

2021 VCE Mathematical Methods Trial Examination 2 Detailed Answers



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SECTION A

ANSWERS

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6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
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SECTION A

Question 1

Answer A

The period is $T = \frac{2\pi}{\frac{\pi}{b}} = 2b$, the range of $\cos(x)$ is $[-1, 1]$, the range of $\cos\left(\frac{\pi x}{b}\right) - 1$ is $[-2, 0]$ but since $a < 0$, reflecting in the x -axis, the range is $[0, 2]$ dilating by a factor of a , the range of $a\left(\cos\left(\frac{\pi x}{b}\right) - 1\right)$ becomes $[0, -2a]$.

Question 2

Answer B

Let $f: [0, 3\pi] \rightarrow \mathbb{R}$, $f(x) = 3\sin\left(\frac{x}{3}\right) - 3$. The period is $T = \frac{2\pi}{\frac{1}{3}} = 6\pi$

The graph of f is transformed by a reflection in the x -axis, the rule is

$g(x) = 3 - 3\sin\left(\frac{x}{3}\right)$, we only have one-half of a cycle.

Now a dilation of factor 3 from the y -axis, replace x with $\frac{x}{3}$.

$g(x) = 3 - 3\sin\left(\frac{x}{9}\right)$, the period is $T = \frac{2\pi}{\frac{1}{9}} = 18\pi$, since we must have one-half of a cycle,

the new domain is $[0, 9\pi]$, then a dilation by a factor of 3 from the x -axis, multiply y by 3

the equation becomes $g: [0, 9\pi] \rightarrow \mathbb{R}$, $g(x) = 9 - 9\sin\left(\frac{x}{9}\right)$

Question 3

Answer C

$$\frac{dy}{dx} = 3\sin\left(\frac{x}{3}\right)$$

$$y = \int 3\sin\left(\frac{x}{3}\right) dx = -9\cos\left(\frac{x}{3}\right) + c$$

to find c use $y\left(\frac{\pi}{2}\right) = 0$

$$0 = -9\cos\left(\frac{\pi}{6}\right) + c, \quad c = 9\cos\left(\frac{\pi}{6}\right) = \frac{9\sqrt{3}}{2}$$

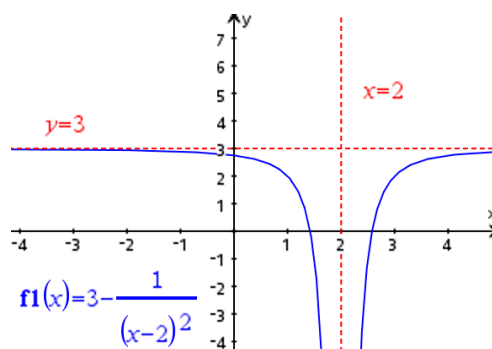
$$y(x) = 9\left(\frac{\sqrt{3}}{2} - \cos\left(\frac{x}{3}\right)\right)$$

$$\text{when } x = 0, \quad y(0) = 9\left(\frac{\sqrt{3}}{2} - 1\right) = \frac{9}{2}(\sqrt{3} - 2)$$

Define $y(x) = \int 3 \cdot \sin\left(\frac{x}{3}\right) dx + c$	Done
solve $\left(y\left(\frac{\pi}{2}\right) = 0, c\right)$	$c = \frac{9 \cdot \sqrt{3}}{2}$
$y(x)$	$\frac{9 \cdot \sqrt{3}}{2} - 9 \cdot \cos\left(\frac{x}{3}\right)$
factor($y(0)$)	$\frac{9 \cdot (\sqrt{3} - 2)}{2}$

Question 4 **Answer D**

$y = b - \frac{1}{(x-a)^2}$ is the only curve which has a vertical asymptote at $x = a$ and a horizontal asymptote at $y = b$.



Question 5 **Answer E**

$$f(x) = \log_e(x+5) \text{ and } g(x) = 4x - x^2$$

$$\begin{aligned} f(g(x)) &= \log_e(g(x)+5) \\ &= \log_e(4x - x^2 + 5) \\ &= \log_e(-(x^2 - 4x - 5)) \\ &= \log_e(-(x-5)(x+1)) \end{aligned}$$

we require $-(x-5)(x+1) > 0$ or

$(x-5)(x+1) < 0$ that is $(-1, 5)$, since $\text{dom } f(g(x)) = \text{dom } g(x) = D_g$

Define $f(x) = \ln(x+5)$ Done

Define $g(x) = 4 \cdot x - x^2$ Done

$f(g(x))$ $\ln(-(x^2 - 4 \cdot x - 5))$

$\text{domain}(f(g(x)), x)$ $-1 < x < 5$

Question 6 **Answer A**

$$f(x) = kx^2 - x^4 = x^2(k - x^2)$$

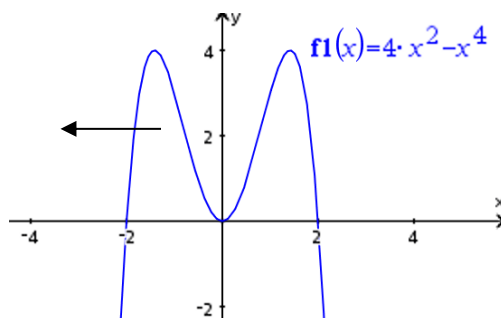
crosses the x -axis at $x = 0$, $x = \pm\sqrt{k}$

$$f'(x) = 2kx - 4x^3 = 2x(k - 2x^2)$$

has stationary points at $x = 0$, $x = \pm\sqrt{\frac{k}{2}}$

since $k > 0$. The function will have an inverse provided that the function is one-one, increasing

function that is $b < -\sqrt{\frac{k}{2}}$.



Question 7 **Answer B**

original curve $\frac{y}{2} = \cos(x)$ image curve $y' = \sin(2x') = \cos\left(\frac{\pi}{2} - 2x'\right)$

$$x = \frac{\pi}{2} - 2x', \quad x' = -\frac{x}{2} + \frac{\pi}{4}, \quad \frac{y}{2} = y' \quad \text{in matrix form} \quad T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \frac{\pi}{4} \\ 0 \end{bmatrix}$$

Question 8

Answer C

$$\int_a^b (f(x) + k) dx = \int_a^b f(x) dx + [kx]_a^b = A + k(b-a) \quad \text{A. is true}$$

$$\frac{1}{k} \int_{ak}^{bk} f\left(\frac{x}{k}\right) dx = \int_a^b f(x) dx = A$$

the area gets dilation by a factor of k parallel to the x -axis. **B.** is true

$$\int_{ak}^{bk} \frac{f(kx)}{k} dx = \frac{1}{k} \int_{ak}^{bk} f(kx) dx \neq A \quad \text{C. is false}$$

$A = \int_{a-k}^{b-k} f(x+k) dx = \int_{a+k}^{b+k} f(x-k) dx = \int_a^b f(x) dx$ the area remains the same under a transformation k units to the left or right parallel to the x -axis. **D.** and **E.** are both true.

Question 9

Answer D

Total $b+r$ marbles, let A be the event two marbles of the same color, drawn without replacement, that is 1 red and 2 blue or 1 blue and 2 red, but there are 3 ways of doing each of these, $BBR, BRB, RBB, RRB, RBR, BRR$,

$$\begin{aligned} \Pr(A) &= \frac{3b(b-1)r}{(b+r)(b+r-1)(b+r-2)} + \frac{3r(r-1)b}{(b+r)(b+r-1)(b+r-2)} \\ &= \frac{3br(b-1) + 3br(r-1)}{(b+r)(b+r-1)(b+r-2)} = \frac{3br(b-1+r-1)}{(b+r)(b+r-1)(b+r-2)} = \frac{3br(b+r-2)}{(b+r)(b+r-1)(b+r-2)} \\ &= \frac{3br}{(b+r)(b+r-1)} \end{aligned}$$

Question 10

Answer E

$$(1) \quad 4y - nx = n^2 \quad \Rightarrow \quad y = \frac{nx}{4} + \frac{n^2}{4}$$

$$(2) \quad ny - mx = n \quad \Rightarrow \quad y = \frac{mx}{n} + 1$$

equal gradients when $\frac{n}{4} = \frac{m}{n}$, $n^2 = 4m$

equal y -intercepts when $\frac{n^2}{4} = 1$, $n = \pm 2$,

so that when $n = \pm 2$ and $m = 1$,

there is an infinite number of solutions,

and when $n = \pm 2$ and $m = -1$ there is a unique solution.

$$eq1 := 4 \cdot y - n \cdot x = n^2 \quad 4 \cdot y - n \cdot x = n^2$$

$$eq2 := n \cdot y - m \cdot x = n \quad n \cdot y - m \cdot x = n$$

$$\text{solve}(eq1 \text{ and } eq2, \{x, y\}) | n = -2 \text{ and } m = 1 \\ x = -2 \cdot (c12 - 1) \text{ and } y = c12$$

$$\text{solve}(eq1 \text{ and } eq2, \{x, y\}) | n = -2 \text{ and } m = -1 \\ x = 0 \text{ and } y = 1$$

$$\text{solve}(eq1 \text{ and } eq2, \{x, y\}) | n = 2 \text{ and } m = 1 \\ x = 2 \cdot (c13 - 1) \text{ and } y = c13$$

$$\text{solve}(eq1 \text{ and } eq2, \{x, y\}) | n = 2 \text{ and } m = -1 \\ x = 0 \text{ and } y = 1$$

Question 11

Answer C

$$\frac{d}{dx}(x^2 e^{-kx}) = x e^{-kx} (2 - kx) = 2x e^{-kx} - kx^2 e^{-kx}$$

$$\int (2x e^{-kx} - kx^2 e^{-kx}) dx = x^2 e^{-kx}$$

$$2 \int x e^{-kx} dx - k \int x^2 e^{-kx} dx = x^2 e^{-kx}$$

$$k \int x^2 e^{-kx} dx = 2 \int x e^{-kx} dx - x^2 e^{-kx}$$

$$\int x^2 e^{-kx} dx = \frac{2}{k} \int x e^{-kx} dx - \frac{x^2}{k} e^{-kx} + c$$

Question 12

Answer C

$$\Pr(X = k) = \frac{e^{-\lambda} \lambda^k}{k!}, \quad \Pr(X = 0) = e^{-\lambda}, \quad \Pr(X = 1) = \lambda e^{-\lambda}, \quad \Pr(X = 2) = \frac{\lambda^2 e^{-\lambda}}{2}$$

$$\Pr(X < 3 | X \geq 1) = \Pr(X \leq 2 | X \geq 1) = \frac{\Pr(1 \leq X \leq 2)}{\Pr(X \geq 1)} = \frac{\Pr(X = 1) + \Pr(X = 2)}{1 - \Pr(X = 0)}$$

$$\Pr(X < 3 | X \geq 1) = \frac{\lambda e^{-\lambda} + \frac{\lambda^2 e^{-\lambda}}{2}}{1 - e^{-\lambda}} = \frac{\lambda e^{-\lambda} \left(1 + \frac{\lambda}{2}\right)}{1 - e^{-\lambda}} = \frac{\lambda e^{-\lambda} (\lambda + 2)}{2(1 - e^{-\lambda})}$$

Question 13

Answer E

	A	A'	
B	p^2	$b^2 - p^2$	b^2
B'	$a^2 - p^2$	$\Pr(A' \cap B')$	$1 - b^2$
	a^2	$1 - a^2$	

$$\Pr(A' \cap B') = 1 - a^2 - (b^2 - p^2) = 1 - b^2 - (a^2 - p^2)$$

$$\Pr(A' \cap B') = (1 - a^2) + (p^2 - b^2) = (1 - b^2) + (p^2 - a^2)$$

$$\text{Given that } (1 - a)(1 + a) + (p - b)(p + b) = (1 - a^2)(1 - b^2) \Leftrightarrow p^2 = a^2 b^2$$

$$(1 - a^2) + a^2 b^2 - b^2 = (1 - a^2) - b^2 (1 - a^2) = (1 - a^2)(1 - b^2)$$

Since $\Pr(A \cap B') \neq 0$, $\Pr(A' \cap B') \neq 0$ **A.** and **B.** are false.

In fact the all the events, A and B, A and B', A' and B', A' and B' are all independent.

$$\Pr(A' \cap B') = (1 - a^2)(1 - b^2) = \Pr(A')\Pr(B')$$

$$\Pr(A \cap B) = p^2 = a^2 b^2 = \Pr(A)\Pr(B)$$

$$\Pr(A \cap B') = a^2 - p^2 = a^2 - a^2 b^2 = a^2 (1 - b^2) = \Pr(A)\Pr(B')$$

$$\Pr(A' \cap B) = b^2 - p^2 = b^2 - a^2 b^2 = (1 - a^2) b^2 = \Pr(A')\Pr(B)$$

Question 14 **Answer D**

$$f: y = \frac{x-a}{x-1}, \quad a \in \mathbb{R} \setminus \{1\} \quad \text{swap } x \text{ and } y$$

$$f^{-1}: x = \frac{y-a}{y-1}, \quad x(y-1) = y-a$$

$$xy - x = y - a, \quad xy - y = x - a, \quad y(x-1) = x - a$$

Since $f^{-1}(x) = \frac{x-a}{x-1} = f(x)$ they are the same graph and therefore have an infinite number of points of intersection.

Question 15 **Answer E**

$$\text{Given } f(a) = b \Rightarrow f^{-1}(b) = a, \quad f'(a) = c$$

$$\text{let } y = g(x) = f^{-1}(x)$$

$$x = f(y) \quad \text{differentiate wrt } y \quad \frac{dx}{dy} = f'(y)$$

$$\text{inverting } \frac{dy}{dx} = \frac{d}{dx}[f^{-1}(x)] = g'(x) = \frac{1}{f'(y)} = \frac{1}{f'(f^{-1}(x))}$$

$$g'(b) = \frac{1}{f'(f^{-1}(b))} = \frac{1}{f'(a)} = \frac{1}{c}$$

Question 16 **Answer A**

$$n = 25, \quad p = 0.1, \quad \hat{p} = \frac{X}{n} = \frac{X}{25} \quad X \stackrel{d}{=} \text{Bi}(n = 25, p = 0.1)$$

$$\left\| \text{binomCdf}(25, 0.1, 0, 4) \quad 0.902006 \right\|$$

$$\Pr(\hat{p} < 0.2) = \Pr\left(\hat{p} < \frac{1}{5}\right) = \Pr(X < 5) = \Pr(X \leq 4) = 0.902$$

Question 17 **Answer A**

$$f(x) = kx, \text{ since it is a probability density function } \int_0^a kx \, dx = 1$$

$$\text{the mean value } \frac{1}{a-0} \int_0^a kx \, dx = \frac{1}{a} \times 1 = 2 \Rightarrow a = \frac{1}{2}$$

$$\int_0^a kx \, dx = \int_0^{\frac{1}{2}} kx \, dx = k \left[\frac{x^2}{2} \right]_0^{\frac{1}{2}} = k \left(\frac{1}{8} - 0 \right) = 1, \quad k = 8$$

Question 18

Answer D

The confidence interval is

$$\left(\hat{p} - 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right)$$

$$\hat{p} = \frac{x}{n}, n = 36$$

$$f(x) = \frac{x}{36} + 1.96\sqrt{\frac{\frac{x}{36}\left(1 - \frac{x}{36}\right)}{36}}$$

$$f'(x) = 0 \Rightarrow x = 35$$

Define $f(x) = \frac{x}{n} + 1.96 \cdot \sqrt{\frac{\frac{x}{n} \cdot \left(1 - \frac{x}{n}\right)}{n}} | n=36$ Done

solve $\left(\frac{d}{dx}(f(x)) = 0, x \right)$ $x = 35.1102$

$f(34)$ 1.01927

$f(35)$ 1.02591

Question 19

Answer B

$$\sum \Pr(X = x) = \log_{81}(a) + \log_{81}(b) = \log_{81}(ab) = 1$$

$$(1) \quad ab = 81$$

$$E(X) = \sum x \Pr(X = x) = 2\log_{81}(b) - 2\log_{81}(a) = 1$$

$$E(X) = 2\log_{81}\left(\frac{b}{a}\right) = 1 \Rightarrow \log_{81}\left(\frac{b}{a}\right) = \frac{1}{2}$$

$$(2) \quad \frac{b}{a} = 9 \quad b = 9a \quad \text{solving (1) and (2)}$$

$$9a^2 = 81, \quad a^2 = 9, \quad a = 3, \quad \text{since } a > 0, b > 0$$

$$a = 3, \quad b = 27$$

eq1: $\log_{81}(a) + \log_{81}(b) = 1$

$\log_{81}(a) + \log_{81}(b) = 1$

eq2: $2 \cdot (\log_{81}(b) - \log_{81}(a)) = 1$

$-2 \cdot (\log_{81}(a) - \log_{81}(b)) = 1$

solve(eq1 and eq2, {a, b}) $a = 3$ and $b = 27$

Question 20

Answer B

$$\Pr(X > b) = 0.03 \quad \Pr(X < c) = 0.1$$

$$\Pr(X < b) = 0.97$$

$$\frac{b - \mu}{\sigma} = 1.88 \quad \text{and} \quad \frac{c - \mu}{\sigma} = -1.28 \quad \frac{b - \mu}{\sigma} = 1.88 \quad \text{and} \quad \frac{\mu - c}{\sigma} = 1.28.$$

invNorm(0.97, 0, 1) 1.88079

invNorm(0.1, 0, 1) -1.28155

END OF SECTION A SUGGESTED ANSWERS

SECTION B

Question 1

a. $f(x) = x^4 - 6x^2 + bx + c$
 $f(3) = 0 \Rightarrow 81 - 54 + 3b + c = 0$, (1) $3b + c = -27$
 $f'(x) = 4x^3 - 12x + b$ M1
 $f'(-2) = 0 \Rightarrow -32 + 24 + b = 0$, $b = 8$
 $b = 8$, $c = -27 - 24 = -51$ A1

b. $f(x) = x^4 - 6x^2 + c$
 $f(x) = x^4 - 6x^2 + c = 0$
 $\Rightarrow x^2 = \frac{6 \pm \sqrt{36 - 4 \times c}}{2} = 3 \pm \sqrt{9 - c}$
 $f'(x) = 4x^3 - 12x = 4x(x^2 - 3) = 0$ for stationary points M1
 $f'(x) = 0 \Rightarrow x = 0$, $\pm\sqrt{3}$
 $f(0) = c$, $f(\sqrt{3}) = c - 9$, $f(-\sqrt{3}) = c - 9$

A2

$y = f(x) = x^4 - 6x^2 + c$	values of c
crosses the x -axis four times, that is, there are four solutions for $f(x) = 0$.	$0 < c < 9$
crosses the x -axis three times, that is, there are three solutions for $f(x) = 0$.	$c = 0$
crosses the x -axis twice, that is, there are two solutions for $f(x) = 0$.	$c = 9$ or $c < 0$
does not cross the x -axis, that is, there are no solutions for $f(x) = 0$.	$c > 9$

c.i. $f(x) = x^4 - 6x^2 - 4x$
 $f'(x) = 4x^3 - 12x - 4 = 0$
 $x = -1.53, -0.35, 1.88$
 $(-1.53, -2.45)$, $(-0.35, 0.68)$, $(1.88, -16.23)$ A1

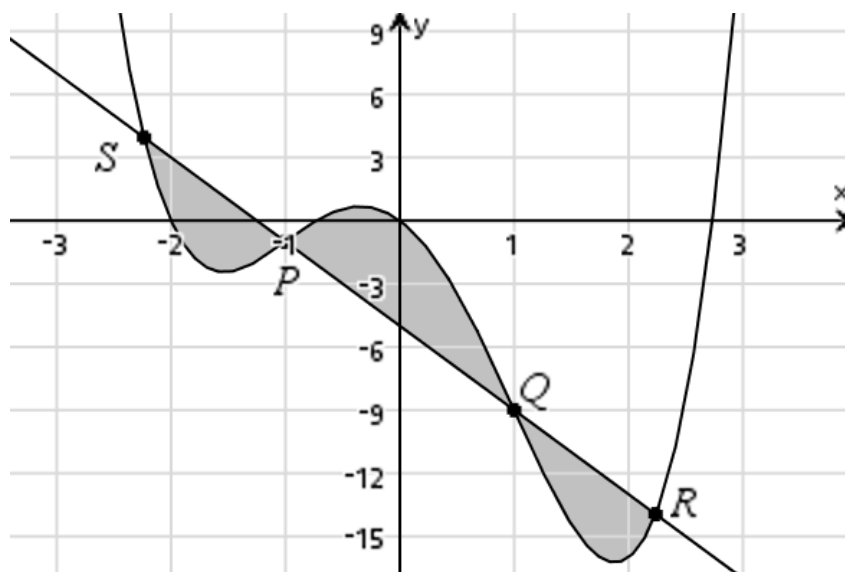
c.ii. $m(x) = f'(x) = 4x^3 - 12x - 4$
 $m'(x) = 12x^2 - 12 = 12(x^2 - 1) = 0$ for maximum and minimum gradient
 $x = \pm 1$ $m(-1) = f'(-1) = -4 + 12 - 4 = 4$, $m(1) = f'(1) = 4 - 12 - 4 = -12$ A1

c.iii. $f(-1) = -1$, $f(1) = 9$, $P(-1, -1)$, $Q(1, -9)$
 gradient joining $m(PQ) = \frac{-9+1}{1-(-1)} = -\frac{8}{2} = -4$ A1

equation of the line joining PQ : $y+1 = -4(x+1)$
 $y = -4x - 5$, and drawing the line on the graph below A1

c.iv. solving $x^4 - 6x^2 - 4x = -4x - 5$
 $x^4 - 6x^2 + 5 = 0$
 $(x^2 - 5)(x^2 - 1) = 0$ M1
 $x = \pm\sqrt{5}$, ± 1
 $x = \pm 1$ are the points P and Q
 $x_S = -\sqrt{5}$, $x_R = \sqrt{5}$ A1

c.v. $f(-\sqrt{5}) = 4\sqrt{5} - 5$, $f(\sqrt{5}) = -4\sqrt{5} - 5$
 $R(\sqrt{5}, -4\sqrt{5} - 5)$, $S(-\sqrt{5}, 4\sqrt{5} - 5)$, $P(-1, -1)$, $Q(1, -9)$
 $d(SP) = \sqrt{(-1+\sqrt{5})^2 + (-4\sqrt{5}+4)^2} = \sqrt{17}(\sqrt{5}-1)$ M1
 $d(QR) = \sqrt{(\sqrt{5}-1)^2 + (4-4\sqrt{5})^2} = \sqrt{17}(\sqrt{5}-1)$
 so $d(SP) = d(QR) = \sqrt{17}(\sqrt{5}-1)$ A1



c.vi. Let $t(x) = -4x - 5$

$$A_1 = \int_{x_p}^{x_q} (t(x) - f(x)) dx = \int_{-\sqrt{5}}^{-1} (-x^4 + 6x^2 - 5) dx = \frac{16}{5} = 3\frac{1}{5} = 3.2$$

M1

$$A_2 = \int_{x_p}^{x_q} (f(x) - t(x)) dx = \int_{-1}^1 (x^4 - 6x^2 + 5) dx = \frac{32}{5} = 6\frac{2}{5} = 6.4$$

$$A_3 = \int_{x_q}^{x_r} (t(x) - f(x)) dx = \int_1^{\sqrt{5}} (-x^4 + 6x^2 - 5) dx = \frac{16}{5} = 3\frac{1}{5} = 3.2$$

A1

$$A_1 = A_3 = \frac{1}{2} A_2, \quad A_1 + A_3 = A_2$$

Define $f1(x) = x^4 - 6 \cdot x^2 + b \cdot x + c$	Done
$f1(3) = 0$	$3 \cdot b + c + 27 = 0$
Define $df1(x) = \frac{d}{dx}(f1(x))$	Done
$df1(-2) = 0$	$b - 8 = 0$
$\text{solve}(3 \cdot b + c + 27 = 0, c) b = 8$	$c = -51$
Define $f1(x) = x^4 - 6 \cdot x^2 + c$	Done
$u^2 - 6 \cdot u + c = 0$	$u^2 - 6 \cdot u + c = 0$
$\text{solve}(u^2 - 6 \cdot u + c = 0, u)$	$u = -(\sqrt{9 - c} - 3) \text{ or } u = \sqrt{9 - c} + 3$
Define $f2(x) = x^4 - 6 \cdot x^2 - 4 \cdot x$	Done
$\text{solve}\left(\frac{d}{dx}(f2(x)) = 0, x\right)$	$x = -1.5321 \text{ or } x = -0.3473 \text{ or } x = 1.8794$
$\text{zeros}\left(\frac{d}{dx}(f1(x)), x\right) \rightarrow xtps$	$\{-1.5321, -0.3473, 1.8794\}$
$f1(xtps)$	$\{-2.4456, 0.6800, -16.2344\}$
$f1(\sqrt{5})$	$-4 \cdot \sqrt{5} - 5$
$f1(-\sqrt{5})$	$4 \cdot \sqrt{5} - 5$
$\sqrt{(\sqrt{5} - 1)^2 + (4 - 4 \cdot \sqrt{5})^2}$	$(\sqrt{5} - 1) \cdot \sqrt{17}$
$f2(x) - (-4 \cdot x - 5)$	$x^4 - 6 \cdot x^2 + 5$
$a1 := \int_{-\sqrt{5}}^{-1} (-x^4 + 6 \cdot x^2 - 5) dx$	$\frac{16}{5}$
$a2 := \int_{-1}^1 (x^4 - 6 \cdot x^2 + 5) dx$	$\frac{32}{5}$
$a3 := \int_1^{\sqrt{5}} (-x^4 + 6 \cdot x^2 - 5) dx$	$\frac{16}{5}$

Question 2

- a. $L = h(y_0 + y_1 + y_2 + y_3 + y_4)$
 $L = 0.2(2 + 2.18 + 2.33 + h + 1.24) = 1.93$
 $h = 1.90 = y_3$, $b = y_5 = ?$
 $R = h(y_1 + y_2 + y_3 + y_4 + y_5)$
 $R = 0.2(2.18 + 2.33 + 1.90 + 1.24 + b) = 1.79$
 $b = 1.30$ A1
- b. $f(1) = 1\sin\left(\frac{7\pi}{4}\right) + 2 = 2 - \frac{\sqrt{2}}{2} = \frac{1}{2}(4 - \sqrt{2})$ A1
- c. $f(x) = x\sin\left(\frac{7\pi x}{4}\right) + 2$
 $f'(x) = \sin\left(\frac{7\pi x}{4}\right) + \frac{7\pi x}{4}\cos\left(\frac{7\pi x}{4}\right)$ A1
solving $f'(x) = 0$ $x \in (0,1)$
gives $x = 0.369$, 0.894 , $f(0.369) = 2.331$, $f(0.894) = 1.124$
closest point $(0.894, 1.124)$ furthest point $(0.369, 2.331)$ A1
- d. $A(0.3, 0.4)$ $R(u, f(u))$
 $d(AR) = s(u) = \sqrt{(u - 0.3)^2 + (f(u) - 0.4)^2}$ M1
solving $\frac{ds}{du} = 0$ gives $u = 0.0162$, 0.3715 , 0.8659 M1
the minimum distance is $s_{\min} = s(0.8659) = 0.9277 \text{ km}$ A1
- e. using average of the left and right rectangles $A = \frac{1}{2}(L + R) = 0.5(1.93 + 1.79) = 1.86 \text{ km}^2$
using the river equation $A = \int_0^1 \left(x\sin\left(\frac{7\pi x}{4}\right) + 2\right) dx = 1.85 \text{ km}^2$
the average of the left and right rectangles over-estimate the modelled area. A1
- f. solving $\int_0^m \left(x\sin\left(\frac{7\pi x}{4}\right) + 2\right) dx = \frac{1.848}{2} = \int_m^1 \left(x\sin\left(\frac{7\pi x}{4}\right) + 2\right) dx$
or solving $\int_0^m \left(x\sin\left(\frac{7\pi x}{4}\right) + 2\right) dx = \int_m^1 \left(x\sin\left(\frac{7\pi x}{4}\right) + 2\right) dx$ M1
gives solving $m = 0.4235$
 $f(m) = 2.3077$, the length of the fence is 2.3077 km A1

$\text{solve}(0.2 \cdot (2+2.18+2.33+h+1.24)=1.93, h)$	$h=1.90$
$\text{solve}(0.2 \cdot (2.18+2.33+1.9+1.24+b)=1.79, b)$	$b=1.30$
Define $fI(x)=x \cdot \sin\left(\frac{7 \cdot \pi \cdot x}{4}\right)+2$	Done
$fI(1)$	$2-\frac{\sqrt{2}}{2}$
$\frac{d}{dx}(fI(x))$	$\frac{7 \cdot \pi \cdot x \cdot \cos\left(\frac{7 \cdot \pi \cdot x}{4}\right)}{4} + \sin\left(\frac{7 \cdot \pi \cdot x}{4}\right)$
$\Delta \text{ solve}\left(\frac{d}{dx}(fI(x))=0, x\right) 0 < x < 1$	$x=0.369 \text{ or } x=0.894$
$fI(0.369)$	2.331
$fI(0.894)$	1.124
Define $s(u)=\sqrt{(u-0.3)^2+(fI(u)-0.4)^2}$	Done
$\Delta \text{ solve}\left(\frac{d}{du}(s(u))=0, u\right) 0 < u < 1$	$u=0.0162 \text{ or } u=0.3715 \text{ or } u=0.8659$
$s(0.0162)$	1.6264
$s(0.3715)$	1.9323
$s(0.8659)$	0.9277
$0.5 \cdot (1.93+1.79)$	1.8600
$\int_0^1 fI(x) \, dx$	1.8480
$\Delta \text{ solve}\left(\int_0^m fI(x) \, dx = \int_m^1 fI(x) \, dx, m\right) 0 < m < 1$	$m=0.4235$
$fI(0.42351)$	2.3077

Question 3

tennis balls $T \stackrel{d}{=} N(57, 0.937^2)$

a.i. $\Pr(T < 56) = 0.1429$ A1

ii. containers $C \stackrel{d}{=} Bi(n = 3, p = 0.1429)$

$\Pr(C \geq 1) = 1 - \Pr(C = 0) = 1 - (1 - 0.1429)^3 = 0.3704$ A1

<code>normCdf(-∞, 56, 57, 0.937)</code>	0.142933
<code>p:=0.1429327019929</code>	0.142933
<code>1-(1-p)³</code>	0.370429
<code>binomCdf(3,p,1,3)</code>	0.370429

b. containers $Cn \stackrel{d}{=} Bi(n = ?, p = 0.1429)$

$\Pr(Cn = 2) + \Pr(Cn = 3) = 0.5$

$\binom{n}{2} 0.1429^2 \times (1 - 0.1429)^{n-2} + \binom{n}{3} 0.1429^3 \times (1 - 0.1429)^{n-3} = 0.5$ M1

solving gives $n = 15.34$ or 18.12

so $n = 16, 17, 18$ A1

<code>nCr(n,2) · p² · (1-p)ⁿ⁻² + nCr(n,3) · p³ · (1-p)ⁿ⁻³ = 0.5</code>
<code>0.000773 · n · (n-1) · (n+15.9889) · (0.857067)ⁿ = 0.5</code>
<code>solve(7.730343401055E-4 · n · (n-1) · (n+15.988898</code>
<code>n=15.3405 or n=18.1177</code>
<code>nCr(n,2) · p² · (1-p)ⁿ⁻² + nCr(n,3) · p³ · (1-p)ⁿ⁻³ n=15</code>
<code>0.497563</code>

<code>nCr(n,2) · p² · (1-p)ⁿ⁻² + nCr(n,3) · p³ · (1-p)ⁿ⁻³ n=16</code>
<code>0.503093</code>
<code>nCr(n,2) · p² · (1-p)ⁿ⁻² + nCr(n,3) · p³ · (1-p)ⁿ⁻³ n=17</code>
<code>0.503952</code>
<code>nCr(n,2) · p² · (1-p)ⁿ⁻² + nCr(n,3) · p³ · (1-p)ⁿ⁻³ n=18</code>
<code>0.50064</code>
<code>nCr(n,2) · p² · (1-p)ⁿ⁻² + nCr(n,3) · p³ · (1-p)ⁿ⁻³ n=19</code>
<code>0.493672</code>

c.i. $E(\hat{P}) = 0.143$

$sd(\hat{P}) = \sqrt{\frac{0.1429 \times (1 - 0.1429)}{49}} = 0.050$ A1

$$\begin{aligned}
 \text{ii. } & \Pr(0.1429 - 2 \times 0.050 \leq T \leq 0.1429 + 2 \times 0.050) \\
 &= \Pr(0.0429 \leq T \leq 0.2429) \times 49 \\
 &= \Pr(2.1 \leq T_b \leq 11.9) \\
 &= \Pr(3 \leq T_b \leq 11) \\
 &= 0.938
 \end{aligned}$$

M1

A1

$49 \cdot p$	7.0037
$\sqrt{\frac{p \cdot (1-p)}{49}}$	0.050001
$sd := 0.05000060095433$	0.050001
$p + 2 \cdot sd$	0.242934
$p - 2 \cdot sd$	0.042932
$49 \cdot 0.04293150008424$	2.10364
$49 \cdot 0.24293390390156$	11.9038
$\text{binomCdf}(49, p, 2.1, 11.9)$	0.938049
$\text{binomCdf}(49, p, 3, 11)$	0.938049

$$\text{d. } \left(\hat{p} - z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right) = (0.0523, 0.2334)$$

M1

since $\hat{p} = 0.1429$, $n = 49$, $sd = 0.05$, $z = ?$

$$z = \frac{0.2334 - 0.0523}{2 \times 0.050} = 1.811, \quad Z \stackrel{d}{=} N(0, 1)$$

$$\Pr(-1.811 \leq Z \leq 1.811) = 0.93$$

A1

93%

$\frac{0.2334 - 0.0523}{2 \cdot sd}$	1.81098												
$\text{normCdf}(-1.811, 1.811, 0, 1)$	0.929859												
$\text{zInterval}_1 \text{Prop } 7, 49, 0.93: \text{stat.results}$													
<table border="1"> <thead> <tr> <th>"Title"</th><th>"1-Prop z Interval"</th></tr> </thead> <tbody> <tr> <td>"CLower"</td><td>0.05228</td></tr> <tr> <td>"CUpper"</td><td>0.233434</td></tr> <tr> <td>"p"</td><td>0.142857</td></tr> <tr> <td>"ME"</td><td>0.090577</td></tr> <tr> <td>"n"</td><td>49.</td></tr> </tbody> </table>		"Title"	"1-Prop z Interval"	"CLower"	0.05228	"CUpper"	0.233434	"p"	0.142857	"ME"	0.090577	"n"	49.
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e. $a \left[\int_0^2 \sin\left(\frac{\pi t}{4}\right) dt + \int_2^4 \frac{4-t}{2} dt \right] = 1$ since the total area is equal to one

$$a \left(\left[-\frac{4}{\pi} \cos\left(\frac{\pi t}{4}\right) \right]_0^2 + \left[\frac{1}{2} \left(4t - \frac{1}{2} t^2 \right) \right]_2^4 \right) = 1 \quad \text{A1}$$

$$a \left(-\frac{4}{\pi} \cos\left(\frac{\pi}{2}\right) + \frac{4}{\pi} \cos(0) + \frac{1}{2} (16 - 8 - 8 + 2) \right) = 1$$

$$a \left(\frac{4}{\pi} + 1 \right) = a \left(\frac{4 + \pi}{\pi} \right) = 1 \quad \text{M1}$$

$$a = \frac{\pi}{4 + \pi}$$

f. $\Pr(T > 3) | \Pr(T > 1) = \frac{\Pr(T > 3)}{\Pr(T > 1)}$

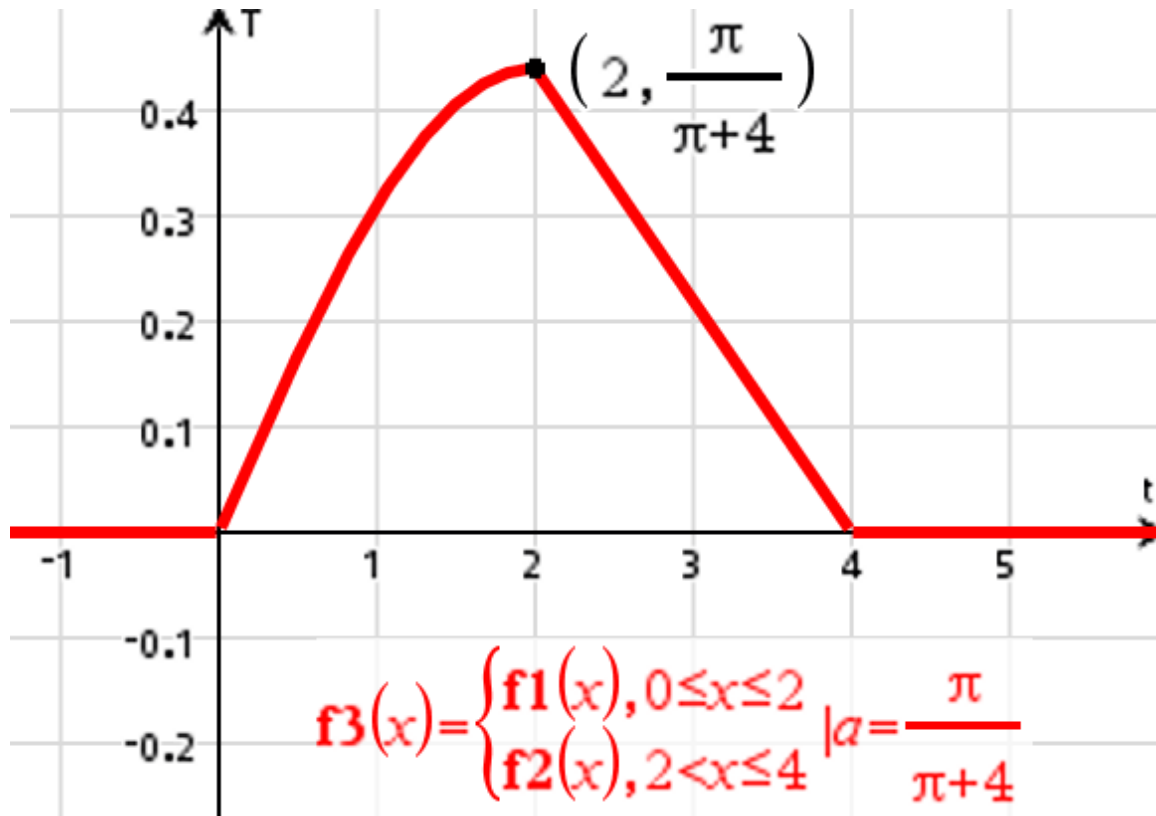
$$= \frac{\int_3^4 \frac{1}{2} (4-t) dt}{\int_1^2 \sin\left(\frac{\pi t}{4}\right) dt + \int_2^4 \frac{1}{2} (4-t) dt} \quad \text{M1}$$

$$= \frac{\pi}{4(\pi + 2\sqrt{2})} \quad \text{A1}$$

Define $f1(x) = a \cdot \sin\left(\frac{\pi \cdot x}{4}\right)$	Done
Define $f2(x) = \frac{a \cdot (4-x)}{2}$	Done
$f1(2) = f2(2)$	true
solve $\left(\int_0^2 f1(x) dx + \int_2^4 f2(x) dx = 1, a \right)$	$a = \frac{\pi}{\pi+4}$
Define $f3(x) = \begin{cases} f1(x), & 0 \leq x \leq 2 \\ f2(x), & 2 \leq x \leq 4 \end{cases} a = \frac{\pi}{\pi+4}$	Done
$\frac{\int_3^4 f3(x) dx}{\int_1^4 f3(x) dx}$	$\frac{\pi}{4 \cdot (\pi + 2 \cdot \sqrt{2})}$

g.

G2



h. $E(T) = \frac{\pi}{4+\pi} \left[\int_0^2 t \sin\left(\frac{\pi t}{4}\right) dt + \int_2^4 \frac{t(4-t)}{2} dt \right] = 1.8862 \text{ hr}$ A1

$$E(T^2) = \frac{\pi}{4+\pi} \left[\int_0^2 t^2 \sin\left(\frac{\pi t}{4}\right) dt + \int_2^4 \frac{t^2(4-t)}{2} dt \right] = 4.2625$$

$\text{var}(T) = E(T^2) - (E(T))^2 = 4.2625 - 1.8862^2 = 0.7047 \text{ hr}^2$ A1

i. $\int_0^m f(t) dt = \frac{1}{2}$

$m = 1.86 \text{ hours} \quad \text{or} \quad m = 111.88 \text{ minutes}$
 $m = 112$

A1

$\int_0^4 f_3(x) \, dx$	1
$\int_0^4 (x \cdot f_3(x)) \, dx$	1.88621
$\int_0^4 (x^2 \cdot f_3(x)) \, dx$	4.2625
$4.262504 - (1.8862093)^2$	0.704718
 solve $\left(\int_0^m f_3(x) \, dx = 0.5, m \right)$	$m=1.86312$
$1.8631166 \cdot 60$	111.787

Question 4

a. original $y = \log_e(x)$ new $y' = \log_3(2x' - 5) - 5$

$$y = \log_e(x) = \frac{\log_3(x)}{\log_3(e)}, \quad \log_3(e)y = \log_3(x)$$

$$y' + 5 = \log_3(2x' - 5), \quad y' + 5 = \log_3(e)y, \quad y' = \log_3(e)y - 5$$

$$x = 2x' - 5, \quad x' = \frac{x}{2} + \frac{5}{2} \quad \text{M1}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \log_3(e) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \frac{5}{2} \\ -5 \end{bmatrix}, \quad a = \frac{1}{2}, \quad b = \log_3(e), \quad p = \frac{5}{2}, \quad q = -5 \quad \text{A1}$$

b. At the point $(u, \log_3(2u - 5) - 5)$

$$y = \log_3(2x - 5) - 5 = \frac{\log_e(2x - 5)}{\log_e(3)} - 5$$

$$\frac{dy}{dx} = \frac{2}{(2x - 5)\log_e(3)} \quad \text{at } x = u \quad m_T = \frac{2}{(2u - 5)\log_e(3)}$$

$$T: y - (\log_3(2u - 5) - 5) = \frac{2}{(2u - 5)\log_e(3)}(x - u)$$

$$y = \frac{2x}{(2u - 5)\log_e(3)} + \log_3(2u - 5) - 5 - \frac{2u}{(2u - 5)\log_e(3)} \quad \text{M1}$$

$$y = \frac{nx}{(2u - 5)} + \log_3(2u - 5) - 5 - \frac{nu}{2u - 5}$$

$$n = \frac{2}{\log_e(3)} = 2\log_3(e) = \log_3(e^2), \quad m = e^2 \quad \text{A1}$$

c. $y = \log_3(2x - k) - k = \frac{\log_e(2x - k)}{\log_e(3)} - k$

$$\frac{dy}{dx} = \frac{2}{(2x - k)\log_e(3)} \quad \text{at } x = v$$

$$m_T = \frac{2}{(2v - k)\log_e(3)} = \tan(45^\circ) = 1$$

$$2v - k = \frac{2}{\log_e(3)} = n$$

$$v = \frac{k}{2} + \frac{1}{\log_e(3)} = \frac{k}{2} + \log_3(e) = \frac{1}{2}(k + \log_3(e^2)) \quad \text{A1}$$

d. The tangent at $x = 3$ is $y = \frac{2x}{\log_e(3)(6-k)} + \log_3(6-k) - k - \frac{6}{\log_e(3)(6-k)}$

if this passes through the origin then $\log_3(6-k) - k - \frac{6}{\log_e(3)(6-k)} = 0$

solving this for k gives $k = 0.5436$,

A1

the gradient of the line is then $\frac{2}{\log_e(3)(6-0.5436)} = 0.3336 = \tan(\theta)$

$\theta = \tan^{-1}(0.3336)$

$\theta = 18.5^\circ$

A1

Define $g(x) = \log_3(2 \cdot x - k) - k$	Done
tangentLine($g(x), x, 3$)	$\frac{(k-6) \cdot \ln(-(k-6)) - k^2 \cdot \ln(3) + 6 \cdot k \cdot \ln(3) + 6}{(k-6) \cdot \ln(3)} - \frac{2 \cdot x}{(k-6) \cdot \ln(3)}$
 solve($\frac{(k-6) \cdot \ln(-(k-6)) - k^2 \cdot \ln(3) + 6 \cdot k \cdot \ln(3) + 6}{(k-6) \cdot \ln(3)} = 0, k$)	$k = 0.5436$
tangentLine($g(x), x, 3$) $k = 0.5435713$	$0.3336 \cdot x + 3.4868E-8$
$\tan^{-1}(0.33363919027)$	18.5

e. $g: y = \log_3(2x - k) - k$ swap x and y

$g^{-1}: x = \log_3(2y - k) - k$

$x + k = \log_3(2y - k)$

$2y - k = 3^{x+k}$

M1

$y = \frac{1}{2}(3^{x+k} + k)$

$\text{dom } g = \left(\frac{k}{2}, b\right) = \text{ran } g^{-1}$

$\text{ran } g = (-\infty, \log_3(2b - k) - k) = \text{dom } g^{-1}$

$g^{-1}: (-\infty, \log_3(2b - k) - k) \rightarrow \mathbb{R}, g^{-1}(x) = \frac{1}{2}(3^{x+k} + k)$

A1

$$\text{f.} \quad g^{-1}(\log_3(2b-k)-k) = \frac{1}{2}(3^{\log_3(2b-k)-k+k} + k)$$

$$g^{-1}(\log_3(2b-k)-k) = \frac{1}{2}(3^{\log_3(2b-k)} + k)$$

$$g^{-1}(\log_3(2b-k)-k) = \frac{1}{2}(2b-k+k)$$

$$g^{-1}(\log_3(2b-k)-k) = b$$

alternatively, at the endpoint of the function

$$g(b) = \log_3(2b-k)-k \Leftrightarrow g^{-1}(\log_3(2b-k)-k) = b \quad \text{A1}$$

$$\text{g.} \quad y = \frac{1}{2}(3^{x+k} + k) = \frac{1}{2}(3^k \times 3^x + k) \text{ at } x=1 \left(1, \frac{1}{2}(3^{k+1} + k)\right)$$

$$\text{Now } \frac{d}{dx}(3^x) = \log_e(3) \times 3^x$$

$$\frac{dy}{dx} = \frac{3^k}{2} \log_e(3) \times 3^x \text{ at } x=1 \quad m_T = \frac{3^{k+1}}{2} \log_e(3)$$

$$T: y - \left(\frac{1}{2}(3^{k+1} + k)\right) = \frac{3^{k+1}}{2} \log_e(3)(x-1) \quad \text{M1}$$

$$y = \frac{3^{k+1}}{2} \log_e(3)x + \frac{1}{2}(3^{k+1} + k) - \frac{3^{k+1}}{2} \log_e(3)$$

$$\text{if this tangent passes through the origin then } \frac{1}{2}(3^{k+1} + k) - \frac{3^{k+1}}{2} \log_e(3) = 0$$

$$\text{solving this for } k \text{ gives } k = 0.529, 1.4415 \quad \text{A1}$$

$$\text{the gradient of the line is } m = \frac{3^{k+1}}{2} \log_e(3)$$

$$\text{when } k = 0.529, m = \frac{3^{0.529+1}}{2} \log_e(3) = 2.9466, \theta = \tan^{-1}(2.9466)$$

$$\text{when } k = 1.4415, m = \frac{3^{1.4415+1}}{2} \log_e(3) = 8.029, \theta = \tan^{-1}(8.029)$$

$$\theta = 71.3^\circ, 82.9^\circ \quad \text{A1}$$

both answers are acceptable.

Define $gi(x) = \frac{1}{2} \cdot (3^{x+k} + k)$

Done

tangentLine($gi(x), x, 1$)

$$\frac{3 \cdot 3^k \cdot \ln(3) \cdot x}{2} - \frac{3 \cdot 3^k \cdot (\ln(3) - 1) - k}{2}$$

⚠ solve($\frac{3 \cdot 3^k \cdot (\ln(3) - 1) - k}{2} = 0, k$)

$k = 0.5290$ or $k = 1.4415$

$\frac{3 \cdot 3^k \cdot \ln(3)}{2} |_{k=0.528981867}$

2.9466

$\tan^{-1}(2.94662)$

71.3

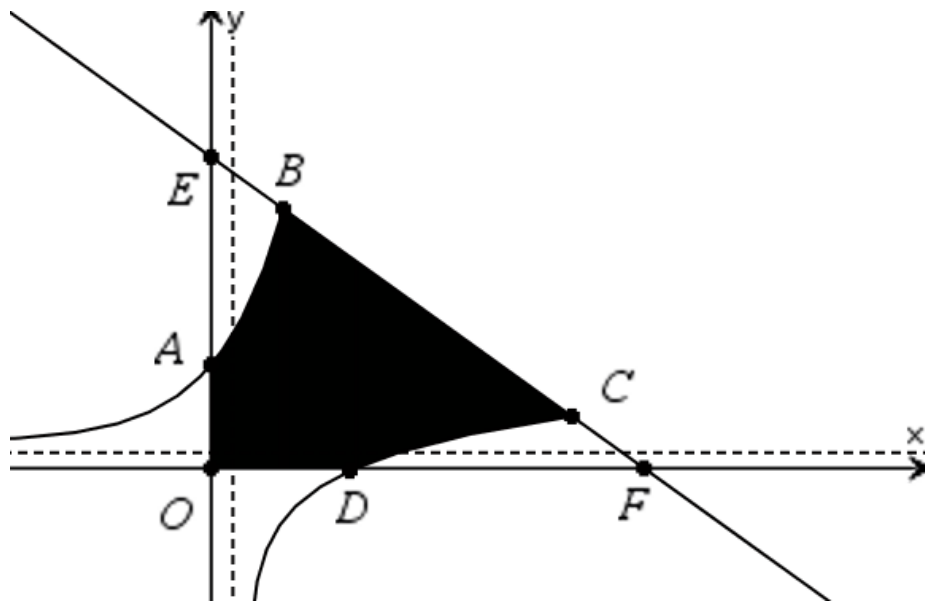
$\frac{3 \cdot 3^k \cdot \ln(3)}{2} |_{k=1.441454872}$

8.0294

$\tan^{-1}(8.02942542)$

82.9

h.



The graph of the function $g(x) = \log_3(2x - k) - k$ has a vertical asymptote at $x = \frac{k}{2}$

it crosses the x-axis when $\log_3(2x - k) - k = 0$, $2x - k = 3^k$, $x = \frac{1}{2}(3^k + k)$

at the point $D\left(\frac{1}{2}(3^k + k), 0\right)$ and has its endpoint at the point, $C(b, g(b))$

$C(b, \log_e(2b - k) - k)$, so we require $b > \frac{1}{2}(3^k + k)$

The graph of the function $g^{-1}(x) = \frac{1}{2}(3^{x+k} + k)$ has a horizontal asymptote at $y = \frac{k}{2}$, it crosses the y -axis at the point $A\left(0, \frac{1}{2}(3^k + k)\right)$ and has its endpoint at the point $B(\log_3(2b-k)-k, b)$. A1

Both graphs are symmetrical about the line $y = x$, which has a gradient of 1, the line perpendicular to this line has a gradient of -1 and has an equation of the form $y = w - x$, now since it must pass through the endpoints of both functions, that is points C and B , $b = w - (\log_3(2b-k)-k)$ so that $w = c = b - k + \log_3(2b-k)$.

The line passing $y = b - k + \log_3(2b-k) - x = c - x$ crosses the x -axis at F and

crosses the y -axis at E , $E(0, c)$, $F(c, 0)$, $c > b > \frac{1}{2}(3^k + k)$. A1

- i. The two non-shaded areas bounded by the coordinates axes are equal, the shaded area is therefore, the area of the triangle OEF , minus twice the area bounded by the graph of $y = g(x)$ between the point D and the line $x = b$, (the line through C) and twice the area bounded by the graph of $y = c - x$ between the line at $x = b$ and the point F .

Let $A_1 = \int_{\frac{1}{2}(3^k+k)}^b g(x) dx$ and $A_2 = \int_b^c (c-x) dx$.

Now $A_2 = \int_b^c (c-x) dx = \left[cx - \frac{x^2}{2} \right]_b^c = \left(c^2 - \frac{c^2}{2} \right) - \left(bc - \frac{b^2}{2} \right) = \frac{1}{2}(c^2 + b^2) - bc$ A1

The area of the triangle OEF is $\frac{1}{2}c^2$, so the shaded area is

$$A = \frac{1}{2}c^2 - 2(A_1 + A_2) = \frac{1}{2}c^2 - 2A_1 - 2A_2$$

$$A = \frac{1}{2}c^2 - 2 \int_{\frac{1}{2}(3^k+k)}^b g(x) dx - 2 \left(\frac{1}{2}(c^2 + b^2) - bc \right) = \frac{c^2}{2} - (c^2 + b^2) + 2bc - 2 \int_{\frac{1}{2}(3^k+k)}^b g(x) dx$$

$$A = 2bc - b^2 - \frac{c^2}{2} - 2 \int_{\frac{1}{2}(3^k+k)}^b g(x) dx, \quad R = 2bc - b^2 - \frac{c^2}{2} \quad \text{A1}$$

END OF SECTION B SUGGESTED ANSWERS

End of detailed answers for the
2021 Kilbaha VCE Mathematical Methods Trial Examination 2

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