



MATHEMATICAL METHODS 2020

Unit 3

Key Topic Test 6 – Exponential & Logarithmic Functions Technology Active

Recommended writing time*: 45 minutes

Total number of marks available: 30 marks

SOLUTIONS

Section A: Multiple Choice

Question 1

Answer: B

$$(3, \infty) \cap (-\infty, 10) \\ (3, 10)$$

Question 2

Answer: C

$$\text{As } x \rightarrow \infty, f(x) \rightarrow 1$$

Question 3

Answer: B

$$81^m \times 9^n \\ = 3^{4m} \times 3^{2n} \\ = 3^{4m+2n}$$

Question 4

Answer: A

$$2\log_b(x^2) = \log_b 9 + 4 \\ \log_b x^4 = \log_b 9 + \log_b b^4 \\ \log_b x^4 = \log_b 9b^4 \\ x^4 = 9b^4 \\ x = \sqrt[4]{9}b \quad (\text{Assume } b \text{ must be positive.})$$

Question 5

Answer: D

$$\text{Domain } f(x) \quad (-\infty, 2) \\ f^{-1}(x) = 2 - e^{-x}$$

Question 6

Answer: C

$$10^2 = mx - 5$$

$$105 = mx$$

$$x = \frac{105}{m}$$

$$2 = \frac{105}{m}$$

$$m = \frac{105}{2}$$

Question 7

Answer: B

CAS solve

Question 8

Answer: D

$$2\log_2 m + \log_2 n - \log_2(m - n)$$

$$= \log_2 m^2 + \log_2 n - \log_2(m - n)$$

$$= \log_2 \frac{m^2 n}{(m - n)}$$

Section B: Extended Response**Question 1**

a. $T(0) = 22^\circ\text{C}$

1 mark

b. $10 = 22 \times 10^{-0.01t}$

$t \approx 34 \text{ hours } 15 \text{ minutes}$

1 mark

c.



1 mark endpoints
1 mark shape

d. $T(20) = \frac{11}{5} \times 10^{\frac{4}{5}}$

1 mark

$T_1(20) = \frac{11}{5} \times 10^{\frac{4}{5}} \quad \text{also}$

$T_1(t) = a \times 10^{\frac{4}{5} + k(t-20)}$

$T_1(20) = a \times 10^{\frac{4}{5}}$

$a = \frac{11}{5}$

1 mark

e. $T_2(24) = 1$

1 mark

$k = -0.2856$

1 mark

Question 2

a. $1000 = A(1 - e^{-k})$ (1)

$120\,000 = A(1 - e^{-2k})$ (2)

1 mark

$$\frac{(2)}{(1)} \quad 120 = \frac{1 - e^{-2k}}{1 - e^{-k}}$$

1 mark

$$120 - 120e^{-k} = 1 - e^{-2k}$$

$$119 - 120e^{-k} + e^{-2k} = 0$$

1 mark

b. $119 - 120e^{-k} + e^{-2k} = 0$

Let $a = e^{-k}$

1 mark

$$119 - 120a + a^2 = 0$$

$$a = 1, 119$$

1 mark

$$e^{-k} = 1, k = 0, \text{ disregard as } k > 0$$

$$e^{-k} = 119$$

$$k = -\log_e(119)$$

1 mark

c. $1000 = A(1 - e^{\log_e(119)})$

$$1000 = A(1 - 119)$$

$$A = -\frac{500}{59}$$

1 mark

d. $2\,000\,000 = -\frac{500}{59}(1 - e^{\log_e 119 \times t})$

$$t = 2.5887 \text{ hours}$$

1 mark

Question 3

a. $C(0) = 20$

$$S(0) = 22.5 \approx 23$$

1 mark

b. $C(1.5) \approx 40$

$$S(1.5) \approx 27$$

1 mark

c. $20 \times 10^{0.2t} = 22.5 \times 10^{0.05t}$

$$10^{0.15t} = \frac{9}{8}$$

1 mark

$$0.15t = \log_{10}\left(\frac{9}{8}\right)$$

1 mark

$$t = \frac{20}{3} \log_{10}\left(\frac{9}{8}\right)$$

1 mark

d. $d(t) = |C(t) - S(t)|$ 1 mark
 $d(0) = 2.5$
 $d(2) \approx 21.91 \dots$

$d(t)$ is greatest after 2 years 1 mark