



Online & home tutors Registered business name: itute ABN: 96 297 924 083

Mathematical Methods

2021

Trial Examination 1 (1 hour)

Instructions

Answer **all** questions.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Question 1

Solve the following equations for x .

a. $(x-1)^{\frac{2}{3}} - (x-2)^{\frac{2}{3}} = 0$ 2 marks

b. $e^{4x-4} = 2e^{2x-2} + 3$ 2 marks

Question 2

Consider the rule of function f , $f(x) = \frac{(x-1)(x^3 + 2x^2 + 2x + 1)}{x^2 - 1}$.

a. Express the rule in simplest form. 2 marks

b. Find $f'(x)$. 1 mark

c. Sketch the graph of f in the interval $[-2, 3]$. Label typical points with coordinates.

3 marks

Question 3

Random variable X has probability distribution $\text{Bi}(n, p)$ and $\Pr(X = 1) = (1 - p)^{n-2}$, $p < 1$. Find the possible values of n and the value(s) of p in terms of n .

3 marks

Question 4

Five samples of size n are taken from a large population. The 70% and 90% confidence intervals for sample proportion $\hat{P}$ are $(0.3896, 0.4104)$ and $(0.3836, 0.4164)$ respectively.

a. Find $\Pr(0.3836 < \hat{P} < 0.4104 \mid \hat{P} > 0.3836)$. 2 marks

b. Given $\Pr(-1.04 < Z < 1.04) \approx 0.70$, estimate the value of n to the nearest hundred.

2 marks

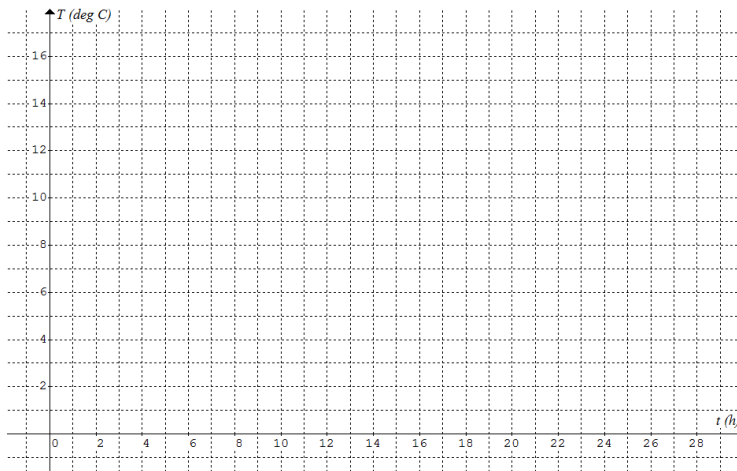
Question 5

The outdoor air temperature (in $^{\circ}\text{C}$) over a period of 24 hours is given by

$$T(t) = 8 - 5 \sin\left(\pi\left(\frac{t}{12} - \frac{1}{6}\right)\right), \quad t \in [0, 24].$$

- a. Sketch a graph of the outdoor temperature $T(t)$.

2 marks



- b. Determine the average temperature over the 24 hour period.

1 mark

- c. Calculate the average rate of change in temperature from its minimum to its maximum.

1 mark

- d. Over the 24 hour period, $T > \alpha$ for 20 hours. Determine the value of α .

2 marks

- e. If you put your clock forward an hour for daylight saving, T in terms of daylight saving time t over the same 24 hour period can be expressed as $T(t) = 8 - 5 \sin\left(\pi\left(\frac{t}{12} + b\right)\right)$.

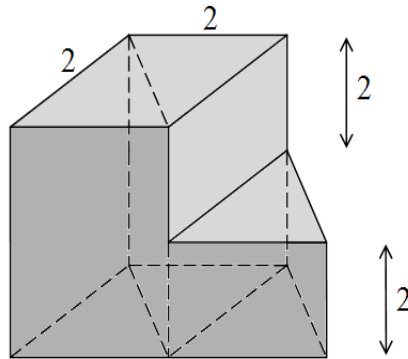
Find the value of b .

2 marks

Question 6

The following diagram shows a large water tank with one section on top of a larger section. Both sections have the same height of 2 metres.

The top and the base planes are parallel, and they consist of two and three congruent equilateral triangles respectively. The area of each triangle is $\sqrt{3} \text{ m}^2$.



Initially ($t = 0$) the tank is full. At time $t \geq 0$ seconds the depth of water is h metres.

a. Write a piecewise function for $V(h) \text{ m}^3$ in simplest factorised form. 2 marks

b. Water is drained at a constant rate of $\frac{1}{500} \text{ m}^3$ per second.

Find $\frac{dh}{dt}$ in m s^{-1} when $h > 2$.

2 marks

Question 7

Given $f(x) = \log_e x$ and $g(x) = \sqrt{ax+b}$ such that $f(e) = g(e)$ and $f'(e) = g'(e)$,

a. find the values of a and b

2 marks

b. and show that $g'(x) > f'(x)$ for $x \in \left(\frac{e}{2}, e\right) \cup (e, \infty)$.

3 marks

Question 8

Function $f(x)$ is odd, periodic and differentiable for $x \in \mathbb{R}$.

The period of f is 7, $f(-2) = 2$ and $f'(-2) = \frac{1}{3}$.

a. Find the value of $f(-5)$.

1 mark

b. Find the value of $f'(-5)$.

1 mark

c. Find the value of $\int_{-5}^9 f(x) dx$.

1 mark

Question 9

Let $f(x) = e^x + \log_e x$, $b = f(a)$ and $g(x) = f^{-1}(x)$.

- a. State the maximal domain of $f(x)$. 1 mark
- b. Find $g'(b)$ in terms of a . 2 marks

End of Exam