

MATHEMATICAL METHODS

Units 3 & 4 – Written examination 1



2019 Trial Examination

SOLUTIONS

Question 1 (4 marks)

a. $u = 2x - 4$ $u' = 2$
 $v = x^2$ $v' = 2x$

$$\frac{dy}{dx} = \frac{2x^2 - 2x(2x - 4)}{(x^2)^2} \quad (1M)$$

$$\frac{dy}{dx} = \frac{2x^2 - 4x^2 + 8x}{x^4}$$

$$\frac{dy}{dx} = \frac{8x - 2x^2}{x^4}$$

$$\frac{dy}{dx} = \frac{x^4}{8 - 2x}$$

$$\frac{dy}{dx} = \frac{8 - 2x}{x^3} \quad (1A)$$

2 marks

b. $u = 1 + x^2$ $u' = 2x$
 $v = \cos x$ $v' = -\sin x$

$$f'(x) = -(1 + x^2) \sin x + 2x \cos x \quad (1M)$$

$$f'(-\pi) = -(1 + (-\pi)^2) \sin(-\pi) + 2(-\pi) \cos(-\pi)$$

$$f'(-\pi) = -(1 + (-\pi)^2) \times 0 + 2(-\pi) \times -1$$

$$f'(-\pi) = 2\pi \quad (1A)$$

2 marks

Question 2 (2 marks)

$$f(x) = \int \frac{4}{(2-x)^2} dx$$

$$f(x) = \frac{-4}{x-2} + c \quad (1M)$$

$$5 = \frac{-4}{-2-2} + c$$

$$c = 4$$

$$f(x) = \frac{-4}{x-2} + 4 \quad (1A)$$

2 marks

Question 3 (4 marks)

a. $u = x \quad u' = 1$
 $v = \cos(3x) \quad v' = -3 \sin(3x)$
 $\frac{dy}{dx} = \cos(3x) - 3x \sin(3x)$

1 mark

b. $\int \cos(3x) - 3x \sin(3x) dx = x \cos(3x) + c \quad (1M)$
 $\int \cos(3x) dx - \int 3x \sin(3x) dx = x \cos(3x) + c$
 $-3 \int \sin(3x) dx = x \cos(3x) - \int \cos(3x) dx + c$
 $\int x \sin(3x) dx = -\frac{1}{3} x \cos(3x) + \frac{1}{3} \int \cos(3x) dx + c \quad (1M)$
 $2 \int x \sin(3x) dx = -\frac{2}{3} x \cos(3x) + \frac{2}{3} \int \cos(3x) dx + c$
 $\int 2x \sin(3x) dx = -\frac{2}{3} x \cos(3x) + \frac{2}{9} \sin(3x) + c \quad (1A)$

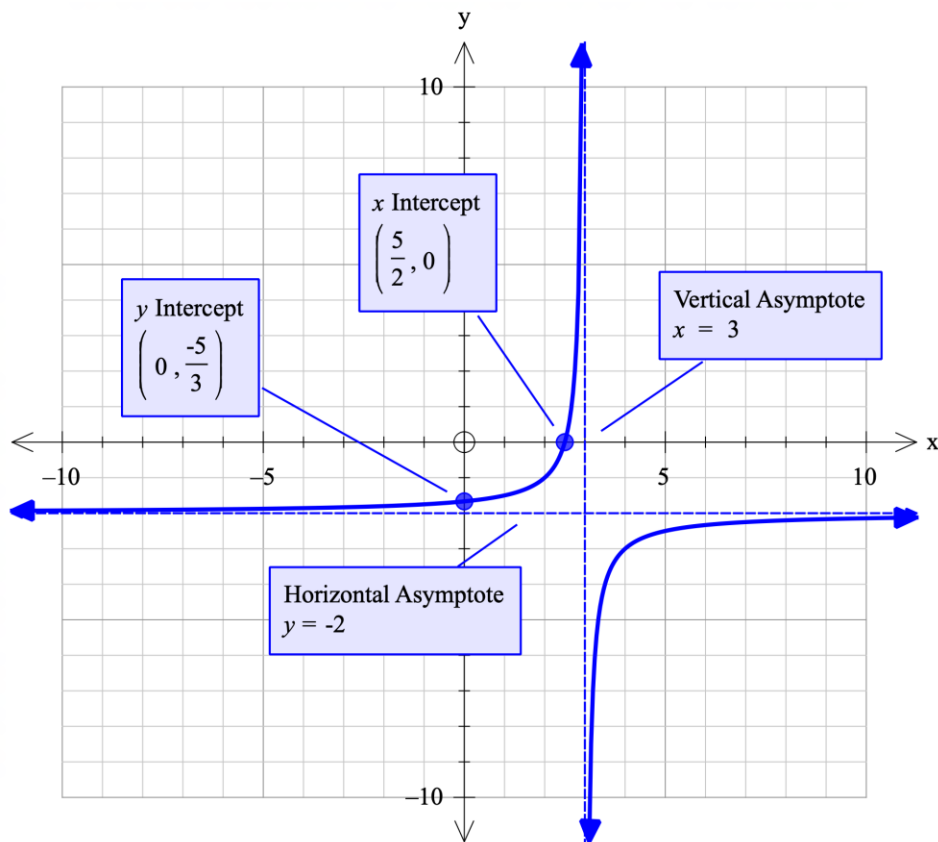
3 marks

Question 4 (5 marks)

a. Let $y = \frac{-1}{x+2} + 3$
 For inverse swap x and y
 $x = \frac{-1}{y+2} + 3 \quad (1M)$
 $x - 3 = \frac{-1}{y+2}$
 $y + 2 = \frac{-1}{x-3}$
 $y = \frac{-1}{x-3} - 2$
 $f^{-1}(x) = \frac{-1}{x-3} - 2$
 $f^{-1}: R \setminus \{3\} \rightarrow R, f^{-1}(x) = \frac{-1}{x-3} - 2 \quad (1A)$

3 marks

b.



(½M each intercept, ½M each asymptote)
2 marks

Question 5 (4 marks)

a. $\cos(x)^2 + 2 \cos(x) \tan(x) + \tan(x)^2 - (\cos(x)^2 - 2 \cos(x) \tan(x) + \tan(x)^2)$ (1M)
 $\cos(x)^2 - \cos(x)^2 + \tan(x)^2 - \tan(x)^2 + 2 \cos(x) \tan(x) + 2 \cos(x) \tan(x)$
 $4 \cos(x) \tan(x)$
 $4 \cos(x) \times \frac{\sin(x)}{\cos(x)}$ (1M)
 $4 \sin(x)$

2 marks

b. $\cos\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}$
 $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ (1M)
 $\frac{x}{2} = \frac{\pi}{6}, \frac{11\pi}{6}$
 $x = \frac{\pi}{3}, \frac{11\pi}{3}$
 For domain of $0 \leq x \leq 3\pi$
 $x = \frac{\pi}{3}$ (1A)

2 marks

Question 6 (3 marks)

$$g'(x) = 3x^2 + 8x - 3$$

$$0 = 3x^2 + 8x - 3$$

$$0 = (x + 3)(3x - 1)$$

$$x = -3 \text{ or } x = \frac{1}{3}$$

$$g(-3) = (-3)^3 + 4(-3)^2 - 3(-3) - 2 = 16, (-3, 16)$$

$$g\left(\frac{1}{3}\right) = \left(\frac{1}{3}\right)^3 + 4\left(\frac{1}{3}\right)^2 - 3\left(\frac{1}{3}\right) - 2 = -\frac{68}{27}, \left(\frac{1}{3}, -\frac{68}{27}\right)$$

(1M)

$g'(x)$	-4	-3	0	$\frac{1}{3}$	1
Sign	+	0	-	0	+
Slope	/	-	\	-	/

$(-3, 16)$ is a maximum (1A)

$\left(\frac{1}{3}, -\frac{68}{27}\right)$ is a minimum (1A)

3 marks

Question 7 (3 marks)

Probability Map to support question 7 (bold is given in question)

	A	A'	
B	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{2}{5}$
B'	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$
	$\frac{3}{10}$	$\frac{7}{10}$	1

a. $\Pr(A' \cap B') = \Pr(B') - \Pr(A \cap B')$

$$\Pr(A' \cap B') = \frac{3}{5} - \frac{1}{5}$$

$$\Pr(A' \cap B') = \frac{2}{5}$$

1 mark

b. $\Pr(B|A') = \frac{\Pr(B \cap A')}{\Pr(A')}$ (1M)

$$\Pr(B|A') = \frac{\frac{10}{7}}{\frac{3}{10}}$$

$$\Pr(B|A') = \frac{3}{7} \quad (1A)$$

2 marks

Question 8 (4 marks)

a. $\int_1^2 (ax^2 - ax) dx = 1$

$$\left[\frac{ax^3}{3} - \frac{ax^2}{2} \right]_1^2 = 1 \quad (1M)$$

$$\left[\frac{a \times 2^3}{3} - \frac{a \times 2^2}{2} \right] - \left[\frac{a \times 1^3}{3} - \frac{a \times 1^2}{2} \right] = 1$$

$$\frac{2a}{3} - \frac{-a}{6} = 1$$

$$\frac{5a}{6} = 1 \quad (1M)$$

$$a = \frac{6}{5}$$

2 marks

b. $\int_1^b \left(\frac{6}{5}x^2 - \frac{6}{5}x \right) dx = \frac{1}{5}$

$$\left[\frac{2x^3}{5} - \frac{3x^2}{5} \right]_1^b = \frac{1}{5}$$

$$\left[\frac{2 \times b^3}{5} - \frac{3 \times b^2}{5} \right] - \left[\frac{2 \times 1^3}{5} - \frac{2 \times 1^2}{5} \right] = \frac{1}{5}$$

$$\frac{2b^3}{5} - \frac{3b^2}{5} + \frac{1}{5} = \frac{1}{5} \quad (1M)$$

$$\frac{2b^3}{5} - \frac{3b^2}{5} = 0$$

$$2b^3 - 3b^2 = 0$$

$$b^2(2b - 3) = 0$$

$$b = 0 \text{ or } b = \frac{3}{2}$$

$$\text{As } b > 1, \text{ then } b = \frac{3}{2} \quad (1M)$$

2 marks

Question 9 (12 marks)

a. Gradient = $\tan \theta$, therefore $m = \tan \frac{\pi}{4} = 1 \quad (1M)$

$$\frac{f(a) - f(b)}{a - b} = 1$$

To find b, let $f(x) = 0$

$$0 = x^3 - 3x^2 + 4$$

$$\text{Using factor theorem } f(2) = 0 \text{ and } f(-1) = 0, \text{ therefore } b = -1 \quad (1A)$$

To find a, sub in $(-1, 0)$

$$\frac{f(a) - 0}{a - (-1)} = 1$$

$$\frac{f(a)}{a + 1} = 1$$

$$f(a) = a + 1$$

$$a^3 - 3a^2 + 4 = a + 1$$

$$a^3 - 3a^2 - a + 3 = 0$$

$$\text{Using factor theorem } f(1) = 0, f(-1) = 0 \text{ and } f(3) = 0, \text{ therefore } a = 1 \quad (1A)$$

3 marks

b. $f'(x) = 3x^2 - 6x$
 $1 = 3x^2 - 6x$ (1M)
 $0 = 3x^2 - 6x - 1$
 $x = \frac{6 \pm \sqrt{(-6)^2 - 4 \times 3 \times -1}}{2 \times 3}$
 $x = \frac{6 \pm \sqrt{36 + 12}}{6}$
 $x = \frac{6 \pm 4\sqrt{3}}{6}$
 $x = \frac{3 \pm 2\sqrt{3}}{3}$ (1A)

2 marks

c. Equation of line segment $m = 1, f(1) = 2$
 $y - 2 = 1(x - 1)$
 $y = x + 1$ (1A)
 Enclosed area
 $\int_{-1}^1 (f(x) - (x + 1))dx + \int_1^3 ((x + 1) - f(x))dx$ (1M)
 $\left[\frac{x^4}{4} - x^3 - \frac{x^2}{2} + 3x \right]_{-1}^1 + \left[-\frac{x^4}{4} + x^3 + \frac{x^2}{2} - 3x \right]_1^3$
 $\left[\frac{7}{4} - \frac{-9}{4} \right] + \left[\frac{9}{4} - \frac{-7}{4} \right]$
 8 square units (1A)

3 marks

d. $-2x^2 + 4 = -\frac{1}{2}$
 $x^2 = \frac{9}{4}, x = \pm \frac{3}{2}$
 As $x < 0, x = -\frac{3}{2}$

1 mark

e. $\int_{-\sqrt{2}}^1 (g(x) - (x + 1))dx$ (1M)
 $= \frac{11}{6} - \left(-\sqrt{6} - \frac{3}{4} \right)$
 $= \frac{31}{12} + \sqrt{6}$ (1M)
 Difference
 $= \frac{31}{12} + \sqrt{6} - 8$
 $= \frac{31}{12} + \sqrt{6} - \frac{96}{12}$
 $= -\frac{65}{12} + \sqrt{6}$ (1A)

marks
3 marks