

2021 VCE Mathematical Methods Trial Examination 1



Kilbaha Education

Quality educational content

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Victorian Certificate of Education 2021

STUDENT NUMBER

Figures
Words

Letter

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MATHEMATICAL METHODS

Trial Written Examination 1

Reading time: 15 minutes

Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software) notes of any kind, blank sheets of paper, and/or correction fluid/tape.

Materials supplied

- Question and answer book of 18 pages.
- Detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (2 + 1 = 3 marks)

- a. For $y = \log_e(\tan(3x))$, find $\frac{dy}{dx}$ in simplified form.

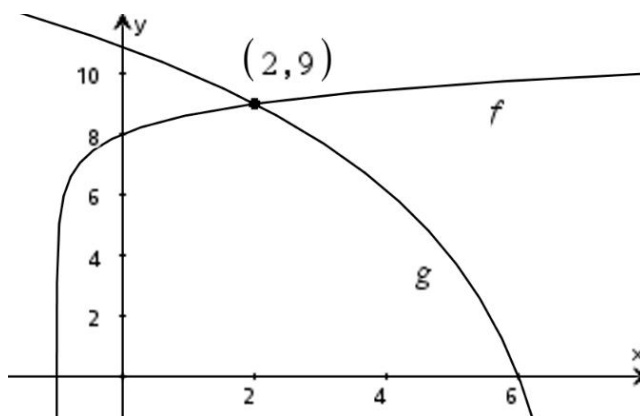
2 marks

- b. Hence find an antiderivative of $\frac{1}{\sin(3x)\cos(3x)}$.

1 mark

Question 3 (4 marks)

Part of the graphs of $f(x) = \log_3(x+a) + b$ and $g(x) = p \log_5(q-x)$ are shown. The graph of $y = f(x)$ crosses the y -axis at $y = 8$, and the graph of $y = g(x)$ crosses the x -axis at $x = 6$. The two curves intersect at the point $(2, 9)$. Determine the values of a , b , p and q .



Consider the function $f(x) = \log_e(kx) - \frac{kx}{2x+3}$ where $k \in \mathbb{R} \setminus \{0\}$.

[illegible]

Question 5 (5 marks)

- a. A veterinarian clinic employs six male and three female veterinarians. On a particular day, three veterinarians are working. Let $\hat{P}$ represent the sample proportion of female veterinarians working on a particular day.

- i. What values can $\hat{P}$ take?

1 mark

- ii. Find $\Pr\left(\hat{P} \geq \frac{1}{2}\right)$.

2 marks

- b. The veterinarian clinic has a large number of registered animals coming to the surgery. A random sample of animals from the surgery is selected, and an approximate 95% confidence interval for p , the proportion of animals which are dogs that attend the surgery was determined to be $\left(\frac{316}{625}, \frac{484}{625}\right)$. Using $z = 1.96 = \frac{49}{25}$, find the sample size from which this approximate 95% confidence interval was obtained.

2 marks

Question 6 (5 marks)

Consider the function $f : D \rightarrow R$, $f(x) = \sqrt{9 - 3x}$

- a.** Find the maximal domain of f , that is find D .

1 mark

- b.** Find the inverse function, f^{-1} .

2 marks

- c. Find the coordinates of **all** the points of intersection between the graphs of $y = f(x)$ and $y = f^{-1}(x)$.

2 marks

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

a. Let $a > 0$, $b > 0$ and let $f(x)$ be an increasing function, such that $f(0) = 0$ and $f(a) = b$. Explain why $\int_0^a f(x) dx = ab - \int_0^b f^{-1}(x) dx$.

3 marks

[illegible]

Question 8 (3 marks)

The probability distribution for the continuous random variable X is given by

$$f(x) = \begin{cases} \frac{k}{\sqrt{16-3x}} & 0 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

Find the value of k .

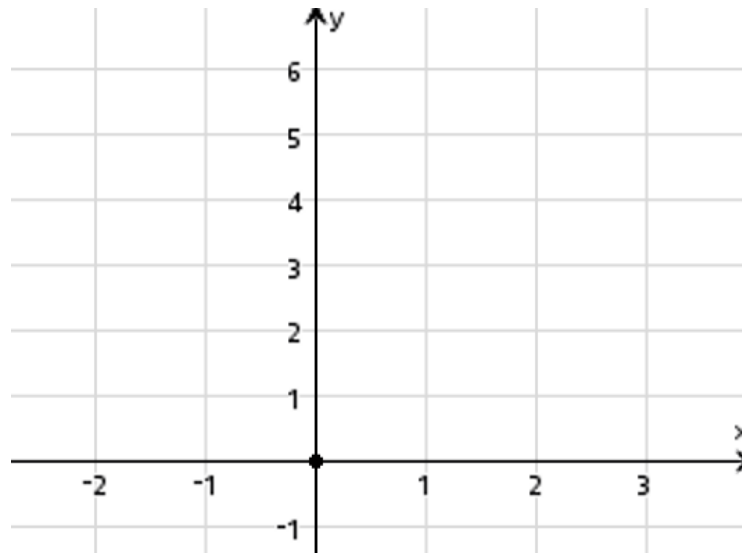
[illegible]

Question 9 (9 marks)

Consider the function $f : [0, \infty) \rightarrow \mathbb{R}$, $f(x) = 4e^{-2x}$

- a. Sketch the graph of the function on the axes below, stating the coordinates of any axial intercepts and equations of any asymptotes.

1 mark



- b. Over the interval $1 \leq x \leq 3$, find the mean value of the function.

1 mark

- c. Find the equation of the tangent to the graph of the function f at the point P where $x = u$. Give your answer in the form $y = mx + c$.

2 marks

- d. This tangent crosses the x -axis at the point Q , and the y -axis at the point R . Write down the coordinates of the points Q and R .

1 mark

- e. If O is the origin, show that the area A of the triangle OQR is given by

$$A(u) = (2u + 1)^2 e^{-2u}$$

1 mark

3 marks

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[illegible]

MATHEMATICAL METHODS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of triangle	$\frac{1}{2}bc \sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$
$\frac{d}{dx}((ax+b)^n) = na(ax+b)^{n-1}$	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e(x) + c, x > 0$
$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$	$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$	
product rule $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$	quotient rule $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
chain rule $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$	

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A B) = \frac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = E(X)$	variance	$\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x p(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x) dx$	$\mu = \int_{-\infty}^{\infty} x f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$

Sample proportions

$\hat{p} = \frac{X}{n}$		mean	$E(\hat{P}) = p$
standard deviation	$\text{sd}(\hat{P}) = \sqrt{\frac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right)$

END OF FORMULA SHEET