

Student Name: _____

MATHEMATICAL METHODS

UNITS 3&4 Exam 1

2020 Written Trial Examination

Reading time: 15 minutes

Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
9	9	40

- Students are to write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 12 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions

- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination

- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

a. If $y = \sqrt{3e^{2x} - e^{4x}}$, find $\frac{dy}{dx}$

2 marks

b. Let $f(x) = x^2 \tan(3x)$

Evaluate $f'(\pi)$

2 marks

Question 2 (4 marks)

Consider the functions $f: R \rightarrow R, f(x) = 3x^2 + x$ and $g: R \rightarrow R, g(x) = 2x - 1$.

- a. State the rule of $f(g(x))$ in the form $ax^2 + bx + c$.

1 mark

- b. Let $h: R \rightarrow R, h(x) = f(g(x))$.

Find the equation of the line perpendicular to the graph, $h(x)$, when $x = \frac{1}{2}$.

Give your answer in the form $ax + by = c$.

3 marks

Question 3 (4 marks)

Let $y = 3x \sin(2x)$.

a. Find $\frac{dy}{dx}$

1 mark

b. Hence, calculate $\int_0^{\pi/4} 3x \cos(2x) dx$.

3 marks

Question 4 (5 marks)

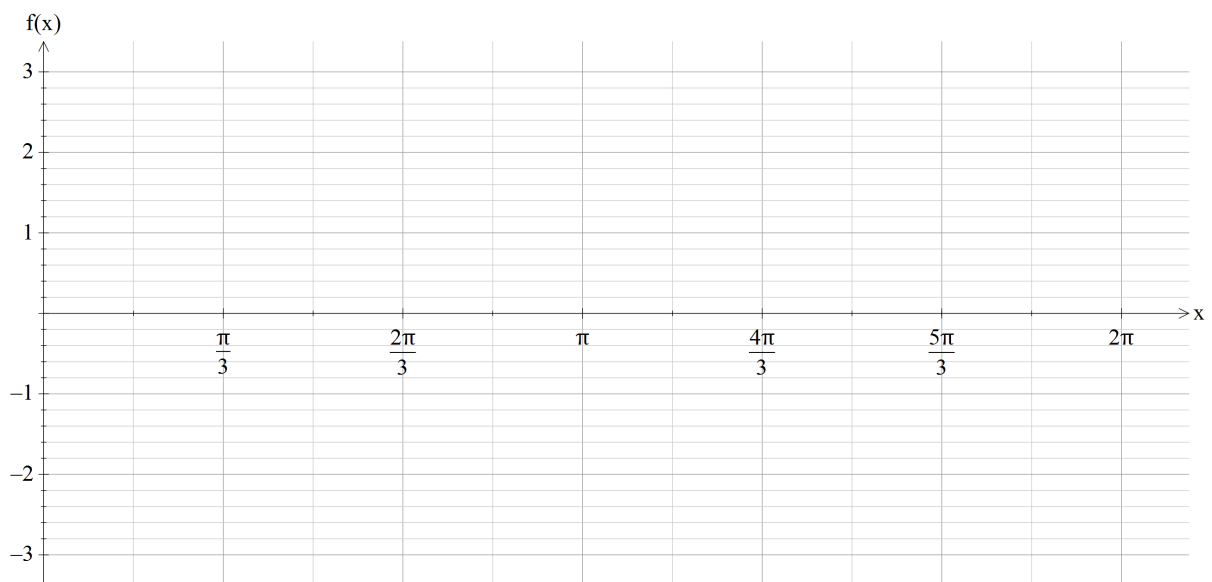
Let $f : [0, 2\pi] \rightarrow \mathbb{R}, f(x) = 2 \sin \left(2 \left(x - \frac{\pi}{12} \right) \right) - 1$.

- a. Find the x – intercepts for $f(x)$

2 marks

- b. Sketch the graph of $f(x)$, labelling any endpoints, axial intercepts and turning points with their coordinates.

3 marks



Question 5 (7 marks)

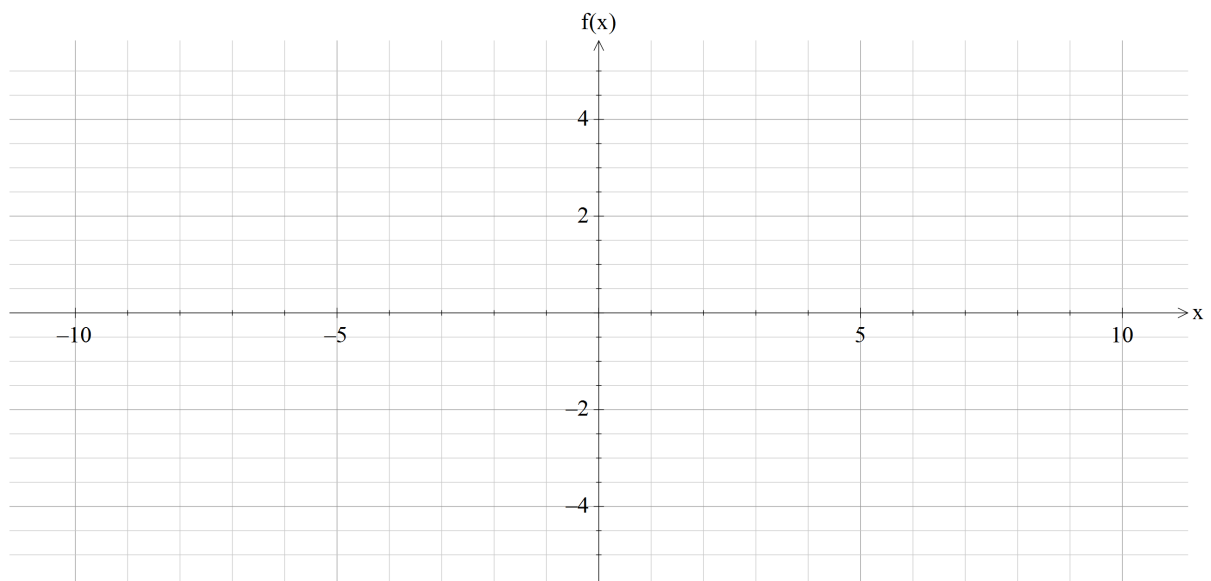
Let $f : R \setminus \{1\} \rightarrow R, f(x) = \frac{5-3x}{1-x}$.

- a. Find the rule of $f^{-1}(x)$, stating the domain.

2 marks

- b. Sketch the graph of $f^{-1}(x)$, labelling axial intercepts with their coordinates and asymptotes with their equations.

3 marks



- c. Find the area bounded by the graph of $f^{-1}(x)$, the x -axis, the line $x = 5$ and $x = 7$.
Express your answer in exact form or in the form $a + \log_e(b)$.

2 marks

Question 6 (3 marks)

A fashion store employs seven people consisting of four females and three males. Only three staff can be rostered on at any time.

- a. Find the probability of the staff on duty all being the same sex.

2 marks

- b. During the day, hundreds of people come through the store. Three in five customers purchase something during the day. On a particular day, 400 customers enter the shop.

1 mark

What is the standard deviation of the Binomial Distribution of customers who buy goods at the store?

Question 7 (3 marks)

A set of linear equations are given as

$$(m+1)x - 2y = k + 1$$

$$-mx + 4y = 2 - k$$

Find the values of m and k which would give infinite solutions.

3 marks

Question 8 (5 marks)

For events A and B from a sample space, $\Pr(A|B) = \frac{2}{q}$ and $\Pr(B|A) = \frac{1}{q+1}$.

Let $\Pr(A \cap B) = \frac{1}{3}$.

- a. Find $\Pr(A)$ in terms of q .

2 marks

- b. Find $\Pr(B)$ in terms of q .

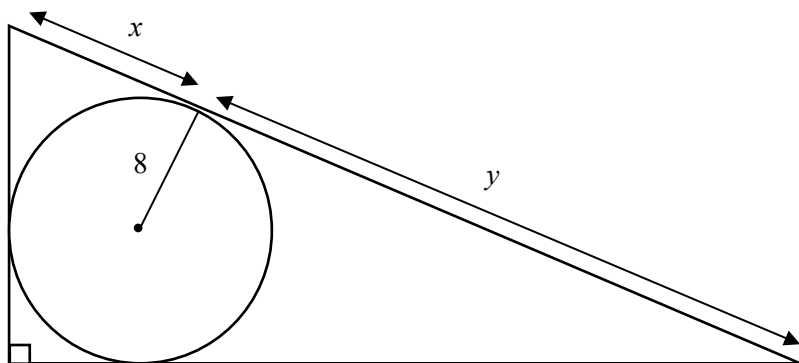
1 mark

- c. Given that A and B are independent events, solve for q .

2 marks

Question 9 (5 marks)

A right-angled triangle fits a circle of radius of 8 units, shown in the figure below.



- a. Show that the y dimension of the right-angles triangle can be expressed as

2 marks

$$y = \frac{128}{x-8} + 8$$

- b. Find the value of x that gives the maximum area of this triangle.

3 marks

Mathematical Methods formulas

Measurement

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc \sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$		$\int x^n dx=\frac{1}{n+1} x^{n+1}+c, n \neq-1$	
$\frac{d}{dx}\left((ax+b)^n\right)=an(ax+b)^{n-1}$		$\int(ax+b)^n dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c, n \neq-1$	
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$		$\int e^{ax} dx=\frac{1}{a} e^{ax}+c$	
$\frac{d}{dx}\left(\log _e(x)\right)=\frac{1}{x}$		$\int \frac{1}{x} dx=\log _e(x)+c, x>0$	
$\frac{d}{dx}(\sin (ax))=a \cos (ax)$		$\int \sin (ax) dx=-\frac{1}{a} \cos (ax)+c$	
$\frac{d}{dx}(\cos (ax))=-a \sin (ax)$		$\int \cos (ax) dx=\frac{1}{a} \sin (ax)+c$	
$\frac{d}{dx}(\tan (ax))=\frac{a}{\cos ^2(ax)}=a \sec ^2(ax)$			
product rule	$\frac{d}{dx}(uv)=u \frac{dv}{dx}+v \frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v \frac{du}{dx}-u \frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du} \frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$	$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$		
$\Pr(A B) = \frac{\Pr(A \cap B)}{\Pr(B)}$	mean	$\mu = E(X)$	