



THE SCHOOL FOR EXCELLENCE (TSFX)

UNIT 3 & 4 MATHEMATICAL METHODS 2020

WRITTEN EXAMINATION 1

Reading Time: 15 minutes

Writing Time: 1 hour

QUESTION AND ANSWER BOOKLET

Student Name:

Structure of Booklet

Number of questions	Number of questions to be answered	Number of marks
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are **NOT** permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials Supplied

- Question and answer book of 11 pages.
- Working space is provided throughout the book.

Instructions

- Write your **name** in the space provided above on this page.
- All written responses must be in English.

Students are **NOT** permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Note: This examination was written for the revised 2020 VCE Mathematics Study Design in which significant deletions of content to Mathematical Methods Area of Study 4 (Probability and Statistics) were made by VCAA.

Answer all questions in the spaces provided.

In questions where more than 1 mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Let $h: (-\infty, a) \rightarrow \mathbb{R}$, $h(x) = \frac{\cos(\pi x)}{\log_e(-3x)}$ and a is a real constant.

- 2 marks

[illegible]

- 1 mark

Question 2 (4 marks)

Let $f : D \rightarrow \mathbb{R}$, $f(x) = \frac{1}{2} + \frac{1}{\sqrt{2-5x}}$ and D is the maximal domain of f .

a. Find $f'(x)$.

1 mark

b. If $g(x)$ is an anti-derivative of $f(x)$ and $g(0) = \sqrt{2}$, find $g(x)$.

3 marks

Question 3 (5 marks)

Let $f: \left(-\infty, \frac{1}{2}\right) \rightarrow \mathbb{R}$, $f(x) = x^2 - 2x$.

- a. Find the rule of f^{-1} .

2 marks

- b. State the domain of f^{-1} .

1 mark

- c. Let g be the function obtained by applying the transformation T to the function f where

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ c \end{bmatrix}$$

and $a, b, c \in \mathbb{R}$.

Find the values of a , b and c given that $g(x) = -2x^2 + 8x - 1$.

2 marks

Question 4 (3 marks)

Let the functions $f : D \rightarrow R$, $f(x) = x^2 - 1$ and $g : (-\infty, 3] \rightarrow R$, $g(x) = \log_e(8 - x)$.

- a.** Find the rule for the function h where $h(x) = g(f(x))$. 1 mark

- b.** Find the maximal domain D for which h exists. 2 marks

Question 5 (3 marks)

Let $f: [-1, 1] \rightarrow \mathbb{R}$, $f(x) = -2x^3 + ax^2 + bx$ where a and b are real constants.

The average rate of change of f over its domain is -3 and the average value of f is -6 .

Find the values of a and b .

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Question 6 (2 marks)

Let X be a normally distributed random variable with mean 24 and standard deviation 5.

Find the value of b if $\Pr(X < b) = \Pr\left(Z > \frac{b}{4}\right)$ where Z is the standard normal random variable.

Question 7 (4 marks)

The Acme Battery Company makes batteries that are sold in packets of 16. The probability that a battery will last for more than 100 hours is $\frac{11}{12}$.

- a. Find the probability that at least 15 batteries in a packet will last for more than 100 hours. Express your answer in the form $\frac{a}{4}\left(\frac{b}{12}\right)^n$ where a , b and n are integers. 2 marks

James buys a packet of batteries made by the Acme Battery Company. The first battery he uses lasts for more than 100 hours.

- b. Find the probability that at least 15 batteries in the packet bought by John last for more than 100 hours. Express your answer in the form $\frac{k(11)^p}{12^m - 1}$ where k , p and m are integers. 2 marks

Question 8 (4 marks)

- a.** Find the derivative of $x^2 \log_e(x)$.

1 mark

- b.** Hence find the area bounded by curve $y = x \log_e(x)$ and the x -axis between $x = 1$ and $x = e$.

3 marks

[illegible]

Question 9 (6 marks)

Let $f(\theta) = 2\cos^2(\theta)$ and $g(\theta) = 3\sin(\theta)$.

a. Solve $f(\theta) = g(\theta)$ for θ .

3 marks

b. Find the maximum difference between the values of $f(\theta)$ and $g(\theta)$.

3 marks

Question 10 (6 marks)

- a.** Solve $3^{2x} < 3^{x+1} + 4$ for x .

3 marks

[illegible]

b. Solve $\log_{2x}(16) < -2$ for x .

3 marks

END OF QUESTION AND ANSWER BOOK