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YEAR 12 *Trial Exam Paper*

2020

MATHEMATICAL METHODS

Written examination 1

Reading time: 15 minutes

Writing time: 1 hour

STUDENT NAME:

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
9	9	40

- Students are to write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 15 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions

- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination

- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

Let $f : \left(-\infty, \frac{1}{3}\right) \rightarrow \mathbb{R}$, $f(x) = \sqrt{1-3x}$.

a. i. Find $f'(x)$.

1 mark

ii. Find an antiderivative of $f(x)$.

1 mark

b. Let $g : (-\pi, \pi) \rightarrow \mathbb{R}$, $g(x) = \frac{\sin(x)}{\cos(x) + 1}$.

Evaluate $g'\left(\frac{\pi}{2}\right)$.

2 marks

Question 2 (3 marks)

Let $f : \left(-\infty, \frac{1}{3}\right) \rightarrow R$, $f(x) = \sqrt{1-3x}$.

a. Find the rule of f^{-1} .

2 marks

b. State the domain of f^{-1} .

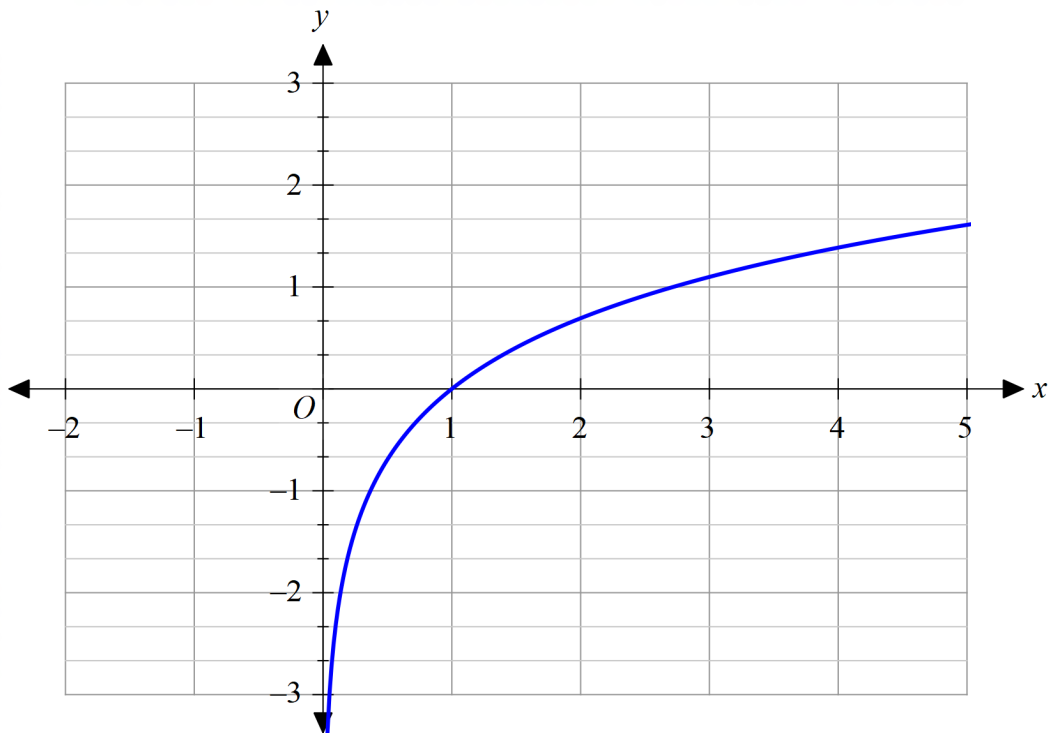
1 mark

Question 3 (4 marks)

- a. Solve $\log_e(x) = 1 - 2 \log_e(x)$.

1 mark

- b. The function $f : (0, \infty) \rightarrow \mathbb{R}$, $f(x) = \log_e(x)$ is shown on the axes below.



Let $g : (0, \infty) \rightarrow \mathbb{R}$, $g(x) = 1 - 2f(x)$.

Sketch the graph of g on the axes above. Label all points of intersection of the graphs of f and g . Label any asymptotes with the appropriate equation and label each of the axis intercepts, with their coordinates.

3 marks

Question 4 (3 marks)

Susan models the time it takes her to get to school each day with the random variable T , normally distributed with a mean of 12 minutes. She estimates that $\Pr(T > 15) = 0.05$.

- a.** Find the probability that it takes between 9 and 15 minutes for Susan to get to school.

1 mark

- b.** Find the probability that it takes Susan more than 15 minutes to get to school, given that it has taken her at least 12 minutes.

2 marks

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Question 5 (6 marks)

Let $f : [0, \infty) \rightarrow \mathbb{R}$, $f(x) = \frac{4}{2x-1} - 1$.

- a. Evaluate $f\left(\frac{3}{2}\right)$.

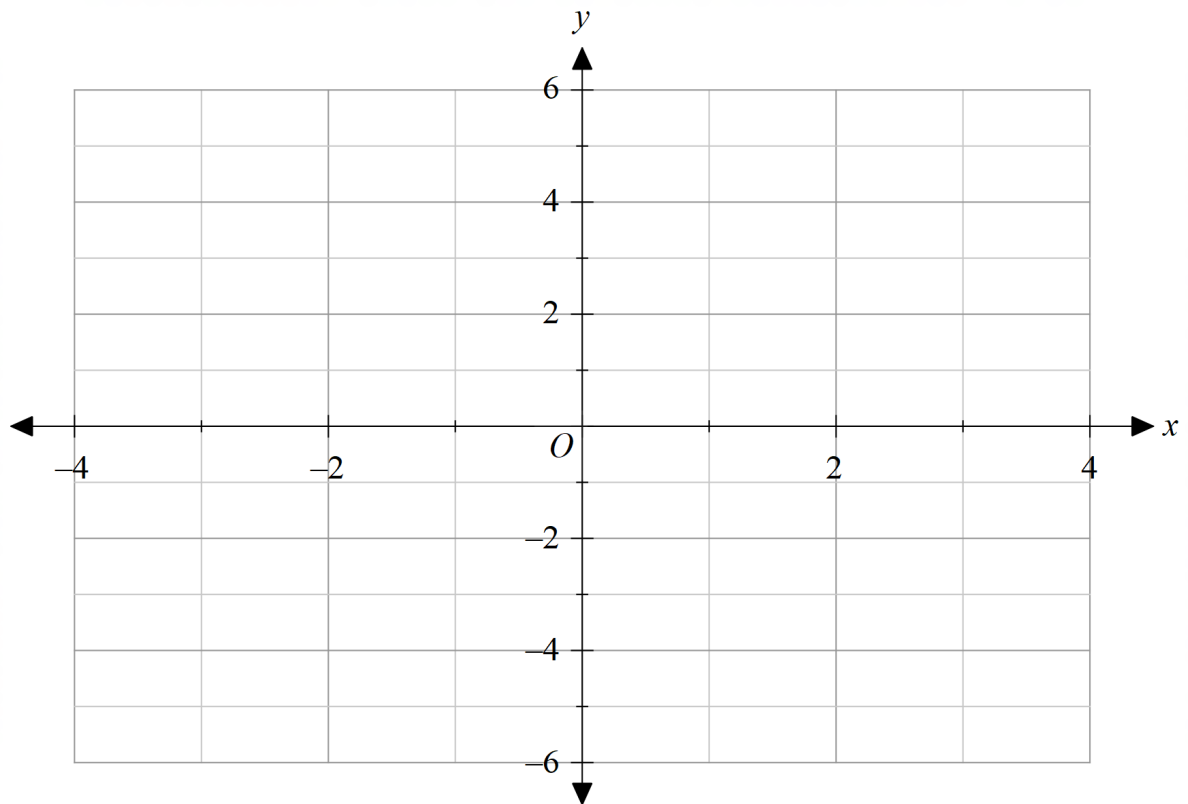
1 mark

- b. Find the equation of the tangent to the graph of f at $x = \frac{3}{2}$.

2 marks

- c. Sketch the graph of $y = f(x)$ on the axes below. Label each of the axis intercepts with their coordinates and label all asymptotes with their equations. Sketch the tangent to the graph of f at $x = \frac{3}{2}$, and label its point of intersection with $f(x)$.

3 marks



Question 6 (2 marks)

A board game uses a customised eight-sided die to resolve conflict within the game. Three sides of the die have a victory symbol. Ten of these dice are rolled. Let N be the number of victory symbols rolled.

- a.** Find $E(N)$.

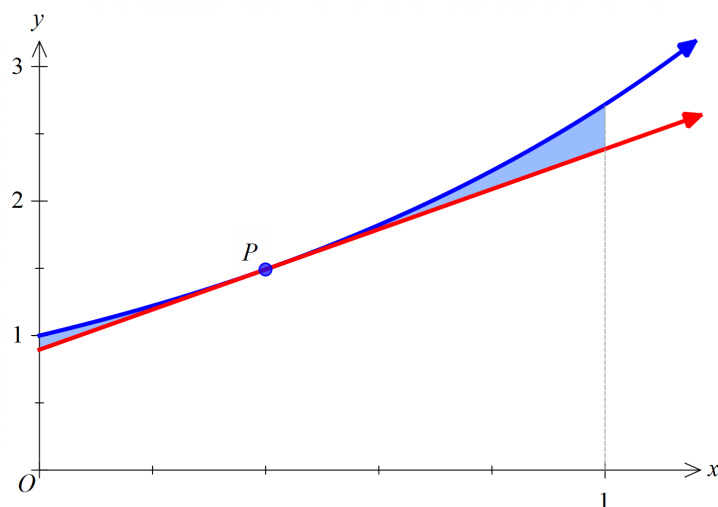
1 mark

- b.** What is the variance of N ?

1 mark

Question 7 (4 marks)

The graph of the relation $y = e^x$ is shown on the axes below. P is a point on the graph of this relation with coordinates (p, e^p) , and the tangent at P is shown.



- a.** Show that the equation of the tangent at P is $y = e^p x + e^p(1 - p)$.

1 mark

- b.** Find the value of p for which the area bounded by $y = e^x$, its tangent at $x = p$ and the lines $x = 0$ and $x = 1$ are minimised.

3 marks

Question 8 (6 marks)

Consider the functions $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = \sin\left(\frac{x}{2}\right)$ and $g : \mathbb{R} \rightarrow \mathbb{R}$, $g(x) = a \sin(\pi x) + a$, where a is a real number.

- a. i.** For which values of a is $g(f(x)) \geq 0$ for all values of x ?

1 mark

- ii.** For which value of x is $g(f(x))$ at its maximum on the interval $[0, \pi]$, when $g(f(x)) \geq 0$?

3 marks

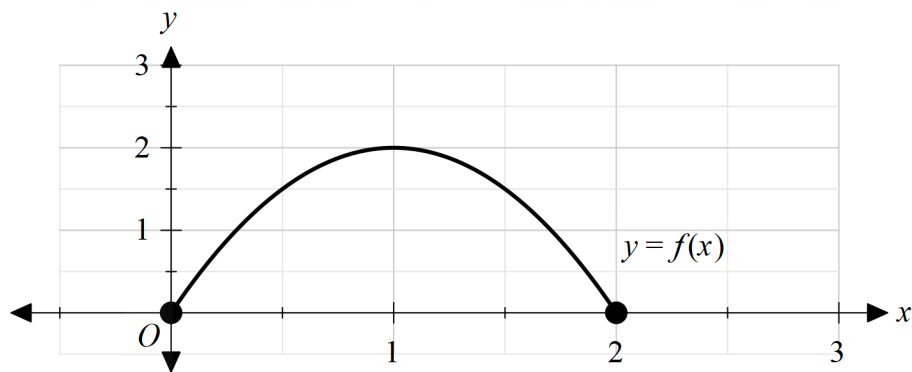
- b.** For which values of a is $f(g(x)) \geq 0$ for all values of x ?

2 marks

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Question 9 (8 marks)

The graph of $f : [0, 2] \rightarrow \mathbb{R}$, $f(x) = -2x^2 + 4x$ is shown below.



- a. Show that the area enclosed by f and the x -axis is equal to $\frac{8}{3}$ square units.

1 mark

Let g be the function obtained by applying the transformation T to the function f , where

$$T : \mathbb{R}^2 \rightarrow \mathbb{R}^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} a & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \text{ and } a > 0 \text{ is a real constant.}$$

- b. State the domain of g in terms of a .

1 mark

- c. For what value of a is the area enclosed by the graph of $y = g(x)$ and the x -axis, over the domain of g , equal to the area enclosed by f and the x -axis?

2 marks

- d. i. Solve $f(x) = 1$.

1 mark

- ii. Show that when $a = 3$, then $f(x) = g(x)$ has a unique solution.

1 mark

- iii. Let D be the intersection of the domains of f and g .
For what values of a is $g(x) > f(x)$ for $x \in D$?

2 marks

END OF QUESTION AND ANSWER BOOK