

The Mathematical Association of Victoria
Trial Examination 2021
MATHEMATICAL METHODS

Trial Written Examination 1 - SOLUTIONS

Question 1

$y = x^2 \log_e \left(\frac{x}{3} \right)$ Use the product rule.

$$\frac{dy}{dx} = 2x \log_e \left(\frac{x}{3} \right) + x^2 \frac{\frac{1}{x}}{\frac{x}{3}} \quad \mathbf{1M}$$

$$= 2x \log_e \left(\frac{x}{3} \right) + x \quad \mathbf{1A}$$

Question 2

$f(x) = \tan(2x)$ Use the chain rule.

$$f'(x) = 2 \sec^2(2x) \quad \mathbf{1M}$$

$$f' \left(\frac{\pi}{3} \right) = 2 \sec^2 \left(\frac{2\pi}{3} \right)$$

$$= 2 \times (-2)^2$$

$$= 8 \quad \mathbf{1A}$$

Question 3

$$3e^x + 4 = e^{-x}$$

$$3e^x + 4 = \frac{1}{e^x}$$

Multiply by e^x .

$$3e^{2x} + 4e^x - 1 = 0 \quad \mathbf{1M}$$

Let $a = e^x$

$$3a^2 + 4a - 1 = 0$$

$$a = \frac{-4 \pm \sqrt{16 + 12}}{6} \quad \mathbf{1M}$$

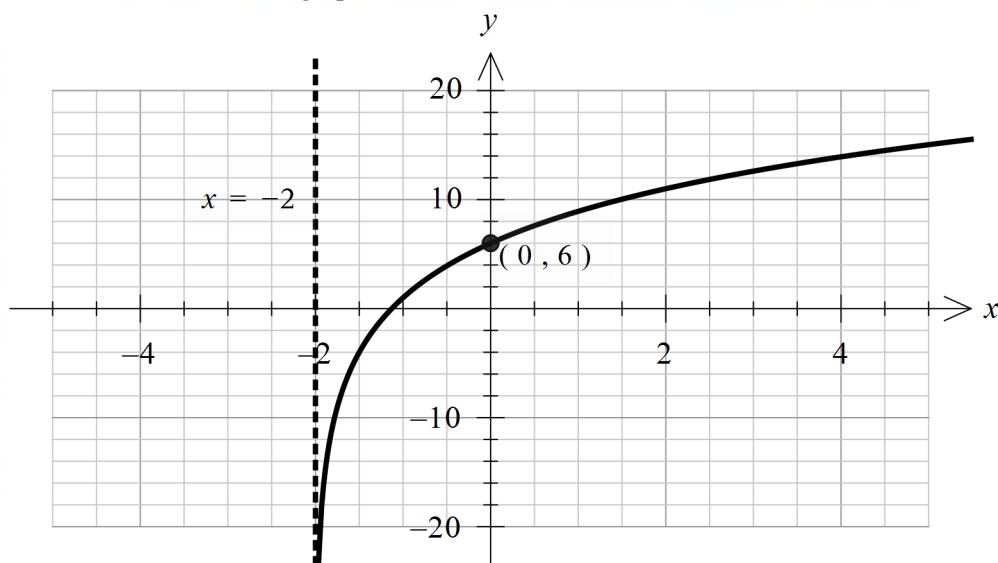
$$= \frac{-4 \pm 2\sqrt{7}}{6}$$

$$= \frac{-2 \pm \sqrt{7}}{3}$$

$$e^x \neq \frac{-2 - \sqrt{7}}{3}$$

$$e^x = \frac{-2 + \sqrt{7}}{3}$$

$$x = \log_e \left(\frac{-2 + \sqrt{7}}{3} \right) \quad \mathbf{1A}$$

Question 4a. $x = -2$ labelled on the graph.**1A**

The graph is of the form $y = a \log_2(x - b) + c$ where a , b and c are real constants.

The equation of the vertical asymptote is $x = b$ and so $b = -2$

b. Using $y = a \log_2(x + 2) + c$ **1A**

and points $(0, 6)$ and $\left(\frac{1}{2^5} - 2, 0\right)$

$$6 = a \log_2(2) + c \text{ simplifies to } 6 = a + c$$

$$0 = a \log_2\left(\frac{1}{2^5} - 2 + 2\right) + c \text{ simplifies to } 0 = a \log_2\left(2^{-\frac{1}{5}}\right) + c \quad \mathbf{1M}$$

$$\text{Solving } a + c = 6 \text{ and } -\frac{1}{5}a + c = 0$$

$$\text{Gives } \frac{6}{5}a = 6, a = 5$$

$$a = 5, b = -2, c = 1$$

1A**Question 5**a. $g(x) = \sqrt{x}$ and $f(x) = \frac{1}{x}$

Test $\text{ran } g \subseteq \text{dom } f$

$$\text{ran}[0, \infty) \not\subseteq \mathbb{R} \setminus \{0\}$$

1A

The function $h(x) = f(g(x))$ does not exist.

b. $g_1(x) = \sqrt{x}$ and $h_1 : D \rightarrow \mathbb{R}, h_1(x) = f(g_1(x))$

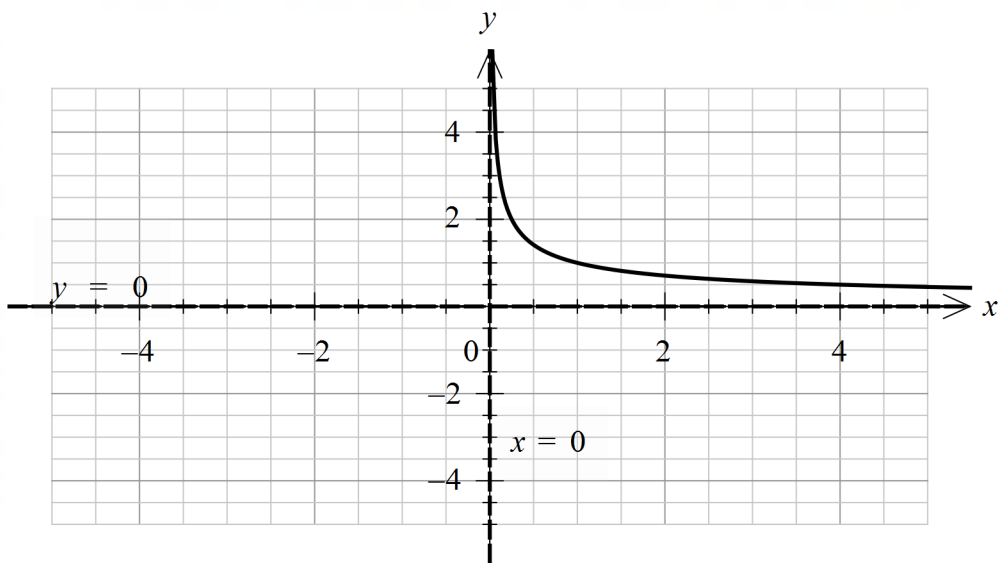
$$D = (0, \infty)$$

1A

$$\text{Rule } h_1(x) = \frac{1}{\sqrt{x}}$$

1A

c. shape **1A**
 asymptotes **1A**



Question 6

a. $\int (3x+1)^{-3} dx = \frac{(3x+1)^{-2}}{3 \times -2} + c$ **1M**

An antiderivative is $\frac{1}{-6(3x+1)^2}$ **1A**

b. Let $f(x) = \frac{1}{-6(3x+1)^2} + c$ and $f(-1) = 2$

$$2 = \frac{1}{-6(-2)^2} + c$$

$$c = \frac{49}{24}$$

$$f(x) = \frac{1}{-6(3x+1)^2} + \frac{49}{24}$$
 1A

Question 7

a. $f: [0, 2\pi] \rightarrow \mathbb{R}, f(x) = -2\sin(2x)\sin\left(x - \frac{\pi}{3}\right)$

Solve $-2\sin(2x) = 0$ for x .

$$2x = 0, \pi, 2\pi, 3\pi, 4\pi$$

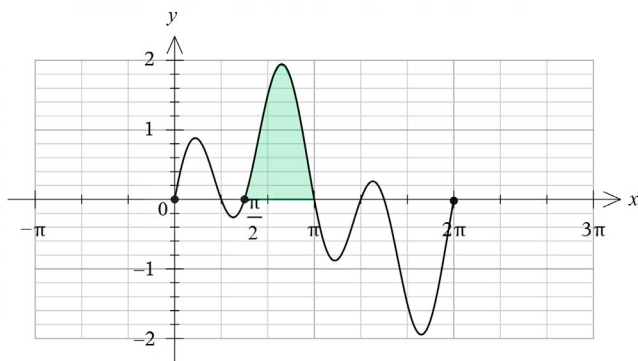
$$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$$
 1A

Solve $\sin\left(x - \frac{\pi}{3}\right) = 0$ for x .

$$x - \frac{\pi}{3} = 0, \pi$$

$$x = \frac{\pi}{3}, \frac{4\pi}{3}$$
 1A

b.



$$\int_{\frac{\pi}{2}}^{\pi} \left(\sin\left(3x + \frac{\pi}{6}\right) - \cos\left(x + \frac{\pi}{3}\right) \right) dx \quad \mathbf{1A}$$

$$= \left[-\frac{1}{3} \cos\left(3x + \frac{\pi}{6}\right) - \sin\left(x + \frac{\pi}{3}\right) \right]_{\frac{\pi}{2}}^{\pi} \quad \mathbf{1M}$$

$$= \left(-\frac{1}{3} \cos\left(3\pi + \frac{\pi}{6}\right) - \sin\left(\pi + \frac{\pi}{3}\right) \right) - \left(-\frac{1}{3} \cos\left(\frac{3\pi}{2} + \frac{\pi}{6}\right) - \sin\left(\frac{\pi}{2} + \frac{\pi}{3}\right) \right)$$

$$= \left(-\frac{1}{3} \cos\left(\frac{19\pi}{6}\right) - \sin\left(\frac{4\pi}{3}\right) \right) - \left(-\frac{1}{3} \cos\left(\frac{5\pi}{3}\right) - \sin\left(\frac{5\pi}{6}\right) \right)$$

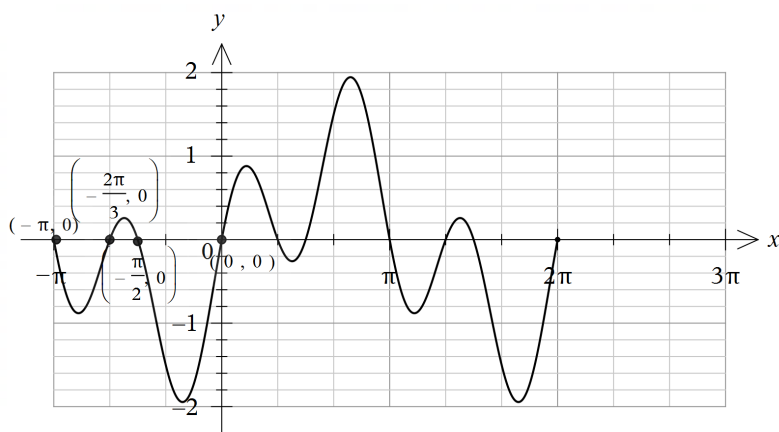
$$= \frac{\sqrt{3}}{6} + \frac{\sqrt{3}}{2} + \frac{1}{6} + \frac{1}{2}$$

$$= \frac{2\sqrt{3} + 2}{3} \quad \mathbf{1A}$$

c. Shape

1A

Coordinates

1A

Question 8

a. $\left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^2 = \frac{9}{256}$

1A

b. $S \sim \text{Bi}\left(4, \frac{1}{4}\right)$

$$\Pr(S > 1) = \Pr(S \geq 2) = 1 - \left({}^4C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^4 + {}^4C_1 \left(\frac{1}{4}\right)^1 \left(\frac{3}{4}\right)^3 \right) \quad \mathbf{1M}$$

$$\Pr(S > 1) = 1 - \left(\frac{81}{256} + 4 \times \left(\frac{1}{4} \right) \left(\frac{27}{64} \right) \right)$$

$$\Pr(S > 1) = 1 - \left(\frac{81}{256} + \frac{27}{64} \right) = \frac{67}{256} \quad \mathbf{1A}$$

c. $0.1 + 0.25 + 0.05 + 0.5 + k = 1$

Giving $k = 0.1$ **1A**

$$\text{Mean} = (1 \times 0.1) + (2 \times 0.25) + (3 \times 0.05) + (4 \times 0.5) + (5 \times 0.1)$$

Giving mean = $\mu = 3.25$

$$\frac{\Pr(X \leq 2)}{\Pr(X < 3.25)} = \frac{0.35}{0.4} = \frac{35}{40} \quad \mathbf{1M}$$

$$\text{Answer} = \frac{7}{8} \quad \mathbf{1A}$$

Question 9

Let $y = g(x) = \frac{x+b}{x+a} = 1 + \frac{b-a}{x+a}$

Inverse swap x and y

$$x = 1 + \frac{b-a}{y+a} \quad \mathbf{1M}$$

$$(x-1)(y+a) = b-a$$

$$y+a = \frac{b-a}{x-1}$$

$$f^{-1}(x) = \frac{b-a}{x-1} - a \quad \mathbf{1A}$$

$$a = -1, b \in \mathbb{R} \setminus \{-1\} \quad \mathbf{1A}$$

Question 10

a. $h(x) = -28x^3 + 4x^2 + 7x - 1$

Rational Root Theorem

Factors of 28: $\pm 1, \pm 2, \pm 4 \dots$ **1M**

Factors of -1 : ± 1

Try $h\left(\frac{1}{2}\right) = 0$

$(2x - 1)$ is a factor

$$\begin{aligned} h(x) &= (2x - 1)(-14x^2 - 5x + 1) \\ &= -(2x - 1)(2x + 1)(7x - 1) \quad \mathbf{1A} \end{aligned}$$

OR

$$\begin{aligned} h(x) &= -28x^3 + 7x + 4x^2 - 1 \\ &= -7x(4x^2 - 1) + 4x^2 - 1 \quad \mathbf{1M} \text{ (grouping)} \\ &= (-7x + 1)(4x^2 - 1) \\ &= -(7x - 1)(2x - 1)(2x + 1) \quad \mathbf{1A} \end{aligned}$$

b. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} m & 0 \\ 0 & n \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

$$x' = mx + 2 \quad \mathbf{1M}$$

$$y' = ny$$

$$p(x) = \frac{1}{2}(x - 3)(x - 1)(7x - 16)$$

$$ny = \frac{1}{2}(mx + 2 - 3)(mx + 2 - 1)(7(mx + 2) - 16)$$

$$ny = \frac{1}{2}(mx - 1)(mx + 1)(7mx - 2)$$

$$y = \frac{1}{2n}(mx - 1)(mx + 1)(7mx - 2)$$

Equate coefficients of x^3 , $h(x) = -28x^3 + 4x^2 + 7x - 1$ **1M**

$$\frac{7m^3}{2n} = -28, \quad n = -\frac{m^3}{8}$$

Equate the constant term.

$$\frac{2}{2n} = -1$$

$$n = -1, \quad m = 2 \quad \mathbf{1A}$$

END OF SOLUTIONS