

The Mathematical Association of Victoria

Trial Exam 2021

MATHEMATICAL METHODS

WRITTEN EXAMINATION 1

STUDENT NAME _____

Reading time: 15 minutes

Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room : any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 12 pages,
- Formula sheet
- Working space is provided throughout the book.

Instructions

- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale .
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (2 marks)

If $y = x^2 \log_e \left(\frac{x}{3} \right)$, find $\frac{dy}{dx}$.

Question 2 (2 marks)

Let $f(x) = \tan(2x)$. Evaluate $f'\left(\frac{\pi}{3}\right)$.

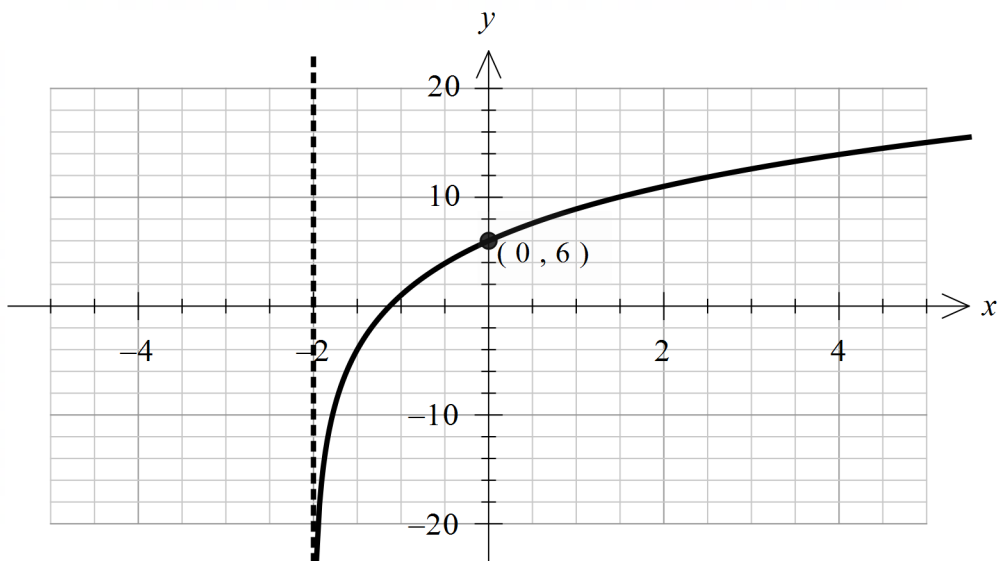
TURN OVER

Question 3 (3 marks)

Solve $3e^x + 4 = e^{-x}$ for x .

Question 4 (4 marks)

The graph of a logarithmic function is shown below.



The graph is of the form $y = a \log_2(x - b) + c$ where a, b, c are real constants.

a. Label the asymptote with its equation on the graph above.

1 mark

b. Given that the graph goes through the point $(0, 6)$ as shown, and the x -intercept has

coordinates $\left(\frac{1}{2^5} - 2, 0\right)$, find the values of a , b and c .

3 marks

TURN OVER

Question 5 (5 marks)

Let $g(x) = \sqrt{x}$ and $f(x) = \frac{1}{x}$ for their maximal domains.

- a. State, with reasons, why the function $h(x) = f(g(x))$ does not exist.

1 mark

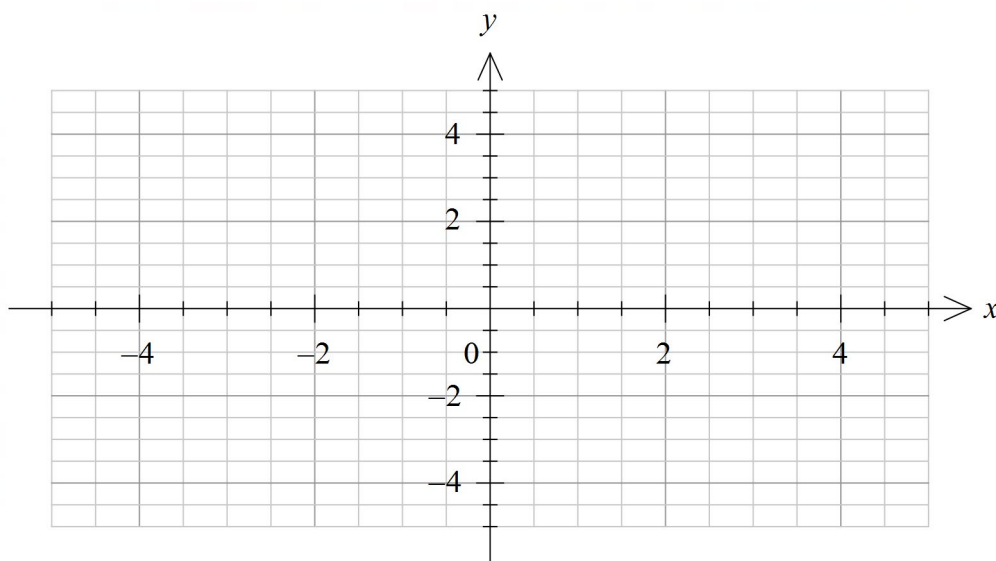
Now consider the function $g_1(x) = \sqrt{x}$.

- b. State the maximal domain, D , of h_1 such that $h_1 : D \rightarrow \mathbb{R}$, $h_1(x) = f(g_1(x))$ exists and give the rule for h_1 .

2 marks

- c. Hence sketch the graph of $y = h_1(x)$ on the axes below. Label the asymptotes with their equations.

2 marks



Question 6 (3 marks)

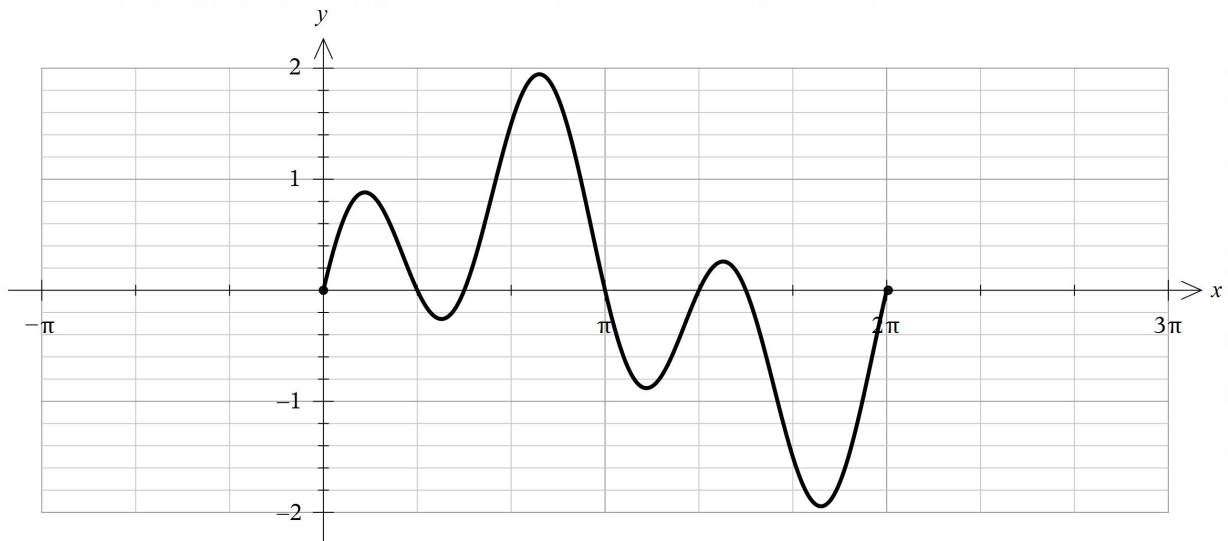
- a. Find an antiderivative of $\frac{1}{(3x+1)^3}$. 2 marks

- b. Hence if $f'(x) = \frac{1}{(3x+1)^3}$ and $f(-1) = 2$ find the rule for $f(x)$. 1 mark

TURN OVER

Question 7 (7 marks)

The graph of $f : [0, 2\pi] \rightarrow \mathbb{R}$, $f(x) = -2\sin(2x)\sin\left(x - \frac{\pi}{3}\right)$ is shown below.



- a. Find the x -axis intercepts.

2 marks

The rule for f can also be expressed as $f(x) = \sin\left(3x + \frac{\pi}{6}\right) - \cos\left(x + \frac{\pi}{3}\right)$.

- b.** Find the area bounded by the x -axis and the curve of f , between $x = \frac{\pi}{2}$ and $x = \pi$. Express

your answer in the form $\frac{a\sqrt{b}+c}{b}$, where a, b and c are integers.

3 marks

[illegible]

- c. Sketch the graph of $h: [-\pi, 0] \rightarrow \mathbb{R}$, $h(x) = -2 \sin(2x) \sin\left(x - \frac{\pi}{3}\right)$ on the set of axes on page 8.

Label the x -axis intercepts with their coordinates. Do not label the turning points.

2 marks

TURN OVER

Question 8 (6 marks)

The Soren family's 4 children, Declan, Faith, Blake and Paris each have an equal chance of 0.25 of going to child care on any one day.

- a. What is the probability that Declan and Faith attend child care, but Blake and Paris do not on a particular day? 1 mark

- b. What is the probability that more than 1 child in the Soren family attends child care? 2 marks

A discrete random variable table below records $\Pr(X = x)$ where the random variable X describes the number of sultanas in a cupcake at morning tea in the child care centre.

x	1	2	3	4	5
$\Pr(X = x)$	0.1	0.25	0.05	0.5	k

- c. Given that a randomly chosen cupcake contains less than the mean number of sultanas, what is the probability it contains at most 2 sultanas? 3 marks

Question 9 (3 marks)

Let $g(x) = \frac{x+b}{x+a}$. For what values of a and b , where a and b are real constants, will $g(x) = g^{-1}(x)$ for all $x \in R$.

TURN OVER

Question 10 (5 marks)Let $h(x) = -28x^3 + 4x^2 + 7x - 1$.

- a. Use the rational root theorem, or otherwise, to factorise $h(x)$ over R .

2 marks

The graph of h is transformed to the graph of p where $p(x) = \frac{7x^3}{2} - 22x^2 + \frac{85x}{2} - 24$, using the transformation $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} m & 0 \\ 0 & n \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix}$, where m and n are integers. In factorised form

$$p(x) = \frac{1}{2}(x-3)(x-1)(7x-16).$$

- b. Find the values of m and n .

3 marks

END OF QUESTION AND ANSWER BOOK