

Student Name: _____



MATHEMATICAL METHODS 2020

Unit 3

Key Topic Test 8 – Transformations Technology Active

Recommended writing time: 45 minutes

Total number of marks available: 30 marks

QUESTION BOOK

* The recommended writing time is a guide to the time students should take to complete this test. Teachers may wish to alter this time and can do so at their own discretion.

Conditions and restrictions

- Students are permitted to bring into the room for this test: a bound reference book, CAS calculator, pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the room for this test: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 9 pages.

Instructions

- Print your name in the space provided on the top of the front page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the room for this test.

SECTION A – Multiple-choice questions

Instructions for Section A

Answer all questions in pencil on the answer sheet provided for multiple-choice questions. Choose the response that is correct for the question.

A correct answer scores 1; an incorrect answer scores 0.

Marks will not be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this book are not drawn to scale

Question 1

The graph $y = x^2$ is transformed by a **dilation** of factor 2 from the x -axis, a **reflection** in the x -axis and a **translation** of 4 units in the positive x direction and of 7 units in the positive y direction.

After the transformations the resulting equation would be:

- A. $y = -2(x - 4)^2 + 7$
- B. $y = (-2x + 8)^2 + 7$
- C. $y = -2(x + 4)^2 - 7$
- D. $y = -2(x + 4)^2 + 7$
- E. $y = 2(-x + 4)^2 - 7$

Question 2

The graph of $y = x^2 - 4x + 3$ is dilated by a factor of 2 from the y -axis, then translated 2 units to the right and 1 unit down. The turning point of the transformed graph has the coordinate:

- A. $(2, -1)$
- B. $(6, -2)$
- C. $(6, -1)$
- D. $(4, -3)$
- E. $(2, -2)$

Question 3

The **vertical asymptote** of the graph of $y = \frac{2}{x-3} + 1$ after it undergoes a dilation factor 2 from the x-axis and a translation 3 units to the right is:

- A. $x = 3$
- B. $x = 9$
- C. $x = 6$
- D. $x = 4.5$
- E. $y = 1$

Question 4

The transformation matrix $\begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$ represents

- A. a dilation by factor 2 from the y-axis and reflection in the y-axis
- B. a dilation by factor 2 from the x-axis and reflection in the x-axis
- C. a dilation by factor $\frac{1}{2}$ from the x-axis and reflection in the y-axis
- D. a dilation by factor 2 from the x-axis and reflection in the y-axis
- E. a dilation by factor $\frac{1}{2}$ from the y-axis and reflection in the x-axis

Question 5

The graph of $f(x) = \frac{2}{x}$ is transformed to graph of $g(x) = \frac{2}{3x-2} + 1$. One sequence of transformations that transform $f(x)$ to $g(x)$ are

- A. dilation by factor 3 from the x-axis, translation of $\frac{2}{3}$ units to the right
- B. dilation by factor 3 from the y-axis, translation of $\frac{2}{3}$ units to the left and translation 1 unit up
- C. dilation of $\frac{1}{3}$ from the y-axis, translation of 2 units to the right and translation of 1 unit up
- D. dilation of $\frac{1}{3}$ from the y-axis, translation of $\frac{2}{3}$ units to the right and translation of 1 unit up
- E. dilation of $\frac{1}{3}$ from the y-axis, translation of 2 units to the left and translation of 1 unit up

Question 6

The following transformation matrix describes $g(x)$ the image of $f(x) = \frac{1}{x}$.

$$\begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 5 \end{bmatrix}$$

The equation for $g(x)$ is:

- A. $g(x) = \frac{1}{x-1} + 5$
- B. $g(x) = -\frac{2}{x-1} + 5$
- C. $g(x) = \frac{1}{2(1-x)} + 5$
- D. $g(x) = -\frac{1}{2(x-1)}$
- E. $g(x) = -\frac{5}{2(x-1)}$

Question 7

A circle with radius 1 and centred at the origin is translated 4 units to the left and 2 units up. The resulting equation is:

- A. $x^2 + y^2 = 1$
- B. $(x + 4)^2 + (y + 2)^2 = 1$
- C. $(x - 4)^2 + (y + 2)^2 = 1$
- D. $(x - 4)^2 + (y - 2)^2 = 1$
- E. $(x + 4)^2 + (y - 2)^2 = 1$

Question 8

The transformation that maps the graph of $y = \sqrt{27x^3 - 1}$ onto the graph of $y = \sqrt{x^3 - 1}$ is a

- A. Dilation by a factor 3 from the y-axis
- B. Dilation by a factor 3 from the x-axis
- C. Dilation by a factor $\frac{1}{3}$ from the y-axis
- D. Dilation by a factor $\frac{1}{3}$ from the x-axis
- E. Dilation by a factor $\frac{1}{27}$ from the y-axis

SECTION B – Extended Response questions**Instructions**

Answer all questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working must be shown. Unless otherwise indicated, the diagrams in this book are not drawn to scale.

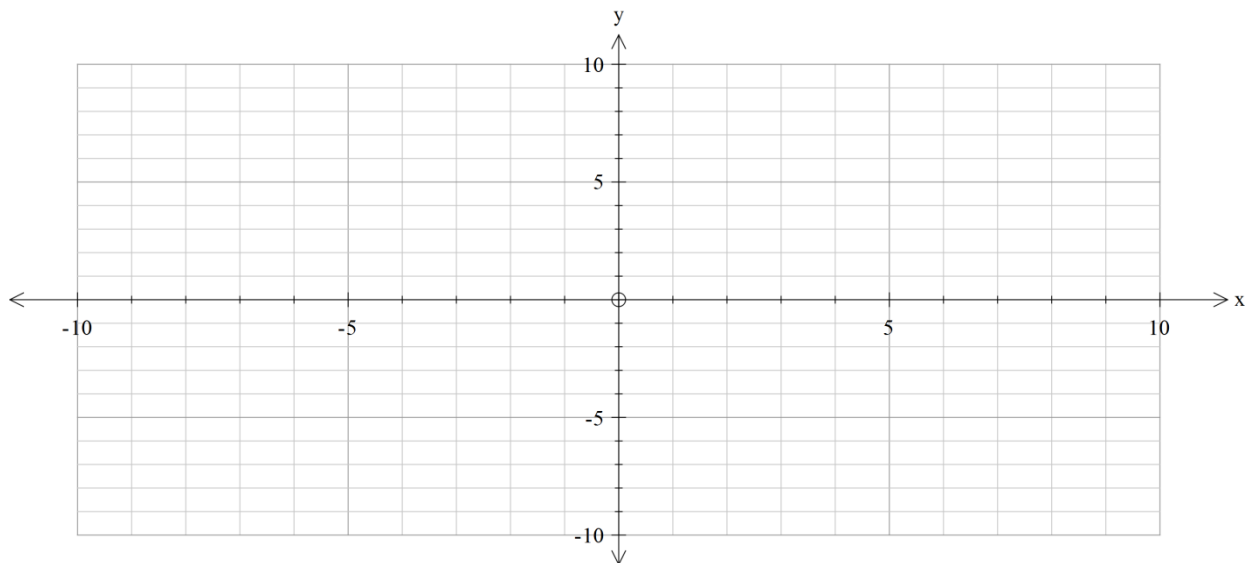
Question 1 (9 marks)

Consider the function $f(x) = \frac{x+2}{x-2}$

- a. Write the function in the form $f(x) = \frac{a}{x-2} + b$

2 marks

- b. Sketch the graph of $y = f(x)$, labelling all asymptotes and intercepts.



3 marks

- c. On the same set of axes, sketch the image of $f(x)$ after a reflection in the y-axis and translation 2 units in the positive x-direction.

2 marks

- d. State the equation of the transformed function.

2 marks

Question 2 (7 marks)

The graph of the function $f(x) = \frac{1}{x^2}$ undergoes the following transformations

- Dilation by factor 3 from the y-axis
- Dilation by factor 2 from the x-axis
- Reflection in y-axis
- Translation of 4 units to the left and 3 units up

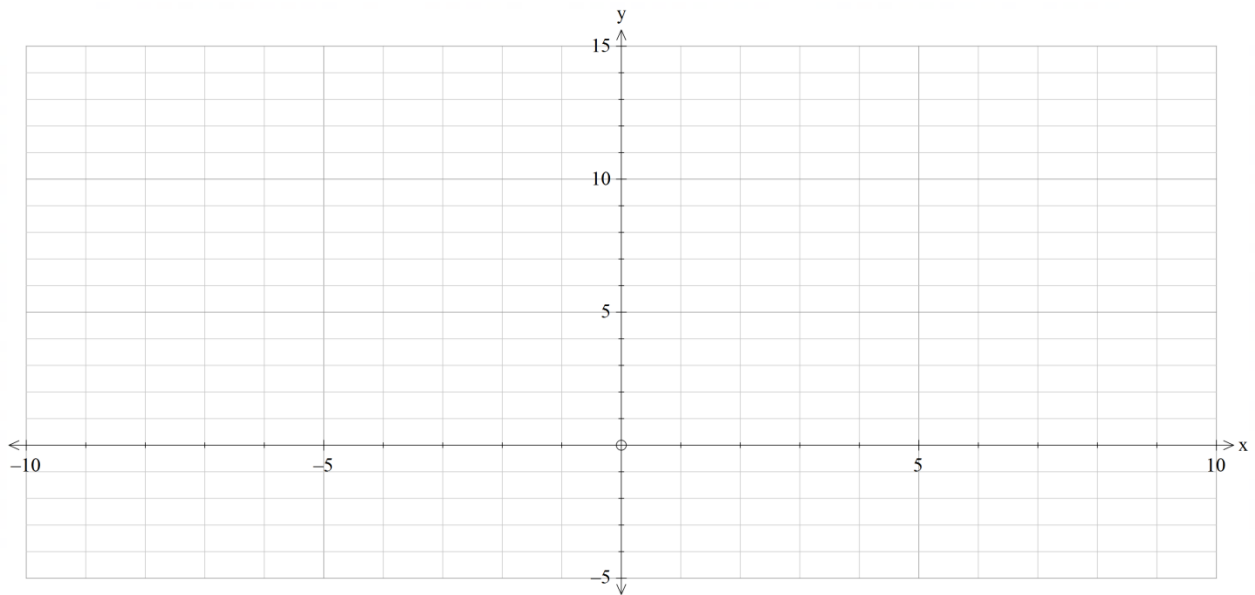
- a. Apply the transformations, in sequence, to find the rule for the transformed function

2 marks

- b.** Find the domain and range of the transformed function.

2 marks

- c.** Sketch the transformed function on the axes below.



3 marks

Question 3 (6 marks)

The graph of the function $f(x) = \frac{1}{2} - 4(x - 5)^2$ is defined over $x \in [5, \infty)$.

- a. State the transformations, in sequence, that transform the graph of $y = x^2$ to the graph of $y = f(x)$.

3 marks

- b. Find the rule for the inverse of the function $f(x)$ and state the domain of this function.

3 marks

END OF KEY TOPIC TEST