VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2021

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

Letter

STUDENT NUMBER

MATHEMATICAL METHODS

Written examination 1

Wednesday 3 November 2021

Reading time: 9.00 am to 9.15 am (15 minutes)
Writing time: 9.15 am to 10.15 am (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 11 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (3 marks)

a. Differentiate $y = 2e^{-3x}$ with respect to x. 1 mark

b. Evaluate $f'(4)$, where $f(x) = x\sqrt{2x+1}$. 2 marks

Question 2 (2 marks)

Let $f'(x) = x^3 + x$.

Find $f(x)$ given that $f(1) = 2$.

DO NOT WRITE IN THIS AREA

Question 3 (5 marks)

Consider the function $g : R \to R, g(x) = 2\sin(2x)$.

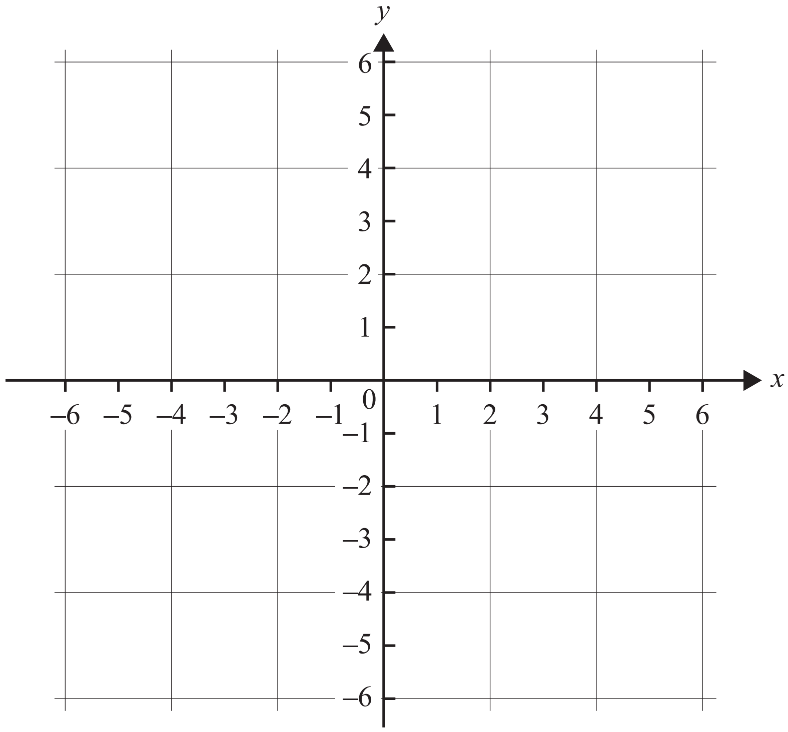
a. State the range of g. 1 mark

b. State the period of g. 1 mark

c. Solve $2\sin(2x) = \sqrt{3}$ for $x \in R$. 3 marks

Question 4 (4 marks)

a. Sketch the graph of $y = 1 - \frac{2}{x-2}$ on the axes below. Label asymptotes with their equations and axis intercepts with their coordinates. 3 marks



b. Find the values of x for which $1 - \frac{2}{x-2} \geq 3$. 1 mark

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 5 (4 marks)

Let $f : R \to R, f(x) = x^2 - 4$ and $g : R \to R, g(x) = 4(x - 1)^2 - 4$.

a. The graphs of f and g have a common horizontal axis intercept at $(2, 0)$.

Find the coordinates of the other horizontal axis intercept of the graph of g. 2 marks

b. Let the graph of h be a transformation of the graph of f where the transformations have been applied in the following order:

- dilation by a factor of $\frac{1}{2}$ from the vertical axis (parallel to the horizontal axis)
- translation by two units to the right (in the direction of the positive horizontal axis)

State the rule of h and the coordinates of the horizontal axis intercepts of the graph of h. 2 marks

Question 6 (6 marks)

An online shopping site sells boxes of doughnuts.

A box contains 20 doughnuts. There are only four types of doughnuts in the box. They are:

- glazed, with custard
- glazed, with no custard
- not glazed, with custard
- not glazed, with no custard.

It is known that, in the box:

- $\frac{1}{2}$ of the doughnuts are with custard
- $\frac{7}{10}$ of the doughnuts are not glazed
- $\frac{1}{10}$ of the doughnuts are glazed, with custard.

a. A doughnut is chosen at random from the box.

Find the probability that it is not glazed, with custard. 1 mark

b. The 20 doughnuts in the box are randomly allocated to two new boxes, Box A and Box B.
Each new box contains 10 doughnuts.
One of the two new boxes is chosen at random and then a doughnut from that box is chosen at random.
Let g be the number of glazed doughnuts in Box A.

Find the probability, in terms of g, that the doughnut comes from Box B given that it is glazed. 2 marks

DO NOT WRITE IN THIS AREA

Question 6 – continued

c. The online shopping site has over one million visitors per day.

It is known that half of these visitors are less than 25 years old.

Let $\hat{P}$ be the random variable representing the proportion of visitors who are less than 25 years old in a random sample of five visitors.

Find $\Pr\left(\hat{P} \geq 0.8\right)$. Do not use a normal approximation. 3 marks

DO NOT WRITE IN THIS AREA

Question 7 (3 marks)

A random variable X has the probability density function f given by

$$f(x) = \begin{cases} \dfrac{k}{x^2} & 1 \le x \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

where k is a positive real number.

a. Show that $k = 2$. 1 mark

b. Find $\mathrm{E}(X)$. 2 marks

DO NOT WRITE IN THIS AREA

Question 8 (5 marks)

The gradient of a function is given by $\frac{dy}{dx} = \sqrt{x+6} - \frac{x}{2} - \frac{3}{2}$.

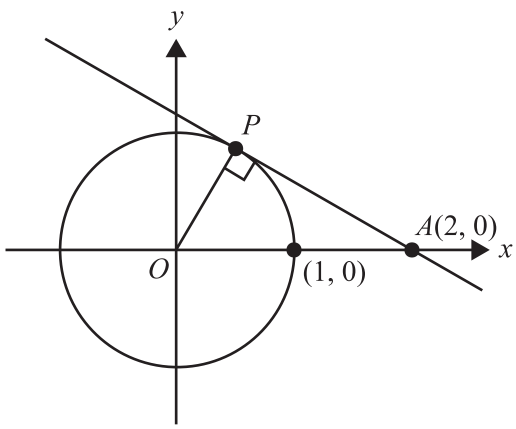
The graph of the function has a single stationary point at $\left(3, \frac{29}{4}\right)$.

a. Find the rule of the function. 3 marks

b. Determine the nature of the stationary point. 2 marks

Question 9 (8 marks)

Consider the unit circle $x^2 + y^2 = 1$ and the tangent to the circle at the point P, shown in the diagram below.



a. Show that the equation of the line that passes through the points A and P is given by $y = -\dfrac{x}{\sqrt{3}} + \dfrac{2}{\sqrt{3}}$. 2 marks

Let $T : R^2 \to R^2,\ T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & q \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$, where $q \in R\backslash\{0\}$, and let the graph of the function h be the transformation of the line that passes through the points A and P under T.

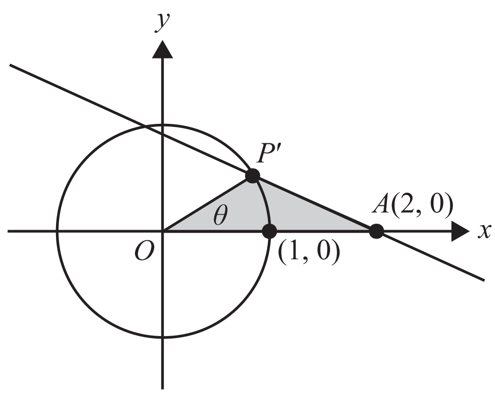
b. **i.** Find the values of q for which the graph of h intersects with the unit circle at least once. 1 mark

ii. Let the graph of h intersect the unit circle twice.

Find the values of q for which the coordinates of the points of intersection have only positive values. 1 mark

Question 9 – continued

c. For $0 < q \leq 1$, let P' be the point of intersection of the graph of h with the unit circle, where P' is always the point of intersection that is closest to A, as shown in the diagram below.



Let g be the function that gives the area of triangle OAP' in terms of θ.

i. Define the function g. 2 marks

ii. Determine the maximum possible area of the triangle OAP'. 2 marks

END OF QUESTION AND ANSWER BOOK

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MATHEMATICAL METHODS

Written examination 1

FORMULA SHEET

Instructions

This formula sheet is provided for your reference.
A question and answer book is provided with this formula sheet.

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Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$		$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, \; n \neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right) = an\left(ax+b\right)^{n-1}$		$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$	
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$		$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$	
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$		$\int \frac{1}{x} dx = \log_e(x) + c, \; x > 0$	
$\frac{d}{dx}\left(\sin(ax)\right) = a \; \cos(ax)$		$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$	
$\frac{d}{dx}\left(\cos(ax)\right) = -a \; \sin(ax)$		$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$	
$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a \; \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A \vert B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X - \mu)^2) = \mathrm{E}(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x - \mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x)dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

END OF FORMULA SHEET