Victorian Certificate of Education – Free Trial Examinations

MATHEMATICAL METHODS

Free Trial Written Examination 2

SUGGESTED SOLUTIONS AND MARKING GUIDE BOOK

Abbreviations and acronyms

WRT – with respect to; PDF – probability density function; PMF – probability mass function;
nDP – correct to n decimal places

Marking instructions

- All multiple-choice questions are worth 1 mark and can only be awarded if the student selects the correct answer.
- The relevant line(s) of working marked give an indication of what statement should be made in order to obtain that mark, but this is subject to the marker's discretion.
- Any mark related to the method can only be awarded if the student presents a convincing (and rigorous) argument.
- The final answer mark (if any) can only be awarded if the student provides the correct answer in either simplest form or the form required.
- Consequential marks can only be obtained for marks related to the method, not for any final answer.
- If elementary mathematical steps and/or logic are broken within a solution, it must be properly justified in order to obtain full marks.

Miscellaneous notes

Some questions may have multiple methods/solutions, including some that are beyond the scope of the course. The solutions provided are the ones that were intended by the examination authors.

Version 2 (April 2019)

SECTION A – Multiple-choice questions

Question 1 (1 mark)

$$m=-\left(\frac{1-0}{-1-3/4}\right)^{-1}=\frac{7}{4}$$

The answer is **D**.

Question 2 (1 mark)

$$\text{Period}=\frac{\pi}{\pi/2}=2$$

$$\begin{aligned}\text{domain}(g)&=\mathbb{R}\setminus\left\{k\,\middle|\,\cos\left(\frac{\pi k}{2}\right)=0\right\}\\&=\mathbb{R}\setminus\{2k-1\,|\,k\in\mathbb{Z}\}\end{aligned}$$

The answer is **B**.

Question 3 (1 mark)

A following derivative sign table shows a stationary point of inflection.

x	<2	$=2$	>2
Graph direction	\	–	\

The answer is **C**.

Question 4 (1 mark)

$$\bar{f}'=\frac{f(4)-f(0)}{4-0}=\frac{1}{4}\log_e(3)+1$$

The answer is **B**.

Question 5 (1 mark)

Method 1:

Noting that the third transformation swaps x and y coordinates, the transformations applied yield the following:

$$\begin{cases}x'=y\\y'=2x+3\end{cases}\Rightarrow\begin{cases}y=x'\\x=\dfrac{y'-3}{2}\end{cases}$$

Thus, the image of $w(x)$ is given by [dropping dash notation]

$$\left(\frac{y-3}{2}\right)^{-3}=x$$

Hence, $y=2x^{-1/3}+3$.

The answer is **A**.

Method 2:

The transformations (1), (2) and (3) transform the graph of $y=w(x)$ as follows:

$$y=w(x)\xrightarrow{(1)}y=w\left(\frac{x}{2}\right)$$

$$\xrightarrow{(2)}y=w\left(\frac{x-3}{2}\right)$$

$$\xrightarrow{(3)}x=w\left(\frac{y-3}{2}\right)=\left(\frac{y-3}{2}\right)^{-3}\quad\text{[functional inverse transformation]}$$

Therefore, $y=2x^{-1/3}+3$.

The answer is **A**.

Question 6 (1 mark)

$$f(x)=\frac{k(x+1-1)}{x+1}=k-\frac{k}{x+1}$$

The asymptotes of the graph of f are $x=-1$ and $y=k$.

Thus the asymptotes of the graph of f^{-1} are given by $x=k$ and $y=-1$.

The answer is **D**.

Question 7 (1 mark)

$p'(x)=3x^2+2kx+3=0$ will have no real solutions for x if

$(2k)^2-4\times3\times3<0 \Rightarrow -3<k<3$.

The answer is **E**.

Question 8 (1 mark)

$$\frac{1}{4-(-8)}\int_{-8}^{4}h(x)\,dx=0 \Rightarrow a=6$$

The answer is **B**.

Question 9 (1 mark)

$$\begin{cases} mx-4y=m \\ 2x-my=1 \end{cases} \Rightarrow \begin{cases} y=\dfrac{m}{4}x-\dfrac{m}{4} \\ y=\dfrac{2}{m}x-\dfrac{1}{m} \end{cases}$$

The linear system will have a unique solution for (x, y) provided

$\frac{m}{4}\neq\frac{2}{m} \Rightarrow m\neq\pm2\sqrt{2}$.

The answer is **C**.

Question 10 (1 mark)

For $f(x)=e^x-1$, we have

$$\begin{aligned} f(x)+f(y)+f(x)f(y)&=e^x-1+e^y-1+\left(e^x-1\right)\left(e^y-1\right) \\ &=e^x+e^y-2+e^{x+y}-e^x-e^y+1 \\ &=e^{x+y}-1 \\ &=f(x+y) \end{aligned}$$

The answer is **C**.

Question 11 (1 mark)

Where $S\sim\text{Bi}(n, p)$, we have

$$\begin{cases} np=6 \\ np(1-p)=5 \end{cases} \Rightarrow n=36 \text{ and } p=\frac{1}{6}$$

Therefore, $\Pr(S>6)=\Pr(S\geq7)=0.393251...$

The answer is **C**.

Question 12 (1 mark)

Using the chain rule,

$$\frac{d}{dx}[\cos(2f(x))]=-\sin(2f(x))\cdot2f'(x)$$

The answer is **E**.

Question 13 (1 mark)

Since f is a PDF, we have

$$\int_{-a}^{a}f(t)\,dt=1 \Rightarrow a=\log_e\left(\sqrt{2}+1\right)$$

The answer is **A**.

Question 14 (1 mark)

$$64 = \int_0^4 \left(4g\left(\frac{x}{2}\right) + ax\right)dx$$

$$= 8\int_0^2 g(x)\,dx + a\int_0^4 x\,dx$$

$$= 32 + 8a$$

Hence, $a = 4$.

The answer is **B**.

Question 15 (1 mark)

Since X is given by a PMF, we require

$\Pr(X=1) + \Pr(X=2) + \Pr(X=3) = 1 \Rightarrow a = \frac{1}{20}$

The answer is **B**.

Question 16 (1 mark)

Method 1:

$y = -f\left(\sqrt{x}\right)$ is defined only for $x > 0$, and using the fact that $g(x) = \sqrt{x}$ is strictly increasing, local extrema occur at the values of x for which

$\sqrt{x} = \frac{1}{2}, 3 \Rightarrow x = \frac{1}{4}, 9$

The minus sign in front of f inverts the quality of the stationary points and their corresponding y-coordinates, and so, we have

a local minimum at $\left(\frac{1}{4}, -\frac{49}{16}\right)$ and a local maximum at $(9, 36)$.

The answer is **D**.

Method 2:

We can guess that the rule of f has the form $f(x) = kx(x+3)(x-1)(x-4)$, where $k > 0$, and with $f(3) = -36$, we have $k = 1$.

$\frac{d}{dx}\left[-f\left(\sqrt{x}\right)\right] = -2x + 3\sqrt{x} - \frac{6}{\sqrt{x}} + 11 = 0 \Rightarrow x = \frac{1}{4}, 9.$

$-f\left(\sqrt{\frac{1}{4}}\right) = \frac{-49}{16}$ and $-f\left(\sqrt{9}\right) = 36$, and using a graph, we have

a local minimum at $\left(\frac{1}{4}, -\frac{49}{16}\right)$ and a local maximum at $(9, 36)$.

The answer is **D**.

Question 17 (1 mark)

The sample proportion used to construct the confidence interval is

$\hat{p} = \frac{0.035434 + 0.064566}{2} = 0.05.$

Hence, $2 \times 1.95996\ldots\sqrt{\frac{0.05 \times 0.95}{n}} = 0.064556 - 0.035434 \Rightarrow n = 860.$

The answer is **D**.

Question 18 (1 mark)

$\int f(x)dx = g'(x)$ gives $\frac{-a}{m}e^{-mx} + c = -bne^{-nx}$.

Comparing components, we have

$$\begin{cases} (1) & c = 0 \\ (2) & -m = -n \\ (3) & \frac{-a}{m} = -bn \end{cases}.$$

Equation (2) gives $\frac{m}{n} = 1 \in \mathbb{N}$, $\frac{n}{m} = 1 \in \mathbb{N}$ and $\frac{m^2}{n^2} = 1 \in \mathbb{N}$.

Equation (3) gives $\frac{a}{b} = mn \in \mathbb{N}$, and $\frac{b}{a} = \frac{1}{mn}$, but $\frac{1}{mn}$ is not necessarily a

a natural number.

The answer is **E**.

Question 19 (1 mark)

Let n be the number of red marbles (and black marbles) in the bag.

$\Pr\left(\hat{P} = 0\right) = \frac{n}{2n} \times \frac{n-1}{2n-1} \times \frac{n-2}{2n-2} \times \frac{n-3}{2n-3} = \frac{1}{33} \Rightarrow n = 6 \quad [n \in \mathbb{N}]$

Thus the total number of marbles in the bag is $2n = 12$.

The answer is **B**.

Question 20 (1 mark)

Using a graph, it is clear that the graphs of f and g intersect once if $a < 0$.

Noting that the graphs will always intersect at $(0, 0)$, we have $g'(0) = a$,

and so, when $a = 2$, $y = f(x)$ is tangent to the graph of g.

Using a graph, it can be seen that when $a = 2$, the graphs of f and g cross,

and also only intersect once for $a > 2$.

Thus, the *maximal* set of values of a is $a \in (-\infty, 0) \cup [2, \infty)$.

The answer is **E**.

SECTION B

Question 1a (1 mark)

Mark	Criteria
1	Provides correct answers

Period $= \frac{2\pi}{1/2} = 4\pi$ **Mark 1**

$\text{range}(f) = [-2, 6]$

Question 1b.i (1 mark)

Mark	Criteria
1	Provides correct answer

$f'(x) = -2\cos\left(\frac{x}{2}\right)$ **Mark 1**

Question 1b.ii (1 mark)

Mark	Criteria
1	Provides correct answer

$f'(x) = 0 \Rightarrow x = \pi, 3\pi, 5\pi, 7\pi$

Using a graph, $f'(x) > 0$ for $x \in (\pi, 3\pi) \cup (5\pi, 7\pi)$ **Mark 1**

Question 1b.iii (2 marks)

Mark	Criteria
1	Writes down expressions for x and y in terms of the images of x and y
2	Provides correct answer

$$\begin{cases} x' = x + \pi \\ y' = ay + b \end{cases} \Rightarrow \begin{cases} x = x' - \pi \\ y = \dfrac{y' - b}{a} \end{cases}$$ **Mark 1**

Hence, noting that $\sin\left(\theta - \frac{\pi}{2}\right) \equiv -\cos(\theta)$, we have

$$\frac{f'(x) - b}{a} = 2 - 4\sin\left(\frac{x - \pi}{2}\right) \Rightarrow f'(x) = 2a + b + 4a\cos\left(\frac{x}{2}\right).$$

By comparing components, we have $a = -\frac{1}{2}$ and $b = 1$. **Mark 2**

Question 1c.i (2 marks)

Mark	Criteria
1	Provides correct answer for tangent where $x = \pi/3$
2	Provides correct answer for tangent where $x = 13\pi/3$

At $x = \frac{\pi}{3}$, we have

$$y_1 = f'\left(\frac{\pi}{3}\right)\left(x - \frac{\pi}{3}\right) + f\left(\frac{\pi}{3}\right) \Rightarrow y_1 = \frac{\pi\sqrt{3}}{3} - \sqrt{3}x.$$ **Mark 1**

At $x = \frac{13\pi}{3}$, we have

$$y_2 = f'\left(\frac{13\pi}{3}\right)\left(x - \frac{13\pi}{3}\right) + f\left(\frac{13\pi}{3}\right) \Rightarrow y_2 = \frac{13\pi\sqrt{3}}{3} - \sqrt{3}x.$$ **Mark 2**

Question 1c.ii (3 marks)

Method 1:

Mark	Criteria
1	Finds the equation of a line perpendicular to both tangents
2	Finds points of intersection between perpendicular line segment and tangents
3	Provides correct answer

A line that is perpedicular to both y_1 and y_2 is the line $y = \frac{1}{\sqrt{3}}x$. **Mark 1**

$$\frac{1}{\sqrt{3}}x = \frac{\pi\sqrt{3}}{3} - \sqrt{3}x \Rightarrow x = \frac{\pi}{4}$$

$$\frac{1}{\sqrt{3}}x = \frac{13\pi\sqrt{3}}{3} - \sqrt{3}x \Rightarrow x = \frac{13\pi}{4}$$

Thus, $y = \frac{1}{\sqrt{3}}x$ intersects y_1 at $\left(\frac{\pi}{4}, \frac{\pi}{4\sqrt{3}}\right)$

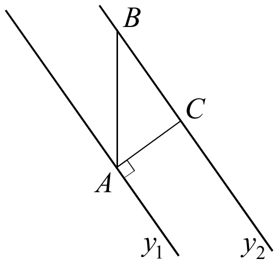
and intersects y_2 at $\left(\frac{13\pi}{4}, \frac{13\pi}{4\sqrt{3}}\right)$. **Mark 2**

Hence, $d = \sqrt{\left(\frac{13\pi}{4} - \frac{\pi}{4}\right)^2 + \left(\frac{13\pi}{4\sqrt{3}} - \frac{\pi}{4\sqrt{3}}\right)^2} = 2\pi\sqrt{3}$ units. **Mark 3**

Method 2:

Mark	Criteria
1	Forms a triangle with a line perpendicular to both tangents and a vertical line
2	Finds a relevant angle in the triangle
3	Provides correct answer

Consider the following triangle shown.



The gradient of a line perpedicular to both y_1 and y_2 is $\frac{1}{\sqrt{3}}$, and so, **Mark 1**

$\angle BAC = \frac{\pi}{3}$. **Mark 2**

Hence, $d = (y_2 - y_1)\cos\left(\frac{\pi}{3}\right) = 2\pi\sqrt{3}$ units. **Mark 3**

Question 2a (2 marks)

Mark	Criteria
1	Finds the values of *m* and *k*
2	Provides a correct method

$\begin{cases} P_1(0) = \frac{1}{5} \\ P_1(4) = \frac{4}{5} \end{cases} \Rightarrow m = 4 \text{ and } k = \log_e(2)$ **Mark 1**

Thus, $P_1(t) = 1 - \frac{4}{e^{\log_e(2)t} + 4} = 1 - \frac{4}{2^t + 4}$, as required. **Mark 2**

Question 2b.i (1 mark)

Mark	Criteria
1	Provides correct answer

$\bar{P}_1' = \frac{P_1(4) - P_1(0)}{4 - 0} = \frac{3}{20} \text{ day}^{-1}$ **Mark 1**

Question 2b.ii (2 marks)

Mark	Criteria
1	Forms equation involving *P'*(*t*) and answer to **Q2b.i**
2	Provides correct answer

Let $P_1'(t) = \frac{4\log_e(2)\cdot 2^t}{(2^t + 4)^2} = \frac{3}{20}$. **Mark 1**

$t = 0.9,\ 3.1$ days (2DP) **Mark 2**

Question 2c.i (1 mark)

Mark	Criteria
1	Provides correct answer

$P_2'(t) = \frac{4\log_e(2)\cdot 2^t}{(2^t + 2)^2} - \frac{3}{50} = 0$ gives maximum coverage after

$t = 5.2$ days (1DP). **Mark 1**

Question 2c.ii (1 mark)

Mark	Criteria
1	Provides correct answer

$\text{maximum}(P_2) = P_2(5.23979...) = 59\%$ (0DP) **Mark 1**

Question 2d (2 marks)

Mark	Criteria
1	Finds values of t for which $P_2(t)=0.5$
2	Provides correct answer

$P_2(t)=\frac{1}{2} \Rightarrow t=3.14596..., \ 8.09264...$ days **Mark 1**

Hence, using a graph, we have more than 50% coverage for

$T=8.09264...-3.14596...=4.9$ days (1DP). **Mark 2**

Question 2e (1 mark)

Mark	Criteria
1	Provides correct answer

$P_2(t)=0 \Rightarrow$ time to eradication is $t=16.7$ days (1DP) **Mark 1**

Question 2f (1 mark)

Mark	Criteria
1	Forms two equations to be solved simultaneously
2	Provides answer for q
3	Provides answer for eradication time

Let α be the value of t for which the bacteria cover the plate maximally.

$$P_3'(t)=\frac{4\log_e(2)\cdot 2^t}{\left(2^t+4\right)^2}-q.$$

Since, $\left(\alpha, \frac{1}{2}\right)$ is the local maximum of P_3, we have $\begin{cases} P_3(\alpha)=1/2 \\ P_3'(\alpha)=0 \end{cases}$ **Mark 1**

$\Rightarrow \alpha=4.75188...$ and $q=0.078$ (3DP) **Mark 2**

Hence, $P_3(t)=0 \Rightarrow$ time to eradication is $t=12.8$ days (1DP). **Mark 3**

Question 3a.i (1 mark)

Mark	Criteria
1	Provides correct answer

Let $W \sim \text{Bi}(32, 0.08)$.

$\Pr(W \geq 1)=0.9306$ (4DP) **Mark 1**

Question 3a.ii (2 marks)

Mark	Criteria
1	Applies conditional probability definition
2	Provides correct answer

$$\Pr(W<4 \mid W \geq 1)=\frac{\Pr(1 \leq W \leq 3)}{\Pr(W \geq 1)}$$ **Mark 1**

$$=\frac{0.679495...}{0.930623...}$$

$=0.7302$ (4DP) **Mark 2**

Question 3b (1 mark)

Mark	Criteria
1	Provides correct answer

Let $B \sim \text{N}\left(300, 8^2\right)$.

$\Pr(B>315)=0.0304$ (4DP) **Mark 1**

Question 3c (3 marks)

Mark	Criteria
1	Applies conditional probability definition
2	Writes expression in terms of a newly defined binomial variable
3	Provides correct answer

$$\Pr\left(\hat{P}<0.05 \mid \hat{P}>0.01\right)=\frac{\Pr\left(0.01<\hat{P}<0.05\right)}{\Pr\left(\hat{P}>0.01\right)}$$ **Mark 1**

$$=\frac{\Pr(2<X<10)}{\Pr(X>2)}, \quad [X\sim\mathrm{Bi}(200,\ 0.0303...)]$$

$$=\frac{\Pr(3\le X\le 9)}{\Pr(X\ge 3)}$$ **Mark 2**

$$=\frac{0.857838...}{0.944098...}$$

$$=0.909 \quad (3\text{DP})$$ **Mark 3**

Question 3d (2 marks)

Mark	Criteria
1	Calculates the standard deviation of $\hat{P}$
2	Provides correct answer

$$\mathrm{sd}\left(\hat{P}\right)=\sqrt{\frac{0.030396...\times 0.969604...}{200}}=0.12139...$$ **Mark 1**

Since $\Pr(Z<1.28155...)=0.9$, where Z denotes the standard normal variable, we have

$b=0.12139...\times 1.28155+0.030396...=0.0460 \quad (4\text{DP})$. **Mark 2**

Question 3e (1 mark)

Mark	Criteria
1	Forms an equation involving σ
2	Provides correct answer

Let $G\sim \mathrm{N}\left(300,\ \sigma^2\right)$

$0.05=\Pr(G>315)$

$$\Rightarrow \Pr(Z>1.64485...)=\Pr\left(Z>\frac{315-300}{\sigma}\right)$$ **Mark 1**

$$\Rightarrow \frac{315-300}{\sigma}=1.64485...$$

Hence, $\sigma=9.1194$ mL $\quad$ (4DP) **Mark 2**

Question 3f (1 mark)

Mark	Criteria
1	Provides correct answer

$$\Pr(E)=[\Pr(G>300)]^3\times\Pr(G<300)$$

$$=\left(\frac{1}{2}\right)^3\times\frac{1}{2}$$

$$=\frac{1}{16}$$ **Mark 1**

Question 3g (1 mark)

Mark	Criteria
1	Provides correct answer

$$\text{CI}: \left(\frac{36}{50}-1.959...\sqrt{\frac{36/50\times 14/50}{50}},\ \frac{36}{50}+1.959...\sqrt{\frac{36/50\times 14/50}{50}}\right)$$

$$=(0.5955,\ 0.8445) \quad (4\text{DP})$$ **Mark 1**

Question 3h.i (1 mark)

Mark	Criteria
1	Provides correct answer

$\mathrm{E}(R) = \int_{250}^{310} r f(r)\,dr = \frac{850}{3}$ mL **Mark 1**

Question 3h.ii (1 mark)

Mark	Criteria
1	Writes down definite integral expression for sd(R)
2	Provides correct answer

$\mathrm{sd}(R) = \sqrt{\int_{250}^{310} \left(r - \frac{850}{3}\right)^2 f(r)\,dr}$ **Mark 1**

$= \frac{10\sqrt{17}}{3}$ mL **Mark 2**

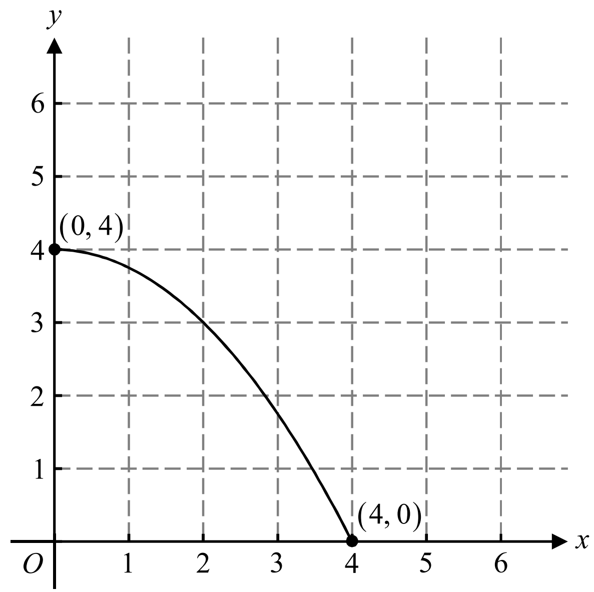
Question 3i (1 mark)

Mark	Criteria
1	Provides correct answer

$\Pr(R > 300) = \int_{300}^{310} f(r)\,dr = 0.1273 \quad (4\text{DP})$ **Mark 1**

Question 4a (1 mark)

Mark	Criteria
1	Sketches current graph shape and labels endpoints correctly



Question 4b.i (2 marks)

Mark	Criteria
1	Provides correct rule of inverse of f
2	Provides correct domain

Let $y = f^{-1}(x)$.

$x = 4 - \frac{1}{4}y^2 \;\Rightarrow\; y = \pm 2\sqrt{4-x}$

However, we require $0 \le f^{-1}(x) \le 4$, so $f^{-1}(x) = 2\sqrt{4-x}$. **Mark 1**

$\mathrm{range}(f) = \mathrm{domain}\left(f^{-1}\right) = [0, 4]$ **Mark 2**

Question 4b.ii (1 mark)

Mark	Criteria
1	Finds values of x for which f and its inverse intersect
2	Writes down a definite integral expression for the area
3	Provides correct answer

Using a graph, f and f^{-1} intersect at both endpoints and once on the line $y = x$.

$f(x) = x \Rightarrow x = 2\sqrt{5} - 2 \quad [x > 0]$ **Mark 1**

Using a graph, we have $f(x) > f^{-1}(x)$ for $0 < x < 2\sqrt{5} - 2$, and so by symmetry about the line $y = x$,

$B = 2\int_0^{2\sqrt{5}-2} \left[f(x) - f^{-1}(x) \right] dx$ **Mark 2**

$= \dfrac{80\sqrt{5} - 176}{3}$ units^2 **Mark 3**

Question 4c.i (1 mark)

Mark	Criteria
1	Provides correct answer

The rectangle has width m and height $f(m)$ and so,

$R(m) = m\,f(m) = 4m - \dfrac{1}{4}m^3.$ **Mark 1**

Question 4c.ii (2 marks)

Mark	Criteria
1	Provides correct answer for m for which $R(m)$ is maximum
2	Provides correct answer for A_R

$R'(m) = 4 - \dfrac{3}{4}m^2 = 0 \Rightarrow m = \dfrac{4}{\sqrt{3}} \quad [0 \le m \le 4]$ **Mark 1**

$A_R = A\left(\dfrac{4}{\sqrt{3}}\right) = \dfrac{32\sqrt{3}}{9}$ units^2 **Mark 2**

Question 4c.iii (2 marks)

Mark	Criteria
1	Finds S
2	Provides correct answer

$S = \int_0^4 f(x)\,dx = \dfrac{32}{3}$ units^2 **Mark 1**

Therefore, $\dfrac{A_R}{S} = \dfrac{1}{\sqrt{3}}.$ **Mark 2**

Question 4c.ii (2 marks)

Mark	Criteria
1	Provides a correct method

$g(0) = 4 - 4 \times 0^k = 4 \Rightarrow (0, 4)$ lies on g

$g(4) = 4 - 4 \times 1^k = 0 \Rightarrow (4, 0)$ lies on g, as required. **Mark 1**

Question 4e (1 mark)

Mark	Criteria
1	Provides a correct method

$$T(z)=\frac{1}{2}\left(4+4-4\left(\frac{z}{4}\right)^k\right)\cdot z$$ **Mark 1**

$$=4z-2z\left(\frac{z}{4}\right)^k, \text{ as required.}$$

Question 4f.i (3 marks)

Mark	Criteria
1	Differentiates T WRT z
2	Finds expression for z in terms k that gives a maximum-sized trapezium
3	Shows required result algebraically

Let $T'(z)=4-2(k+1)\left(\frac{z}{4}\right)^k=0.$ **Mark 1**

Then, $\left(\frac{z}{4}\right)^k=\frac{2}{k+1} \Rightarrow z=4\left(\frac{2}{k+1}\right)^{1/k} \quad [k>1 \text{ and } 0\le z\le 4]$ **Mark 2**

$$A_{\mathrm{T}}(k)=T\left[4\left(\frac{2}{k+1}\right)^{1/k}\right]$$

$$=16\left(\frac{2}{k+1}\right)^{1/k}-8\left(\frac{2}{k+1}\right)^{1/k}\times\frac{2}{k+1}$$

$$=\left(16-\frac{16}{k+1}\right)\left(\frac{2}{k+1}\right)^{1/4}$$ **Mark 3**

$$=\frac{16k}{k+1}\left(\frac{2}{k+1}\right)^{1/k}, \text{ as required.}$$

Question 4f.ii (1 mark)

Mark	Criteria
1	Provides a correct answer

Method 1:

Using a graph, A_{T} is strictly increasing and is bounded above, so

$p=\lim_{k\to\infty} A_{\mathrm{T}}(k)=16.$ **Mark 1**

Method 2:

The trapezium is bounded by the coordinate axes and the graph of g.

Since g is strictly decreasing for all $k>1$, it is too bounded inside the square described by $0\le x\le 4$ and $0\le y\le 4$, which has area 16 units^2.

Thus, $p=16.$ **Mark 1**

Question 4g.i (1 mark)

Mark	Criteria
1	Provides a correct answer

$A=\int_0^4 g(x)\,dx=\frac{16k}{k+1}$ units^2 **Mark 1**

Question 4g.i (1 mark)

Mark	Criteria
1	Provides a correct answer

$\frac{d}{dk}\left[\frac{A_{\mathrm{T}}}{A}\right]=0 \Rightarrow k=3.31107...$ **Mark 1**

$A_{\mathrm{T}}(3.31107...)=9.7446$ units^2 (4DP) **Mark 2**

END OF SUGGESTED SOLUTIONS AND MARKING GUIDE BOOK