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YEAR 12 *Trial Exam Paper*

2022

MATHEMATICAL METHODS

Written examination 1

Reading time: 15 minutes

Writing time: 1 hour

STUDENT NAME:

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 11 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions

- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

a. Let $y = x(2 - x)^3$.

Find $\frac{dy}{dx}$.

2 marks

b. Let $f(x) = 2e^{-\frac{1}{2}(x-1)}$.

Evaluate $f'(1)$.

2 marks

Question 2 (4 marks)

a. Let $y = \frac{1}{\sin(x)+1}$.

Show that $\frac{dy}{dx} = \frac{-\cos(x)}{(\sin(x)+1)^2}$.

1 mark

b. Let $f'(x) = \frac{\cos(x)}{(\sin(x)+1)^2}$.

Find $f(x)$ given that $f\left(\frac{\pi}{2}\right) = 0$.

3 marks

Question 3 (4 marks)

In a board game, a player takes a turn by rolling two fair dice. One die has four sides, numbered 1 to 4, and the other has six sides, numbered 1 to 6. A turn is called a success if at least one die rolls a four or higher.

- a.** Find the probability that a player's turn is a success.

1 mark

- b.** If both dice roll a four or more, the turn is called an exceptional success.
Find the probability that a turn that is a success is also an exceptional success.

2 marks

- c.** Let $\hat{P}$ be the proportion of a player's first two turns in the game that are a success.
Find $\Pr(\hat{P} > 0)$.

1 mark

Question 4 (3 marks)

- a.** Solve $e^{2x} - 5e^x + 4 = 0$ for $x \in R$.

2 marks

- b.** Let $f : [k, \infty) \rightarrow R$ $f(x) = e^{2x} - 5e^x + 4$ and $g : [0, \infty) \rightarrow R$, $g(x) = \sqrt{2x} + 1$.
Find the lowest value of k for which $g(f(x))$ is defined.

1 mark

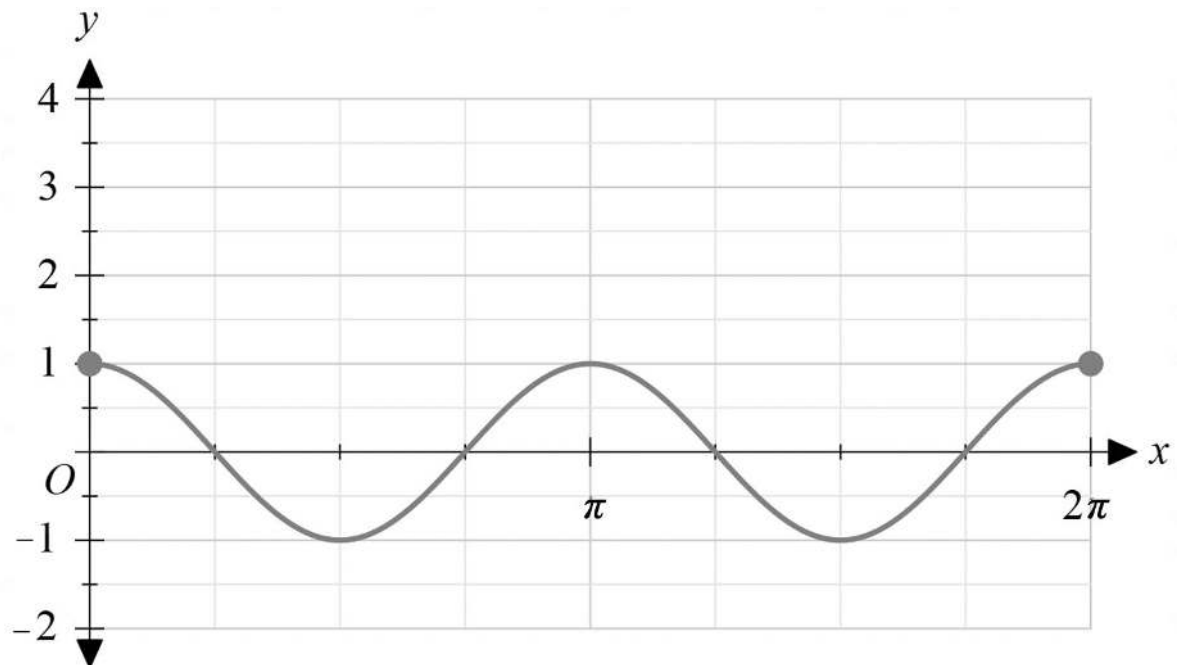
Question 5 (5 marks)

Let $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = \cos(2x)$ and $g: \mathbb{R} \rightarrow \mathbb{R}$, $g(x) = 2\sin(x) + 1$.

- a. The graph of $y = f(x)$ for $x \in [0, 2\pi]$ is shown on the axes below.

Sketch the graph of $y = g(x)$ for $x \in [0, 2\pi]$ on the axes below and label the coordinates of all points of intersection with the graph of $y = f(x)$.

2 marks



- b. The transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ with rule $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}$ maps the graph of $y = g(x)$ onto the graph of $y = f(x)$, where $a, b, d \in \mathbb{R}$ and $c \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Find the values of a, b, c and d .

3 marks

Question 6 (3 marks)

Let X be a normally distributed random variable with a mean of 10 and a standard deviation of 5.

- a. Find $\Pr(X < 10)$.

1 mark

- b. Let Y be a normally distributed random variable with a mean of 15 and a standard deviation of σ .

Find the value of σ if $\Pr(X < 12) = \Pr(Y > 12)$.

2 marks

Question 7 (3 marks)

Let $f : [0, a] \rightarrow \mathbb{R}$ $f(x) = \frac{k}{x+1}$.

If f is a probability density function, find k in terms of a .

Question 8 (8 marks)

In a game, the results of three independent and identical Bernoulli trials with a probability of success p are used to determine the result for a player.

- If all three trials are failures, then the player gains no points.
 - If only one trial is a success, then the player loses two points.
 - If only two trials are a success, then the player gains one point.
 - If all three trials are a success, then the player gains three points.
- a. Show that the expected number of points that would be gained by a player is given by the expression $-6p^3 + 15p^2 - 6p$.

2 marks

- b. Find the values of p for which a player would expect to gain points in the game.

3 marks

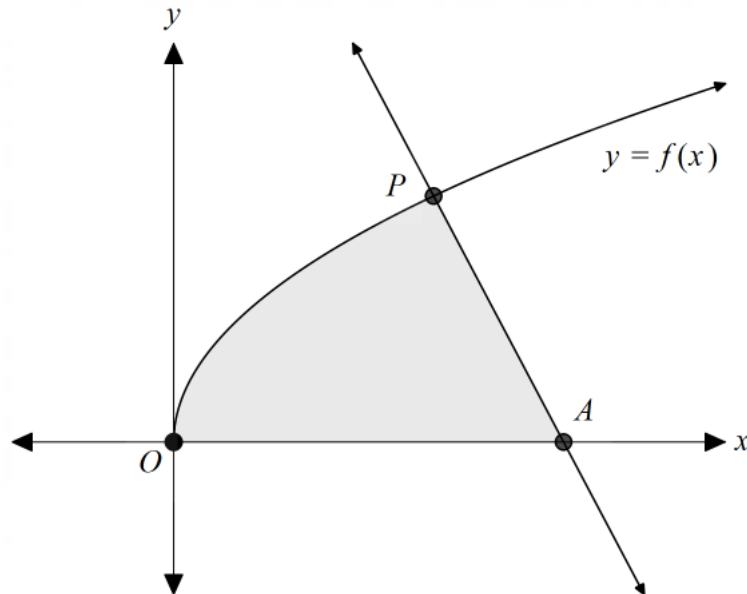
- c. Find the value of p for which a player would expect to lose the most points.

3 marks

Question 9 (6 marks)

Let $f : [0, \infty) \rightarrow \mathbb{R}$ $f(x) = \sqrt{x}$. Let P be the point $(p, \sqrt{p})$ on f with $p > 0$. Let A be the x -intercept of the line perpendicular to f at P .

The shaded region in the diagram is enclosed by the horizontal axis, the graph of $y = f(x)$ and the graph of the line perpendicular to f at P .



- a. Show that the x -coordinate of A is $p + \frac{1}{2}$.

2 marks

- b. Find an expression for the area of the shaded region in terms of p .

2 marks

- c. Show that the area of the shaded region strictly increases as p increases.

2 marks

END OF QUESTION AND ANSWER BOOK