



Trial Examination 2020

VCE Mathematical Methods Units 3&4

Written Examination 1

Suggested Solutions

Question 1 (3 marks)

a. $y = \frac{1}{(1-2x)^2}$
 $= (1-2x)^{-2}$

$$\frac{dy}{dx} = -2 \times -2 \times (1-2x)^{-3}$$

$$= \frac{4}{(1-2x)^3}$$

A1

b. Let $f(x) = x^3 \cos(2x)$.

$$f'(x) = 3x^2 \cos(2x) - 2x^3 \sin(2x)$$

M1

$$f'\left(\frac{\pi}{4}\right) = 3\left(\frac{\pi}{4}\right)^2 \cos\left(\frac{\pi}{2}\right) - 2\left(\frac{\pi}{4}\right)^3 \sin\left(\frac{\pi}{2}\right)$$

$$= 0 - \frac{2\pi^3}{64}$$

$$= -\frac{\pi^3}{32}$$

A1

Question 2 (2 marks)

$$\int_{-1}^0 \frac{3}{1-3x} dx = \left[-\frac{1}{3} \times 3 \log_e(1-3x) \right]_{-1}^0$$

M1

$$= -[\log_e(1-3x)]_{-1}^0$$

$$= -(\log_e(1) - \log_e(4))$$

$$= \log_e(4)$$

$$\therefore b = 4$$

A1

Question 3 (6 marks)

a. $f'(x) = 1 + e^{-\frac{x}{2}}$

$$f'(\log_e(9)) = 1 + e^{-\frac{\log_e(9)}{2}}$$

$$= 1 + e^{\log_e 9^{-0.5}}$$

M1

$$= 1 + e^{\log_e \frac{1}{3}}$$

$$= 1 + \frac{1}{3}$$

$$= \frac{4}{3}$$

A1

b. $f(x) = \int 1 + e^{-\frac{x}{2}} dx$

$$= x - 2e^{-\frac{x}{2}} + c$$

A1

$$f(-2) = -2e \rightarrow -2e = -2 - 2e + c$$

$$c = 2$$

$$f(x) = x - 2e^{-\frac{x}{2}} + 2$$

A1

c. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{pmatrix} 1 \\ 4 \end{pmatrix}$

$$x' = 2x + 1$$

$$\rightarrow x = \frac{1}{2}(x' - 1)$$

M1

$$y' = y + 4$$

$$\rightarrow y = y' - 4$$

$$y' - 4 = 1 + e^{-\frac{1}{2} \times \frac{1}{2}(x' - 1)}$$

$$y = e^{-\frac{1}{4}(x' - 1)} + 5$$

A1

Question 4 (5 marks)

a. $25^m - \frac{1}{5^{1-2m}} = 48$

$$5^{2m} - \frac{1}{5} \times 5^{2m} = 48$$

M1

$$\frac{4}{5} \times 5^{2m} = 48$$

$$5^{2m} = 60$$

M1

$$\log_e(5^{2m}) = \log_e(60)$$

$$2m(\log_e(5)) = \log_e(60)$$

$$m = \frac{\log_e(60)}{2\log_e(5)}$$

$$= \frac{\log_e(60)}{\log_e(25)}$$

A1

b.
$$\frac{5}{\log_e(x) + 2} = \log_e(x) - 2$$

$$(\log_e(x) + 2)(\log_e(x) - 2) = 5$$

$$(\log_e(x))^2 - 4 = 5 \quad \text{M1}$$

$$(\log_e(x))^2 = 9$$

$$\log_e(x) = \pm 3$$

$$x = e^3 \text{ or } x = e^{-3} \quad \text{A1}$$

Question 5 (7 marks)

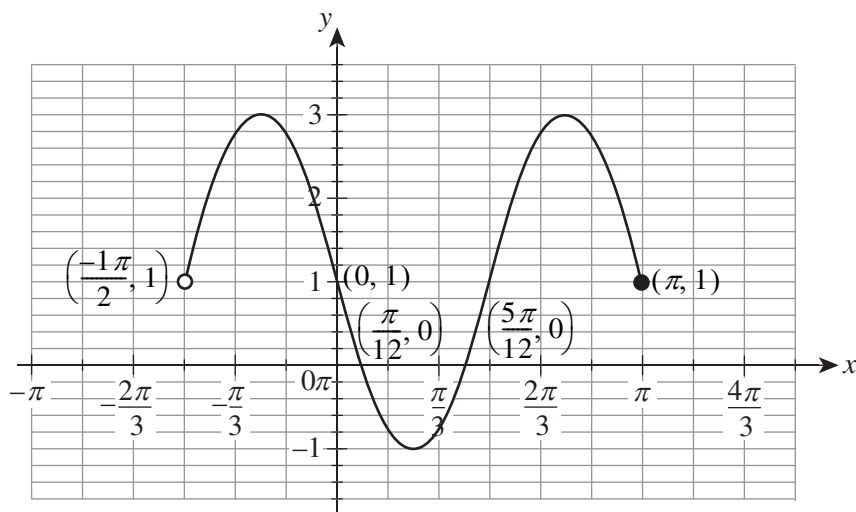
a. $1 - 2 \sin(2x) = 0$

$$\sin(2x) = \frac{1}{2}$$

$$2x = \frac{\pi}{6}, \frac{5\pi}{6} \quad \text{M1}$$

$$x = \frac{\pi}{12} \text{ or } x = \frac{5\pi}{12} \quad \text{A1}$$

b.



correct intercepts A1

correct endpoints A1

correct shape A1

c. $f'(x) = -4 \cos(2x)$

$$f'(0) = -4 \cos(0) \quad \text{M1}$$

$$= -4$$

$$= 1$$

$$\therefore y = -4x + 1 \text{ is the tangent line at the } y\text{-intercept} \quad \text{A1}$$

Question 6 (8 marks)

- a. i. $\Pr(\text{at least one faulty}) = 1 - \Pr(\text{none faulty})$

$$= 1 - \frac{5}{8} \times \frac{4}{7} \quad \text{A1}$$

$$= 1 - \frac{5}{14} = \frac{9}{14} \quad \text{M1}$$

- ii. Let X be the number of faulty batteries.

$$\Pr(X = 1 | X \geq 1) = \frac{\Pr(X = 1 \cap X \geq 1)}{\Pr(X \geq 1)} = \frac{\Pr(X = 1)}{\Pr(X \geq 1)}$$

$$\Pr(X = 1) = \frac{3}{8} \times \frac{5}{7} + \frac{5}{8} \times \frac{3}{7} = \frac{15}{28} \quad \text{M1}$$

$$\Pr(X \geq 1) = \frac{9}{14}$$

$$\begin{aligned} \Pr(X = 1 | X \geq 1) &= \frac{\frac{15}{28}}{\frac{9}{14}} \\ &= \frac{5}{6} \quad \text{A1} \end{aligned}$$

- b. Let Y be the random variable for faulty batteries.

$$Y \sim Bi\left(4, \frac{1}{5}\right)$$

$$\Pr(Y \geq 2) = 1 - \Pr(Y = 0) - \Pr(Y = 1)$$

$$= 1 - \left(\frac{4}{5}\right)^4 - {}^4C_1 \left(\frac{4}{5}\right)^3 \left(\frac{1}{5}\right) \quad \text{M1}$$

$$= 1 - \frac{256}{625} - \frac{256}{625}$$

$$= \frac{113}{625} \quad \text{A1}$$

- c. i. $\Pr(Z > -1.5) = \Pr(Z < 1.5)$

$$z = \frac{x - \bar{x}}{\mu}$$

$$1.5 = \frac{b - 540}{70}$$

$$b = 540 + 1.5 \times 70$$

$$= 645 \quad \text{A1}$$

- ii. $\Pr(X > 470 | X < 540) = \frac{\Pr(470 < X < 540)}{\Pr(X < 540)}$

$$= \frac{\Pr(-1 < Z < 0)}{\Pr(Z < 0)}$$

$$= \frac{0.5 - 0.16}{0.5}$$

$$= 0.68 \quad \text{A1}$$

Question 7 (5 marks)

a. $f: R \rightarrow R, f(x) = e^{3x} - 2$

Let $y = e^{3x} - 2$.

For inverse, swap x and y :

$$x = e^{3y} - 2$$

$$3y = \log_e(x + 2)$$

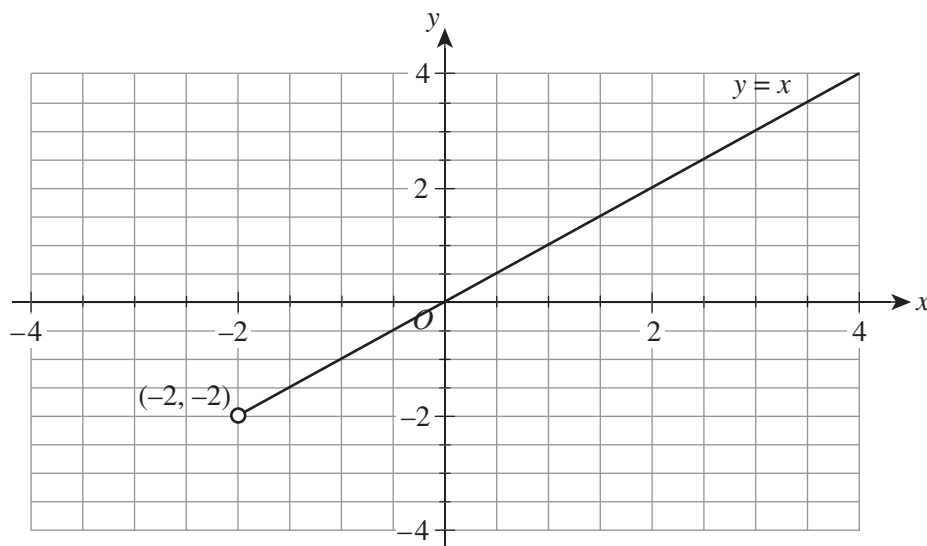
$$f^{-1}(x) = \frac{1}{3} \log_e(x + 2)$$

A1

The domain is $(-2, \infty)$.

A1

b.



correct line, domain and end point $(-2, -2)$ A1

c. $f^{-1}(3x) = \frac{1}{3} \log_e(3x + 2)$

$$f(f^{-1}(3x)) = e^{\ln\left(\frac{1}{3x+2}\right)} - 2$$

M1

$$= \frac{1}{3x+2} - 2$$

$$= \frac{1}{3x+2} - \frac{2(3x+2)}{3x+2}$$

$$= \frac{-6x-3}{3x+2}$$

A1

Question 8 (4 marks)

a. $f(x) = 6\sqrt{x} - x - 5$

$$f'(x) = 3x^{-\frac{1}{2}} - 1$$

$$= \frac{3}{\sqrt{x}} - 1$$

Let $f'(x) = 0$.

$$\frac{3}{\sqrt{x}} - 1 = 0$$

$$x = 9$$

M1

The domain is strictly decreasing for $x \in [9, \infty)$.

A1

- b.** The maximum area of the triangle ABC occurs when point C is the turning point of $f(x)$ at $x = 9 \rightarrow f(9) = 4$.

Point C is $(9, 4)$.

Let $f(x) = 0$.

$$6\sqrt{x} - x - 5 = 0$$

Let $a = \sqrt{x} \rightarrow 6a - a^2 - 5 = 0$.

$$a^2 - 6a + 5 = 0$$

$$(a - 5)(a - 1) = 0$$

$$(\sqrt{x} - 5)(\sqrt{x} - 1) = 0$$

M1

$$x = 1 \text{ or } x = 25$$

Area of $ABC = \frac{1}{2}bh$

$$= \frac{1}{2}(25 - 1) \times 4$$

$$= 48 \text{ square units}$$

A1