

Student Name: _____



MATHEMATICAL METHODS 2020

Unit 3

Key Topic Test 6 –Exponential & Logarithmic Functions Technology Active

Recommended writing time: 45 minutes

Total number of marks available: 30 marks

QUESTION BOOK

* The recommended writing time is a guide to the time students should take to complete this test. Teachers may wish to alter this time and can do so at their own discretion.

Conditions and restrictions

- Students are permitted to bring into the room for this test: a bound reference book, CAS calculator, pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the room for this test: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 10 pages.

Instructions

- Print your name in the space provided on the top of the front page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the room for this test.

SECTION A – Multiple-choice questions

Instructions for Section A

Answer all questions in pencil on the answer sheet provided for multiple-choice questions. Choose the response that is correct for the question.

A correct answer scores 1; an incorrect answer scores 0.

Marks will not be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this book are not drawn to scale

Question 1

The domain of the function $f(x) = \log_e(x - 3) + \log_e(10 - x) - 4$ is:

- A. R
- B. $(3, 10)$
- C. $(3, \infty)$
- D. $(-\infty, 10)$
- E. $[3, 10]$

Question 2

Which of the following is not true about the function $f(x) = 2e^{-x} + 1$?

- A. The graph of the function will have an asymptote with equation $y = 1$
- B. The graph of the function will pass through the point $(0, 3)$
- C. As $x \rightarrow \infty, f(x) \rightarrow \infty$
- D. The graph will not have an x-intercept
- E. The range is $(1, \infty)$

Question 3

$81^m \times 9^n$ is equal to;

- A. 3^{8mn}
- B. 3^{4m+2n}
- C. 3^{6mn}
- D. 9^{4m+2n}
- E. 3^{6m}

Question 4

If $2\log_b(x^2) = \log_b 9 + 4$, then x could be equal to

- A. $\sqrt{3}b$
- B. $3b$
- C. $9b$
- D. $\sqrt{3}$
- E. 36

Question 5

For the function $f(x) = \log_e(2 - x)$, the inverse function $f^{-1}(x)$ and its range are:

- A. $f^{-1}(x) = e^x, R$
- B. $f^{-1}(x) = -e^{x+2}, (-\infty, 2]$
- C. $f^{-1}(x) = -e^x - 2, (-\infty, 2]$
- D. $f^{-1}(x) = 2 - e^x, (-\infty, 2)$
- E. $f^{-1}(x) = 2 - e^x, (2, \infty)$

Question 6

If $x = 2$ is a solution to $\log_{10}(mx - 5) = 2$, then m is equal to:

- A. 100
- B. $105m$
- C. $\frac{105}{2}$
- D. $\log_{10}(105)$
- E. $\log_{10}2 + 5$

Question 7

The solution to the equation $2x = 10^{-0.1x}$ is closest to:

- A. 0.40
- B. 0.45
- C. 0.82
- D. 0.31
- E. 0.36

Question 8

$2\log_2 m + \log_2 n - \log_2(m - n)$ can be simplified to:

- A. $\frac{\log_2 m^2 n}{\log_2(m - n)}$
- B. $2\log_2\left(\frac{m^2 n}{m - n}\right)$
- C. $\log_2(m^2 - m + 2n)$
- D. $\log_2\left(\frac{m^2 n}{m - n}\right)$
- E. $\log_2\left(\frac{2mn}{m - n}\right)$

SECTION B – Extended Response questions**Instructions**

Answer all questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working must be shown. Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Question 1 (8 marks)

After moving house, a refrigerator is switched on and the temperature inside is monitored. The temperature, T , inside the refrigerator in degrees Celsius at time t minutes after the refrigerator is switched on, is given by

$$T(t) = 22 \times 10^{-0.01t}, t \in [0, 48]$$

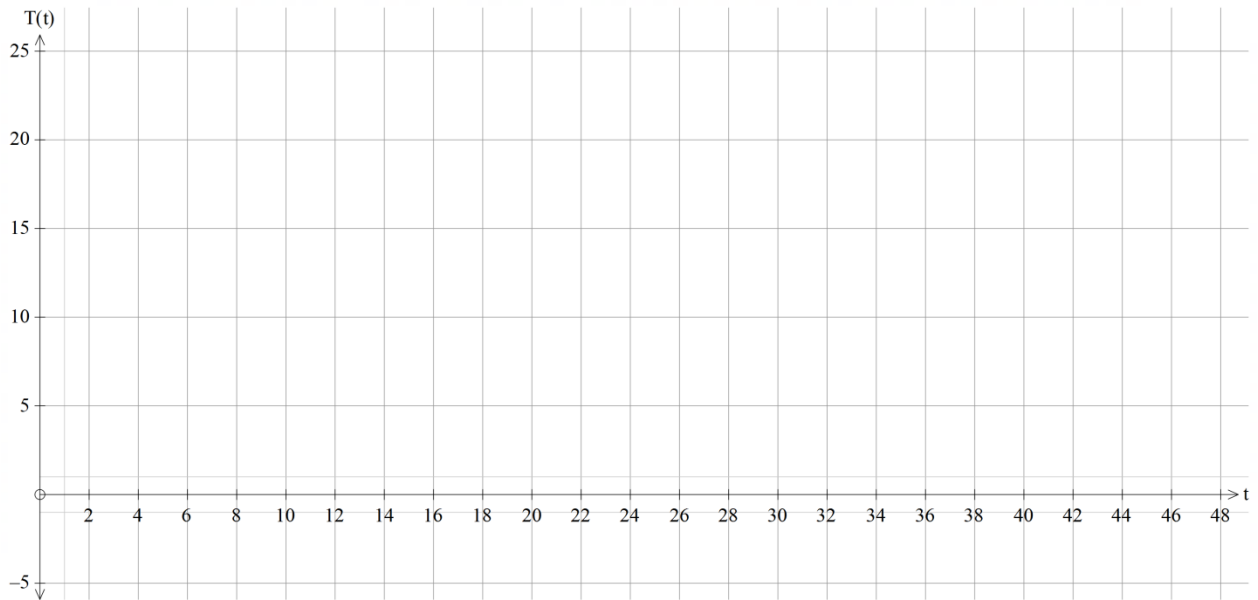
- a. Find the temperature inside the refrigerator when it is turned on.

1 mark

- b. Find the time when the temperature inside the refrigerator first reaches 10°C . Answer to the nearest minute.

1 mark

- c. Sketch the graph of $T(t) = 22 \times 10^{-0.01t}$, $t \in [0, 48]$ on the set of axes below, clearly marking any intercepts and/or endpoints correct to 4 decimal places.



2 marks

If instead, after 20 hours, ice is added to the fridge to speed up the cooling process. The temperature inside the fridge now after 20 hours is given by;

$$T_2(t) = a \times 10^{\frac{4}{5} + k(t-20)}, t \in [20, 48]$$

- d. If there is no immediate drop in temperature when the ice is added, show that $a = \frac{11}{5}$.

2 marks

- e. If the temperature inside the fridge reaches 1°C 24 hours after it is first switched on, find the value of k . Give your answer correct to 4 decimal places.

2 marks

Question 2 (7 marks)

The number of bacteria in a jar after t minutes is given by $n(t) = A(1 - e^{-kt})$ where A and k are constants, and $k > 0$.

When $t = 1, n = 1000$ and when $t = 2, n = 120\,000$.

- a. Show that $119 + e^{-2k} - 120e^{-k} = 0$

2 marks

- b. Show algebraically that $k = -\log_e(119)$

3 marks

- c. Find the exact value of A

1 mark

- d. After how many hours is the number of bacteria 2 000 000 ? Answer to 4 decimal places.

1 mark

Question 3 (7 marks)

The population of Cheetahs, C , in a zoo is given by $C(t) = 20(10^{0.2t})$, where t is the number of years since the park opened in 2000.

The population of Snow Leopards, S , is given by $S(t) = 22.5(10^{0.05t})$.

- a. What is the initial population of Cheetahs and Snow Leopards? (Round your answer to the nearest integer)

1 mark

- b. Find the population of each species (to the nearest integer) after 18 months

1 mark

- c. Show that the population of Snow Leopards and Cheetahs is equal at $t = \frac{20}{3} \log_{10}\left(\frac{9}{8}\right)$

3 marks

- d. If $d(t)$ is the difference in population of the two animals over first 2 years, when is $d(t)$ a maximum?

2 marks

END OF KEY TOPIC TEST