial intercepts with their coordinates and any asymptotes with their equations.

3 marks

**TURN OVER**

Question 4 (3 marks)

Solve $2\log_3(x+2) - \log_3(2x^2 + x - 6) = 2$ for x .

Question 5 (4 marks)

Consider the polynomial $p(x) = 2x^3 - 4x^2 + 6x - 12$.

a. Factorise $p(x)$ over R , and explain why there is only one linear factor.

2 marks

b. Show that the graph of p has no stationary points.

2 marks

Question 6 (3 marks)

Given that $\cos(x) = -\frac{1}{3}$ for $x \in \left[\pi, \frac{3\pi}{2}\right]$, evaluate

a. $\sin\left(x + \frac{3\pi}{2}\right)$ 1 mark

b. $\cos\left(x + \frac{\pi}{2}\right)$ 2 marks

TURN OVER

Question 7 (7 marks)

There are three red balls and two blue balls in a bag. Three balls are drawn from the bag with replacement. Let X represent the number of red balls drawn.

- a. Complete the following probability distribution table for X . 2 marks

x	0	1	2	3
$\Pr(X = x)$	$\frac{8}{125}$	$\frac{36}{125}$		

- b. Given that at least one red ball is drawn, find the probability that only one of the balls is red. 1 mark

- c. Find $\text{sd}(X)$. 2 marks

n balls are now drawn from the bag with replacement.

- d. What is the least number of balls that need to be drawn to ensure that the probability of getting at least one red ball is greater than 0.9? 2 marks

Question 8 (4 marks)

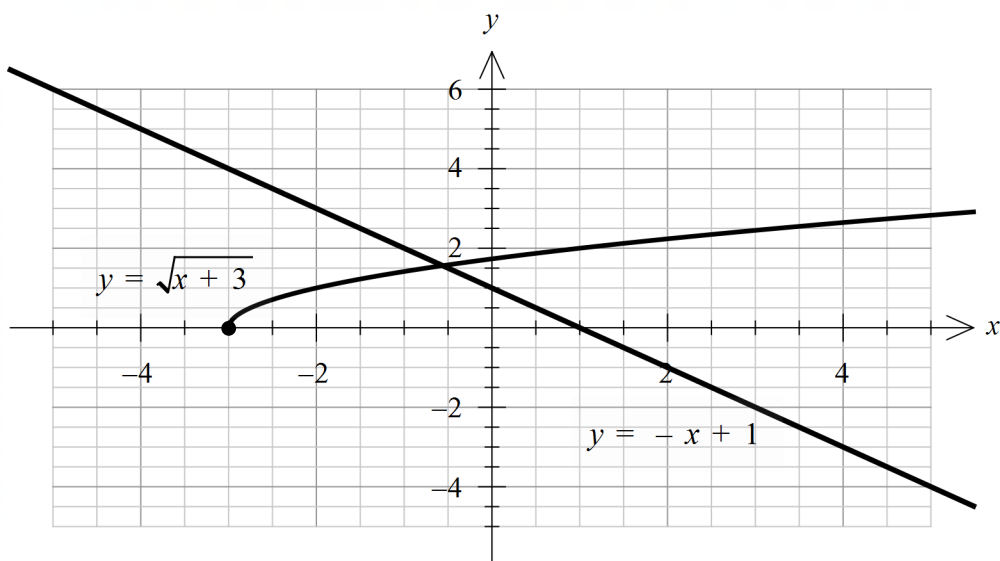
- a. Show that $\frac{d}{dx}\left(\frac{1}{2}(2x-1)\log_e(2x-1) - x\right) = \log_e(2x-1)$. 1 mark

- b. Hence find the area bounded by the graph of $f(x) = \log_e(2x-1) + 1$, the x -axis and the line $x = 2$. 3 marks

TURN OVER

Question 9 (6 marks)

The graphs of $f : [-3, \infty) \rightarrow \mathbb{R}, f(x) = \sqrt{x+3}$ and $g : \mathbb{R} \rightarrow \mathbb{R}, g(x) = -x + 1$ are shown below.



The graph of $f(x) + g(x)$ has one turning point and no other stationary points.

- a. Verify that the x -coordinate of the turning point of the graph of $f(x) + g(x)$ is $-\frac{11}{4}$. 1 mark

- b. Sketch the graph of $f(x) + g(x)$ on the set of axes above. Label the endpoint and the turning point with their coordinates. 3 marks

Let $h: \left[-\frac{11}{4}, \infty\right) \rightarrow \mathbb{R}$, where $h(x) = \sqrt{x+3} - x + 1$.

- c. Find the coordinates of the point where $h(x) = h^{-1}(x)$.

2 marks

END OF QUESTION AND ANSWER BOOK