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# ***Mathematical Methods***

## ***2020***

### ***Trial Examination 1 (1 hour)***

## Instructions

Answer **all** questions.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

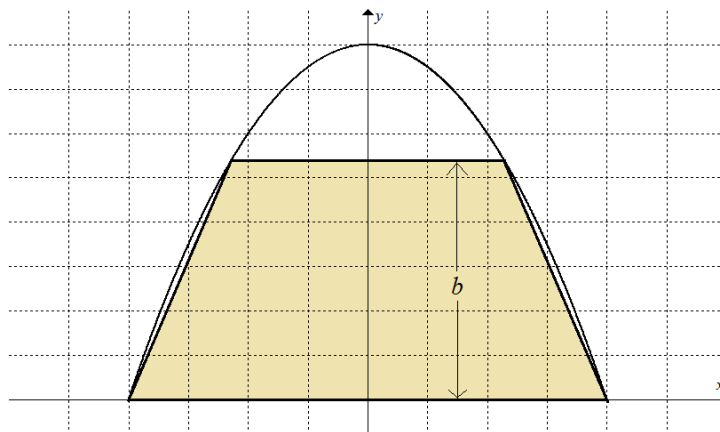
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

### Question 1

The following diagram shows a trapezium (height =  $b$ ) bounded by a parabola and the  $x$ -axis.

All corners of the trapezium are on the parabola.

The turning point of the parabola is  $(0, 4)$ , and the  $x$ -intercepts are  $(-2, 0)$  and  $(2, 0)$ .



- a. Write down the equation of the parabola in turning point form. 1 mark
- b. Express the area of the trapezium in terms of  $b$ . 3 marks
- c. Determine the value of  $b$  for maximum area of the trapezium. 2 marks

## Question 2

Consider polynomial  $P(x) = x^6 + 64$ .

a i. By long division find the quotient and the remainder when  $P(x)$  is divided by  $x^2 + 4$ . 2 marks

a ii. Hence or otherwise express  $P(x)$  as the product of 3 quadratic functions of  $x$ . 2 marks

b. Show that the turning points of the three quadratic functions form the vertices of an equilateral triangle. 2 marks

### Question 3

The curve  $y = 2x^3 - e^x$  undergoes transformation  $T$  given by

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \text{ where } \begin{bmatrix} x' \\ y' \end{bmatrix} \text{ is the image of } \begin{bmatrix} x \\ y \end{bmatrix}.$$

$$\text{Define } \begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{2} \left( \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} x' \\ y' \end{bmatrix} \right) + \begin{bmatrix} -1 \\ 1 \end{bmatrix}.$$

Find the relationship between  $X$  and  $Y$  (i.e. an equation satisfied by  $X$  and  $Y$ ).

3 marks

### Question 4

a. Solve  $5e^{-4x} + 2e^{-2x} = 3$  for  $x$ .

2 marks

b. Given  $\log_3 5 + \log_4 x = 1$ , express  $\log_e x$  in the form  $\frac{(\log_{10} a)(\log_{10} b)}{(\log_{10} c)(\log_{10} d)}$  where  $a, b, c, d \in \mathbb{R} \setminus \{0\}$ .

2 marks

c. Given  $(e, e)$  and  $(e^2, e^2)$  are points on  $y = Ae^{kx}$  where  $k, A \in \mathbb{R} \setminus \{0\}$ , find the values of  $k$  and  $A$ .

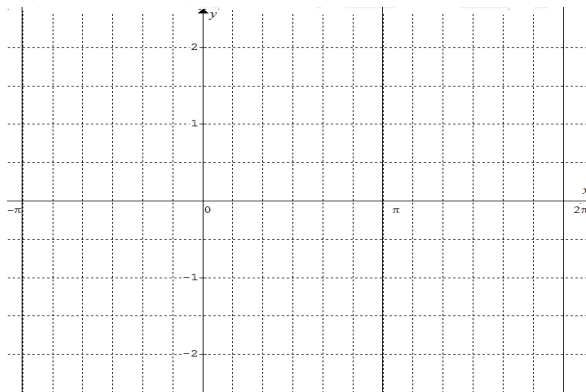
2 marks

### Question 5

Consider  $f(x) = 2\sin x + \cos\left(x + \frac{5\pi}{6}\right)$ .

- a. Sketch accurately the graphs of  $y = 2\sin x$  and  $y = \cos\left(x + \frac{5\pi}{6}\right)$  for  $-\pi \leq x \leq 2\pi$  on the same set of axes below.

2 marks



- b. Use the graphs to find the solutions to  $2\sin x + \cos\left(x + \frac{5\pi}{6}\right) = 0$  in the interval  $-\pi \leq x \leq 2\pi$ .

1 mark

- c. Write a general solution to  $2\sin x + \cos\left(x + \frac{5\pi}{6}\right) = 0$ .

1 mark

### Question 6

Consider  $f(x) = \frac{\cos x}{\sin x}$ ,  $x \in (0, \pi)$ .

- a. Find the derivative of  $f(x)$ .

1 mark

- b. Hence or otherwise evaluate  $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos^2 x}{\sin^2 x} dx$ .

2 marks

### Question 7

Consider events  $A$  and  $B$  such that  $\Pr(A) = \frac{3}{5}$  and  $\Pr(B' \cap A) = \frac{1}{4}$ .

- a. Evaluate  $\Pr(A \cap B)$ . 1 mark
- b. Given  $\Pr(A|B) = \frac{3}{5}$ , evaluate  $\Pr(A \cup B)$ . 1 mark
- c. Are  $A'$  and  $B'$  independent? Explain. 1 mark

### Question 8

Two tellers A and B in a bank serve customers.

On a particular day the proportion of 4 random customers choosing the queue served by Teller A waiting longer than 5 minutes is 0.36 and the proportion of 144 random customers choosing the queue served by Teller B waiting longer than 5 minutes is 0.64.

Assume both queues have the same length when a customer enters the bank.

- a. Which queue should a customer choose knowing these statistics?  
Explain by quoting the approximate 95% confidence intervals. 2 marks

- b. The probabilities that a customer going to Teller A and Teller B are  $\frac{1}{2}$  and  $\frac{1}{3}$  respectively, and the probability of not doing banking on the day after entering the bank due to long queues is  $\frac{1}{6}$ .

Using 0.36 for A and 0.64 for B as point estimates, determine the probability to 1 decimal place that the customer spends more than 5 minutes in waiting. 1 mark

### Question 9

Consider  $y = x^{\frac{m}{n}}$  and  $y = x^{\frac{m+2}{n+2}}$  where  $m$  is a positive even integer, and  $n$  a positive odd integer, and  $m > n$ .

a. Show that  $x^{\frac{m+2}{n+2}} > x^{\frac{m}{n}}$  for  $x \in (-1, 0)$  or  $x \in (0, 1)$ . 3 marks

b i. Find the area  $A$  of the regions bounded by  $y = x^{\frac{m}{n}}$  and  $y = x^{\frac{m+2}{n+2}}$  in terms of  $m$  and  $n$ . 2 marks

b ii. Show that  $A \rightarrow 0$  as  $n \rightarrow \infty$ . 1 mark

**End of Exam**