

Trial Examination 2020

VCE Mathematical Methods Units 1&2

Written Examination 1

Suggested Solutions

Question 1 (3 marks)

a. degree of the polynomial $P(x) = 3$ A1

b.
$$\begin{aligned} P(x) - Q(x) &= 3x^3 - 2x^2 + 4x - 5 - (-4x^2 - 3) \\ &= 3x^3 - 2x^2 + 4x - 5 + 4x^2 + 3 \\ &= 3x^3 + 2x^2 + 4x - 2 \end{aligned}$$
 A1

c.
$$\begin{aligned} P(-2) &= 3(-2)^3 - 2(-2)^2 + 4(-2) - 5 \\ &= -24 - 8 - 8 - 5 \\ &= -45 \end{aligned}$$
 A1

Question 2 (3 marks)

Using re-expression:

$$\begin{aligned} x^3 - 7x^2 + 7x + 15 &= x^2(x - 3) - 4x(x - 3) - 5(x - 3) && \text{A1} \\ &= (x - 3)(x^2 - 4x - 5) && \text{M1} \end{aligned}$$

Therefore, $b = -4$, $c = -5$ and $r = 0$. A1

Note: Students may use other acceptable methods.

Question 3 (1 mark)

No, the set of ordered pairs is not a function.

Any one of:

- There is an x -value with two different y -values: $(-3, 5)$ and $(-3, 10)$.
- The relation contains two sets of ordered pairs with the same first coordinate: $(-3, 5)$ and $(-3, 10)$.
- This set of ordered pairs does not pass the vertical line test as there is more than one value of y for the x -value of -3 .

A1

Question 4 (7 marks)**a. i.** x -intercept, $y = 0$

$$-2(x-5)^2 + 18 = 0$$

$$-2(x-5)^2 = -18$$

$$2(x-5)^2 = 18$$

$$(x-5)^2 = 9$$

$$x-5 = \pm 3$$

$$x = 2 \text{ or } 8$$

The x -intercepts are $(2, 0)$ and $(8, 0)$.

A1

 y -intercept, $x = 0$

$$y = -2(0-5)^2 + 18$$

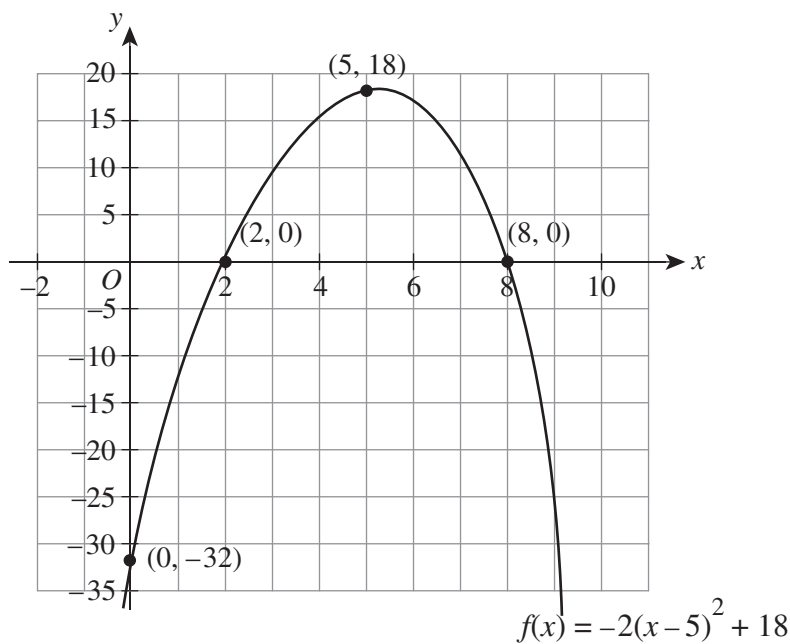
$$= -2 \times 25 + 18$$

$$= -50 + 18$$

$$= -32$$

The y -intercept is $(0, -32)$.

A1

ii.*correct shape and turning point $(5, 18)$* A1*correct x -intercepts $(2, 0)$ and $(8, 0)$ and y -intercept $(0, -32)$* A1

- b.** domain: $x \in R$ A1
 range: $y \in (-\infty, 18]$ A1
- c.** domain: $x \in (-\infty, 5]$ **OR** $x \in [5, \infty)$ A1

Question 5 (4 marks)Let $y = f(x)$

$$\text{Let } T = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix}$$

$$T^{-1} = \frac{1}{6-0} \begin{bmatrix} -3 & 0 \\ 0 & -2 \end{bmatrix}$$

$$T^{-1} = \frac{1}{6} \begin{bmatrix} -3 & 0 \\ 0 & -2 \end{bmatrix} \quad \text{M1}$$

$$\begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$TX = X'$$

$$T^{-1}TX = T^{-1}X'$$

$$\frac{1}{6} \begin{bmatrix} -3 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{6} \begin{bmatrix} -3 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix} \quad \text{M1}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{6} \begin{bmatrix} -3 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$x = -\frac{1}{2}x' \quad y = -\frac{1}{3}y' \quad \text{M1}$$

$$-\frac{1}{3}y' = -2\left(-\frac{1}{2}x'\right)^2 + 3\left(-\frac{1}{2}x'\right) + 5$$

$$-\frac{1}{3}y' = -2\left(\frac{1}{4}x'^2\right) - \frac{3}{2}x' + 5$$

$$-\frac{1}{3}y' = -\frac{1}{2}x'^2 - \frac{3}{2}x' + 5$$

$$y' = \frac{3}{2}x'^2 + \frac{9}{2}x' - 15 \quad \text{A1}$$

OR

$$X' = TX$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$x' = -2x \Rightarrow x = \frac{-x'}{2} \quad \text{M1}$$

$$y' = -3y \Rightarrow y = \frac{-y'}{3} \quad \text{M1}$$

$$y = -2x^2 + 3x + 5$$

$$\frac{-y'}{3} = -2\left(\frac{x'}{2}\right)^2 + 3\left(\frac{-x'}{2}\right) + 5 \quad \text{M1}$$

$$\frac{-y}{3} = \frac{x^2}{2} - \frac{3x}{2} + 5$$

$$y = -\frac{3}{2}x^2 + \frac{9x}{2} - 15 \quad \text{A1}$$

Question 6 (3 marks)

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{3(x+h)^2 - 4(x+h) + 5 - (3x^2 - 4x + 5)}{h} \quad \text{M1} \end{aligned}$$

$$= \lim_{h \rightarrow 0} \frac{3(x^2 + 2xh + h^2) - 4x - 4h + 5 - 3x^2 + 4x - 5}{h}$$

$$= \lim_{h \rightarrow 0} \frac{3x^2 + 6xh + 3h^2 - 4h - 3x^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{3h^2 + 6xh - 4h}{h} \quad \text{M1}$$

$$= \lim_{h \rightarrow 0} \frac{h(3h + 6x - 4)}{h}$$

$$= \lim_{h \rightarrow 0} (3h + 6x - 4)$$

$$= 6x - 4 \quad \text{A1}$$

Question 7 (8 marks)

a. $f(x) = -2x^3 - 5x^2 + x - 2$

$$f'(x) = -6x^2 - 10x + 1$$

A1

b. i. gradient:

$$f'(x) = -6x^2 - 10x + 1$$

$$f'(-2) = -6(-2)^2 - 10(-2) + 1$$

$$= -24 + 20 + 1$$

$$= -3$$

A1

equation of the tangent at $(-2, -8)$:

$$y - y_1 = m(x - x_1)$$

$$y + 8 = -3(x - (-2))$$

$$y + 8 = -3x - 6$$

$$y = -3x - 14$$

A1

Consequential on answer to Question 7a.

ii. gradient:

$$f'(-2) = -3$$

$$m(\text{perpendicular}) = \frac{1}{3}$$

A1

equation of the perpendicular line at $(-2, -8)$:

$$y - y_1 = m(x - x_1)$$

$$y + 8 = \frac{1}{3}(x - (-2))$$

$$y + 8 = \frac{1}{3}x + \frac{2}{3}$$

$$y = \frac{1}{3}x + \frac{2}{3} - 8$$

$$y = \frac{1}{3}x + \frac{2}{3} - \frac{24}{3}$$

$$y = \frac{x}{3} - \frac{22}{3}$$

A1

OR $y = \frac{1}{3}(x - 22)$

Consequential on answer to Question 7b.i.

$$\begin{aligned}
 \text{c. } \int_{-1}^1 f(x) dx &= \int_{-1}^1 (-2x^3 - 5x^2 + x - \log_{10} 100) dx \\
 &= \int_{-1}^1 (-2x^3 - 5x^2 + x - 2) dx \\
 &= \left[-\frac{2x^4}{4} - \frac{5x^3}{3} + \frac{x^2}{2} - 2x \right]_{-1}^1 && \text{M1} \\
 &= \left[-\frac{x^4}{2} - \frac{5x^3}{3} + \frac{x^2}{2} - 2x \right]_{-1}^1 \\
 &= \left(-\frac{1}{2} - \frac{5}{3} + \frac{1}{2} - 2 \right) - \left(-\frac{1}{2} + \frac{5}{3} + \frac{1}{2} + 2 \right) && \text{M1} \\
 &= -\frac{5}{3} - 2 - \frac{5}{3} - 2 \\
 &= -\frac{10}{3} - 4 \\
 &= -\frac{22}{3} && \text{A1}
 \end{aligned}$$

OR $-7\frac{1}{3}$

Question 8 (2 marks)

$$\begin{aligned}
 \frac{3^{4 \times 2x} \times 3^{2(x+3)}}{3^{-2x} \times 1} &= 3 \times 3 \\
 \frac{3^{8x} \times 3^{2x+6}}{3^{-2x}} &= 3^2 \\
 3^{8x+2x+6+2x} &= 3^2 \\
 3^{12x+6} &= 3^2 && \text{M1} \\
 12x+6 &= 2 \\
 12x &= -4 \\
 x &= -\frac{1}{3} && \text{A1}
 \end{aligned}$$

Question 9 (2 marks)

$$3^{\log_3(9)} - \log_2 \sqrt{32} = 9 - \log_2(2^{\frac{5}{2}})$$

M1

$$= 9 - \log_2 \left(2^{\frac{5}{2}} \right)$$

$$= 9 - \frac{5}{2}$$

$$= \frac{13}{2}$$

A1

OR $6\frac{1}{2}$ **Question 10** (4 marks)**a.**

		Possible outcome	Probability
	red	red, red	$\frac{1}{2} \times \frac{4}{9} = \frac{2}{9}$
		red, blue	$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$
		red, green	$\frac{1}{2} \times \frac{2}{9} = \frac{1}{9}$
	blue	blue, red	$\frac{3}{10} \times \frac{5}{9} = \frac{1}{6}$
		blue, blue	$\frac{3}{10} \times \frac{2}{9} = \frac{1}{15}$
		blue, green	$\frac{3}{10} \times \frac{2}{9} = \frac{1}{15}$
	green	green, red	$\frac{1}{5} \times \frac{5}{9} = \frac{1}{9}$
		green, blue	$\frac{1}{5} \times \frac{1}{3} = \frac{1}{15}$
		green, green	$\frac{1}{5} \times \frac{1}{9} = \frac{1}{45}$

*correct second branch of tree A1**correct possible outcomes A1**correct probabilities A1*

b. $\Pr(\text{green given red first}) = \Pr(\text{green second})\Pr(\text{red first})$

$$= \frac{\Pr(\text{green second} \cap \text{red first})}{\Pr(\text{red first})}$$

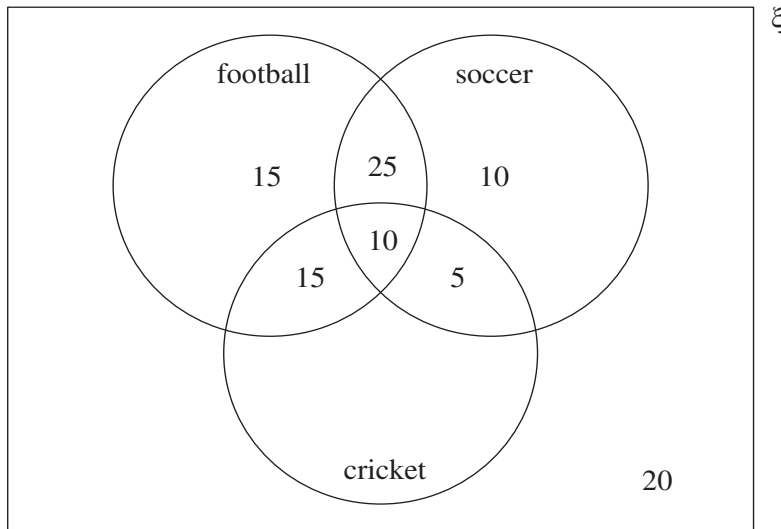
$$= \frac{\frac{1}{9}}{\frac{2}{9} + \frac{1}{6} + \frac{1}{9}}$$

$$= \frac{\frac{2}{18}}{\frac{4}{18} + \frac{3}{18} + \frac{2}{18}}$$

$$= \frac{2}{18} \times \frac{18}{9}$$

$$= \frac{2}{9}$$

A1

Question 11 (3 marks)

ξ

universal symbol outside diagram and 20 students outside circles A1
 correct 'football' circle A1
 correct 'soccer' circle A1