Neap

Trial Examination 2020

VCE Mathematical Methods Units 1&2

Written Examination 2

Suggested Solutions

SECTION A – MULTIPLE-CHOICE QUESTIONS

1	A
2	D
3	B
4	C
5	A
6	D
7	E
8	E
9	A
10	E
11	B
12	D
13	C
14	C
15	D
16	E
17	C
18	E
19	A
20	A

Question 1 A

$Q(-1) = 3(-1)^3 - a(-1)^2 + b(-1) + c = -6$ (equation 1)

$Q(3) = 3(3)^3 - a(3)^2 + b(3) + c = 66$ (equation 2)

$Q(-3) = 3(-3)^3 - a(-3)^2 + b(-3) + c = -126$ (equation 3)

Solving simultaneous equations using CAS gives $a = 4$, $b = 5$, and $c = 6$.

Question 2 D

The correct period is π. The amplitude is 3. Translation is one unit up. Therefore, a possible equation for the graph is $y = 3\sin(2x) + 1$.

Question 3 B

The function can be sketched using CAS. For the function to have an inverse, it must be a one-to-one function. Therefore, to suit the specified domain, the largest possible value of q is $= -1$.

Question 4 C

Solving for x manually gives:

$$3x^2 - 6x + 4 = x$$

$$3x^2 - 7x + 4 = 0$$

$$(3x - 4)(x - 1) = 0$$

$$x = 1 \text{ or } \frac{4}{3}$$

OR

Solving for x using CAS gives $x = 1$ or $\frac{4}{3}$.

For either method, by inspection:

$f(x) \geq x$ when $x \leq 1$, and $x \geq \frac{4}{3}$.

Question 5 A

Sketching the graph using CAS gives a semi-circle with domain $-6 \leq x \leq 6$.

Question 6 D

The function is a truncus graph, so **A** and **B** are incorrect. **E** represents an inverted truncus, so is incorrect. The sketch shows a vertical asymptote at $x = 2$. Therefore, the denominator of the truncus fraction is $(x - 2)^2$. The horizontal asymptote is at $y = -4$. Therefore, **D** is correct and **C** is incorrect.

Question 7 **E**

Completing the square:

$$x^2 + y^2 - 10x + 4y = -25$$
$$x^2 - 10x + 5^2 - 5^2 + y^2 + 4y + 2^2 - 2^2 = -25$$
$$(x-5)^2 - 25 + (y+2)^2 - 4 = -25$$
$$(x-5)^2 + (y+2)^2 = 4$$

This is a circle in the form $(x-h)^2 + (y-k)^2 = r^2$, where the centre is (h, k) and radius r. Therefore, the centre is $(5, -2)$ and radius is 2.

Question 8 **E**

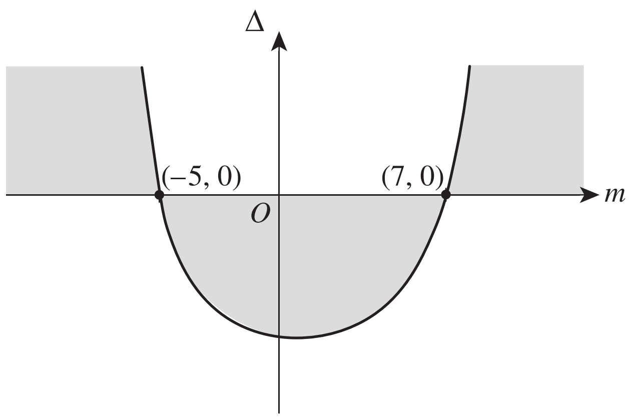
Equating the equations $3x^2 + mx - 2 = x - 5$ and rearranging into the form $ax^2 + bx + c = 0$ gives:

$$3x^2 + mx - x - 2 + 5 = 0$$
$$3x^2 + x(m-1) + 3 = 0$$

Solving for the discriminant equalling zero ($\Delta = 0$) gives:

$$b^2 - 4ac = 0$$
$$(m-1)^2 - 4(3)(3) = 0$$
$$(m-1)^2 - 36 = 0$$
$$(m-1)^2 = 36$$
$$m - 1 = \pm 6$$
$$m = -5 \text{ or } 7$$

Graph of the discriminant against m:



Reading from the graph, **E** is correct.

Question 9 A

Using CAS, in Statistics, x-values are entered into List 1 and y-values are entered into List 2. Calculating the regression line in the form $y = a \times b^x$ gives **A**.

Question 10 E

Solving the equation for x using CAS gives the general solutions:

$x = \frac{2\pi n}{3} - \frac{\pi}{18}$ and $x = \frac{2\pi n}{3} + \frac{7\pi}{18}$, where $n \in Z$.

For $n = -2$, $x = -\frac{25\pi}{18}, -\frac{17\pi}{18}$.

For $n = -1$, $x = -\frac{13\pi}{18}$ and $x = -\frac{5\pi}{18}$.

For $n = 0$, $x = \frac{\pi}{18}$ and $x = \frac{7\pi}{18}$.

For $n = 1$, $x = \frac{11\pi}{18}$ and $x = \frac{19\pi}{18}$.

Only the six values $\frac{-17\pi}{18}, \frac{-13\pi}{18}, \frac{-5\pi}{18}, \frac{\pi}{18}, \frac{7\pi}{18}, \frac{11\pi}{18}$ are within the specified domain.

OR

Solving using the specified domain gives the six solutions.

Question 11 B

$$
\begin{aligned}
f(x) &= -2x^3 + \frac{2}{x^2} + 3x + 4 \\
&= -2x^3 + 2x^{-2} + 3x + 4 \\
f'(x) &= -6x^2 - 4x^{-3} + 3 \\
&= -6x^2 - \frac{4}{x^3} + 3
\end{aligned}
$$

Question 12 D

Calculating the antiderivative using CAS gives $x^4 - \frac{2}{3}x^3 + 3x^2$, where $c = 0$. Therefore, **D** is correct, where $c = -3$.

Question 13 C

$f(-1) = -3$ and $f'(-1) = 0$, so $(-1, -3)$ is a stationary point. Therefore, **D** and **E** are incorrect.

$f'(x) < 0$ for $x < -1$, so when x is less than -1, the gradient of the graph is negative. $f'(x) > 0$ for $x > -1$, so when x is greater than -1, the gradient of the graph is positive. Therefore, the point $(-1, -3)$ is a local minimum, so **C** is correct.

Question 14 C

Either using CAS, or manually, finding $\frac{dy}{dx} = 4x + 3$ gives $y = 2x^2 + 3x + c$.

Since $y = 9$ when $x = 2$, substituting these values into the equation and solving for c gives:

$$9 = 2(2)^2 + 3 \times 2 + c$$
$$9 = 8 + 6 + c$$
$$c = -5$$

So, $y = 2x^2 + 3x - 5$.

Question 15 D

Finding the gradient of the line gives $\frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-(-2)}{2-1} = -3$.

Solving using CAS gives

$$\begin{aligned} m &= \tan(\theta) \\ -3 &= \tan(\theta) \\ \tan^{-1}(-3) &= \theta \\ \theta &= -71.5650° \end{aligned}$$

However, this is not the required angle; the angle that is made with the positive direction of the x-axis is $180 - 71.5650 = 108.435°$, correct to three decimal places.

Question 16 E

For independent events, $\Pr(A) \times \Pr(B) = \Pr(A \cap B)$.

$$\begin{aligned} 0.55 \times \Pr(B) &= 0.363 \\ \Pr(B) &= 0.66 \end{aligned}$$

Question 17 C

$$\begin{aligned} \Pr(A|B) &= \frac{\Pr(A \cap B)}{\Pr(B)} \\ &= \frac{0.363}{0.66} \\ &= 0.55 \end{aligned}$$

Question 18 E

Pr(both Jane and Ken have cereal for breakfast) $= 0.65 \times 0.55 = 0.3575$

Question 19 A

Pr(only one of them has cereal for breakfast) = Pr(Jane has cereal and Ken has toast) + Pr(Ken has cereal and Jane has toast)

$= 0.65 \times 0.45 + 0.55 \times 0.35$

$= 0.4850$

Question 20 A

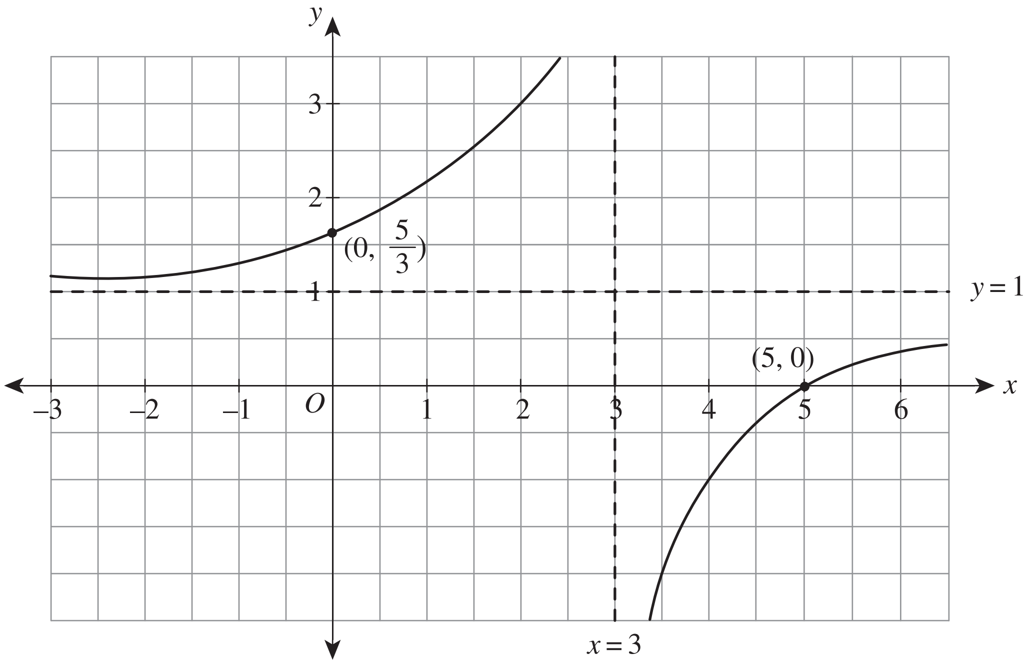
	N	N'	
F	0.45	0.18	0.63
F'	0.36	0.01	0.37
	0.81	0.19	1

$\Pr(F' \cap N')$ is 0.01, so **A** is correct.

SECTION B

Question 1 (15 marks)

a.



Key points and asymptotes can be found using CAS or manually.

correct shape A1

correct x- and y-intercepts labelled (5, 0) *and* $\left(0, \frac{5}{3}\right)$ A1

correct x-asymptote labelled $(x = 3)$ A1

correct y-asymptote labelled $(y = 1)$ A1

b. one-to-one A1

c. Let $y = \frac{2}{x-3} + 1$.

Therefore, for the inverse function:

$$x = -\frac{2}{y-3} + 1 \qquad \text{M1}$$

$$x - 1 = -\frac{2}{y-3}$$

$$1 - x = \frac{2}{y-3} \qquad \text{M1}$$

$$y - 3 = \frac{2}{1-x} \qquad \text{M1}$$

$$f^{-1}(x) = \frac{2}{1-x} + 3$$

d. domain: $x \in R\backslash\{1\}$ A1

range: $y \in R\backslash\{3\}$ A1

e. The first transformation is a vertical translation of 1 unit down (in the negative direction of the y-axis). A1

The second transformation is a horizontal translation of 3 units left (in the negative direction of the x-axis). A1

The third transformation is a reflection in the x-axis, and the fourth transformation is a dilation by a factor of $\frac{1}{2}$ from the y-axis. A1

f.

$$f(x) = -\frac{2}{x-3} + 1$$

$$f(4) = -\frac{2}{4-3} + 1 \qquad \text{M1}$$

$$= -\frac{2}{1} + 1$$

$$= -1 \qquad \text{A1}$$

Question 2 (15 marks)

a. The amplitude is 3. A1

b.

$$P = \frac{2\pi}{n}$$

$$= \frac{2\pi}{\frac{\pi}{4}}$$

$$= 8 \qquad \text{A1}$$

c.

$$D(t) = 20 + 3\cos\left(\frac{\pi t}{4}\right),\ 0 \le t \le 16$$

$$D(0) = 20 + 3\cos\left(\frac{\pi \times 0}{4}\right) \qquad \text{(Solve using CAS)}$$

$$= 20 + 3\cos(0)$$

$$= 23 \text{ metres} \qquad \text{A1}$$

d. Sketching the graph $D(t)$ using CAS (trace function):

Maximum depth is 23 metres. A1

Minimum depth is 17 metres. A1

OR

Using $D(t) = 20 + 3\cos\left(\frac{\pi t}{4}\right)$, the mid-line of the graph is at $y = 20$.

From **part a.**, the amplitude is 3 metres. Therefore:

$$\begin{aligned} \text{maximum depth} &= 20 + 3 \\ &= 23 \text{ metres} \end{aligned}$$ A1

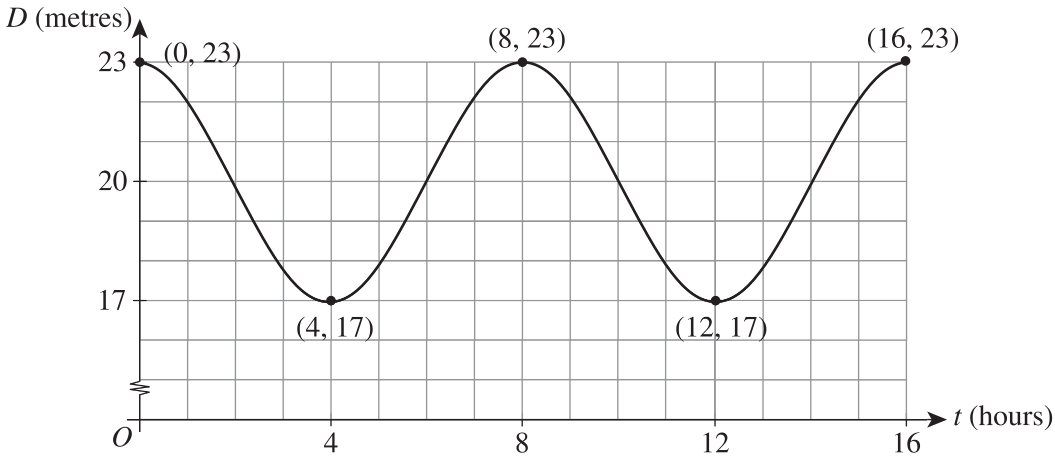
$$\begin{aligned} \text{minimum depth} &= 20 - 3 \\ &= 17 \text{ metres} \end{aligned}$$ A1

e. **i.** Sketching the graph over the 16 hours using CAS, we can see there are **three** high tides. A1

ii. The high tides occur at 6.00 am, 2.00 pm and 10.00 pm. A1

Note: All **three** *times must be provided.*

f.



correct start and endpoints labelled (0, 23) *and* (16, 23) A1

correct maximum and minimum points labelled (4, 17), (8, 23) *and (12, 17)* A1

correct shape A1

g. **i.** $D(t) = 20 + 3\cos\left(\frac{\pi t}{4}\right),\ 0 \le t \le 16$ M1

$$19 = 20 + 3\cos\left(\frac{\pi t}{4}\right)$$

$$t = 2.43,\ 5.57,\ 10.43 \text{ and } 13.57$$ A1

ii. From **part g.(i).** $t = 2.43,\ 5.57,\ 10.43$ and 13.57.

The depth (D) of water, in metres, is given at time t hours after 6.00 am. Therefore, $t = 8.43,\ 11.57,\ 16.43$ and 19.57 (in 24-hour form).

Reading the graph, at 6.00 am the depth of water is 23 metres. Therefore, the ship can sail to and from the pier during the times:

6.00–8.43

11.57–16.43

19.57–22.00

Converting the decimal part of the time to minutes (12-hour system form) by multiplying by $\frac{60}{100}$ gives:

6:00–8:26 am A1

11:34 am–4:26 pm A1

7:34–10:00 pm. A1

Award one mark for each correct time range provided.

Question 3 (14 marks)

a. Using CAS, solving for x gives:

$0 = -x^4 - 4x^3 - x^2 + 8x + 4$ M1

$x = -2.81, -2.00, -0.53, 1.34$ A1

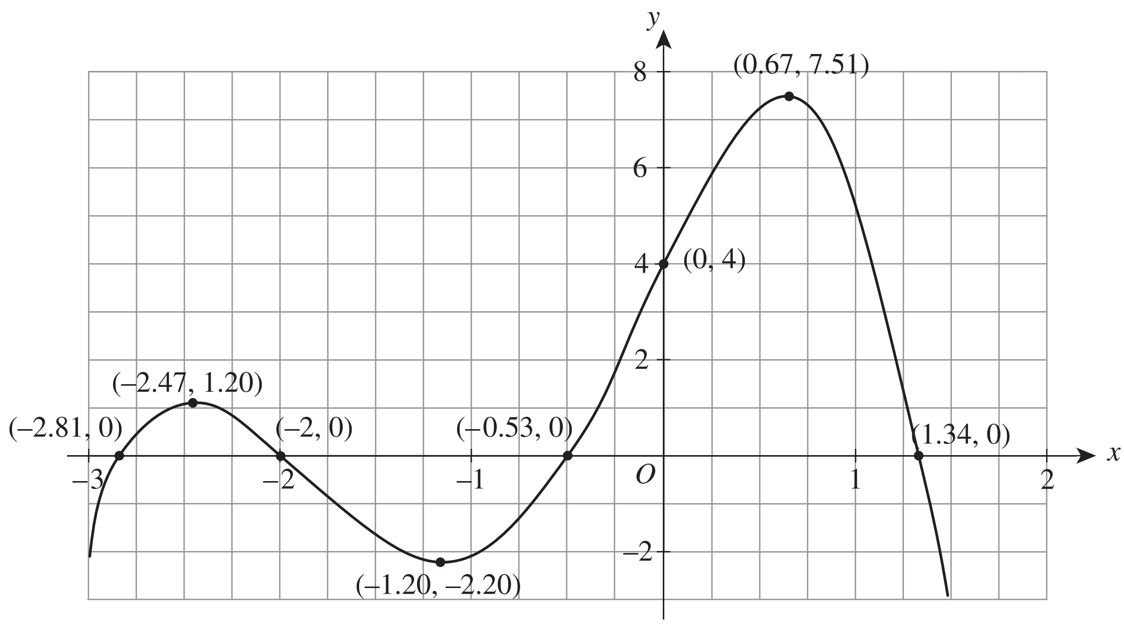
b. Sketching the graph using CAS to find the maximum and minimum points gives:

$(-2.47, 1.20)$ maximum A1

$(-1.20, -2.20)$ minimum A1

$(0.67, 7.51)$ maximum. A1

c.



correct x-intercepts labelled $(-2.81, 0), (-2, 0), (-0.53, 0)$ *and* $(1.34, 0)$ A1

correct y-intercept labelled $(0, 4)$ A1

correct maximum and minimum labelled $(-2.47, 1.20), (-1.20, -2.20)$ *and* $(0.67, 7.51)$ A1

correct shape A1

d.

$f(x) = -x^4 - 4x^3 - x^2 + 8x + 4$

$f(0) = 4$

$f(0.5) = 7.1875$ M1

$$\text{average rate of change} = \frac{f(0.5) - f(0)}{0.5 - 0} = \frac{7.1875 - 4}{0.5} = 6.375$$ A1

e. Finding $f'\left(\frac{1}{2}\right)$ to find the instantaneous rate of change gives:

$f'(x) = -4x^3 - 12x^2 - 2x + 8$ M1

$f'\left(\frac{1}{2}\right) = -4\left(\frac{1}{2}\right)^3 - 12\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right) + 8$

$f'\left(\frac{1}{2}\right) = \frac{7}{2}$ A1

OR $3\frac{1}{2}$

f. Solving using CAS gives:

$$\int_{-1.2}^{1.2} f(x)dx = \int_{-1.2}^{1.2} (-x^4 - 4x^3 - x^2 + 8x + 4)dx$$

$= 7.453$ A1

Question 4 (16 marks)

a. **i.** $\Pr(\text{red, blue, green}) = 6 \times \left(\frac{4}{12} \times \frac{5}{12} \times \frac{3}{12}\right)$

$= \frac{5}{24}$ A1

ii. $\Pr(\text{green, green, green}) = \frac{3}{12} \times \frac{3}{12} \times \frac{3}{12}$

$= \frac{1}{64}$ A1

iii. $\Pr(\text{green, green, green}) = \frac{3}{12} \times \frac{3}{12} \times \frac{3}{12} = \frac{1}{64}$

$\Pr(\text{red, red, red}) = \frac{4}{12} \times \frac{4}{12} \times \frac{4}{12} = \frac{1}{27}$

$\Pr(\text{blue, blue, blue}) = \frac{5}{12} \times \frac{5}{12} \times \frac{5}{12} = \frac{125}{1728}$

correctly provides probabilities of all **three** *colour*s M1

$\Pr(\text{three marbles of the same colour}) = \frac{1}{64} + \frac{1}{27} + \frac{125}{1728} = \frac{1}{8}$ A1

b. **i.** red red, red blue, red green, blue red, blue blue, blue green, green red, green blue, green green

$9 - 3 = 6$ A1

ii. $\Pr(\text{green, green}) = \frac{3}{12} \times \frac{2}{11} = \frac{1}{22}$ A1

iii. $\Pr(\text{two marbles of the same colour}) = \frac{4}{12} \times \frac{3}{11} + \frac{5}{12} \times \frac{4}{11} + \frac{3}{12} \times \frac{2}{11} = \frac{19}{66}$ A1

iv. $1 - \Pr(\text{two marbles of the same colour}) = 1 - \frac{4}{12} \times \frac{3}{11} + \frac{5}{12} \times \frac{4}{11} + \frac{3}{12} \times \frac{2}{11}$ M1

$= \frac{47}{66}$ A1

OR

$\Pr((\text{red, blue}) + (\text{blue, red}) + (\text{red, green}) + (\text{green, red}) + (\text{green, blue}) + (\text{blue, green}))$

$= \frac{4}{12} \times \frac{5}{11} + \frac{5}{12} \times \frac{4}{11} + \frac{4}{12} \times \frac{3}{11} + \frac{3}{12} \times \frac{4}{11} + \frac{3}{12} \times \frac{5}{11} + \frac{5}{12} \times \frac{3}{11}$ M1

$= \frac{47}{66}$ A1

v. $\Pr(\text{green second} \mid \text{green first}) = \frac{\Pr(\text{green} \cap \text{green})}{\Pr(\text{green first})} = \frac{\frac{2}{22}}{\frac{1}{4}} = \frac{2}{11}$ A1

c. **i.** combinations $= \dfrac{12!}{5! \times 4! \times 3!} = 27\ 720$ A1

ii. combinations $= \dfrac{9!}{5! \times 3!}$ M1

$= 504$ A1

iii. combinations $= 3! = 6$ A1

d. **i.** $12C_5 = 792$ A1

ii. $10C_3 = 120$ A1