

Victorian Certificate of Education
2019

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

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MATHEMATICAL METHODS

Written examination 1

Friday 31 May 2019

Reading time: 2.00 pm to 2.15 pm (15 minutes)

Writing time: 2.15 pm to 3.15 pm (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
8	8	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 12 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions

- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination

- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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DO NOT WRITE IN THIS AREA

Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

a. Let $y = \frac{2e^{2x} - 1}{e^x}$.

Find $\frac{dy}{dx}$.

2 marks

b. Let $f(x) = x^2 \cos(3x)$.

Find $f'\left(\frac{\pi}{3}\right)$.

2 marks

TURN OVER

Question 2 (2 marks)

Find $f(x)$ given that $f(1) = -\frac{7}{4}$ and $f'(x) = 2x^2 - \frac{1}{4}x^{-\frac{2}{3}}$.

Question 3 (4 marks)

- a. Evaluate $\int_2^7 \frac{1}{x+\sqrt{3}} dx$ and $\int_2^7 \frac{1}{x-\sqrt{3}} dx$.

2 marks

- b. Show that $\frac{1}{2} \left(\frac{1}{x-\sqrt{3}} + \frac{1}{x+\sqrt{3}} \right) = \frac{x}{x^2-3}$.

1 mark

- c. Use your answers to **part a.** and **part b.** to evaluate $\int_2^7 \frac{x}{x^2-3} dx$ in the form $\frac{1}{a} \log_e(b)$, where a and b are positive integers.

1 mark

TURN OVER

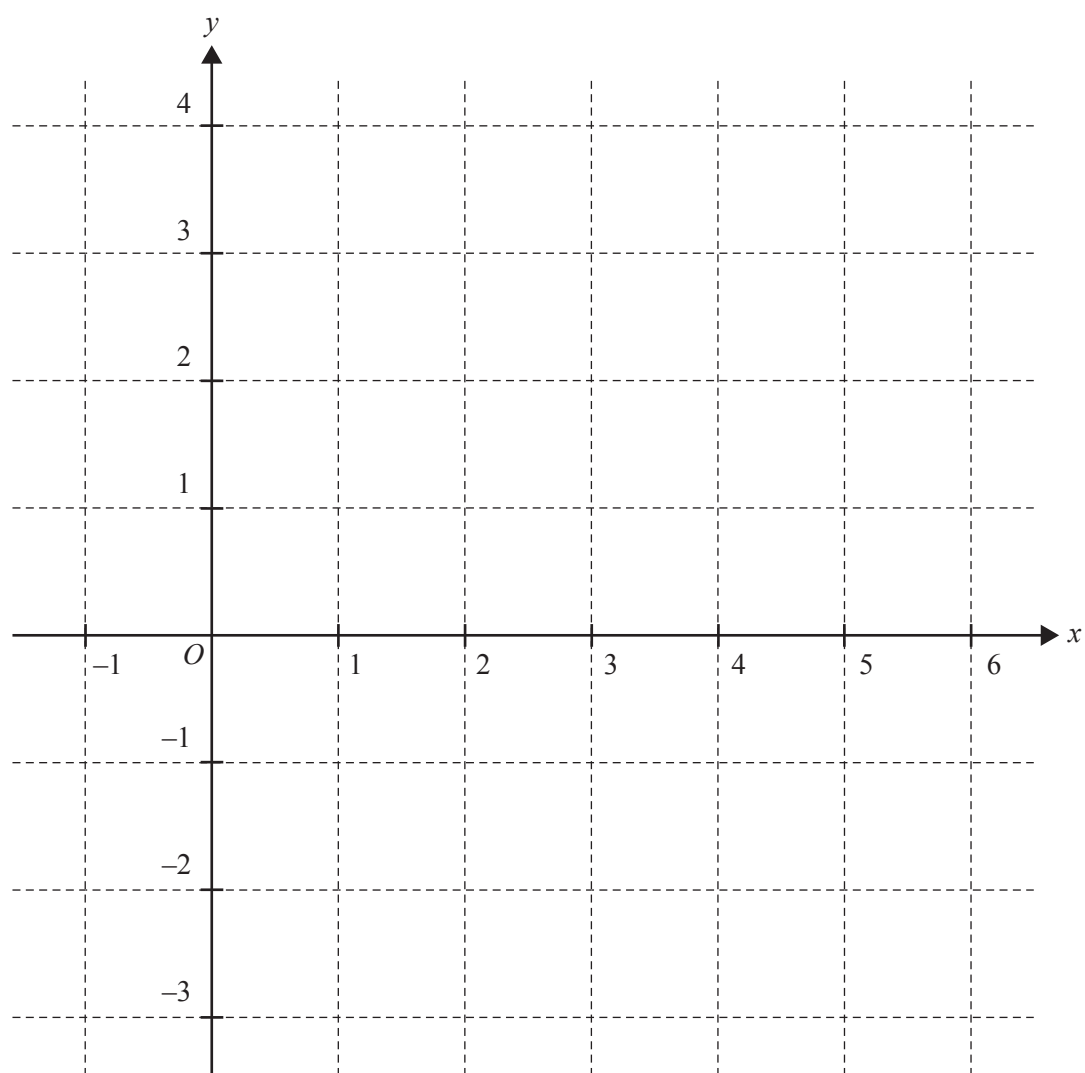
Question 4 (8 marks)

A function g has rule $g(x) = \log_e(x-3) + 2$.

- a.** State the maximal domain of g and the range of g over its maximal domain. 2 marks

- b. i.** Find the equation of the tangent to the graph of g at $(4, 2)$. 2 marks

- ii.** On the axes on page 7, sketch the graph of the function g , labelling any asymptote with its equation. Also draw the tangent to the graph of g at $(4, 2)$. 4 marks



Question 5 (5 marks)

Let $h: \left[-\frac{3}{2}, \infty\right) \rightarrow \mathbb{R}$, $h(x) = \sqrt{2x+3} - 2$.

- a. Find the value(s) of x such that $[h(x)]^2 = 1$.

2 marks

- b. Find the domain and the rule of the inverse function h^{-1} .

3 marks

Question 6 (4 marks)

Jacinta tosses a coin five times.

- a. Assuming that the coin is fair and given that Jacinta observes a head on the first two tosses, find the probability that she observes a total of either four or five heads.

2 marks

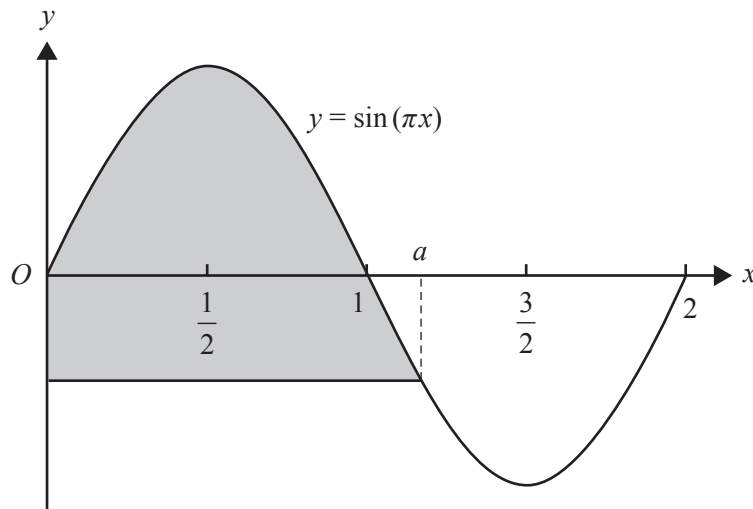
- b. Albin suspects that the coin Jacinta tossed is not actually a fair coin and he tosses it 18 times. Albin observes a total of 12 heads from the 18 tosses.

Based on this sample, find the approximate 90% confidence interval for the probability of observing a head when this coin is tossed. Use the z value $\frac{33}{20}$.

2 marks

Question 7 (8 marks)

The shaded region in the diagram below is bounded by the vertical axis, the graph of the function with rule $f(x) = \sin(\pi x)$ and the horizontal line segment that meets the graph at $x = a$, where $1 \leq a \leq \frac{3}{2}$.



Let $A(a)$ be the area of the shaded region.

- a. Show that $A(a) = \frac{1}{\pi} - \frac{1}{\pi} \cos(a\pi) - a \sin(a\pi)$.

3 marks

- b. Determine the range of values of $A(a)$.

2 marks

- c. i. Express in terms of $A(a)$, for a specific value of a , the area bounded by the vertical axis, the graph of $y = 2\left(\sin(\pi x) + \frac{\sqrt{3}}{2}\right)$ and the horizontal axis.

2 marks

- ii. Hence, or otherwise, find the area described in **part c.i.**

1 mark

Question 8 (5 marks)

A fair standard die is rolled 50 times. Let W be a random variable with binomial distribution that represents the number of times the face with a six on it appears uppermost.

- a.** Write down the expression for $\Pr(W = k)$, where $k \in \{0, 1, 2, \dots, 50\}$. 1 mark

- b.** Show that $\frac{\Pr(W = k + 1)}{\Pr(W = k)} = \frac{(50 - k)}{5(k + 1)}$. 2 marks

- c.** Hence, or otherwise, find the value of k for which $\Pr(W = k)$ is the greatest. 2 marks

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MATHEMATICAL METHODS

Written examination 1

FORMULA SHEET

Instructions

This formula sheet is provided for your reference.
A question and answer book is provided with this formula sheet.

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Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc \sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$		
$\frac{d}{dx}((ax+b)^n) = an(ax+b)^{n-1}$	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$		
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$		
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e(x) + c, x > 0$		
$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$		
$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$	$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$		
$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A B) = \frac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = E(X)$	variance	$\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x p(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x) dx$	$\mu = \int_{-\infty}^{\infty} x f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$

Sample proportions

$\hat{p} = \frac{X}{n}$		mean	$E(\hat{p}) = p$
standard deviation	$\text{sd}(\hat{p}) = \sqrt{\frac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right)$