TRIUMPH PUBLICATIONS

Fortify Sample Exam 1A

MATHEMATICAL METHODS

Written examination 1

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer booklet of 15 pages.
- Formula sheet.
- Working space is provided throughout the book.

Instructions
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the space provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

a. Differentiate $x \log_e(x^2)$ with respect to x.

2 marks

b. Let $h(x) = 2e^{\sin(3x)}$.

Evaluate $h'\left(\frac{\pi}{18}\right)$.

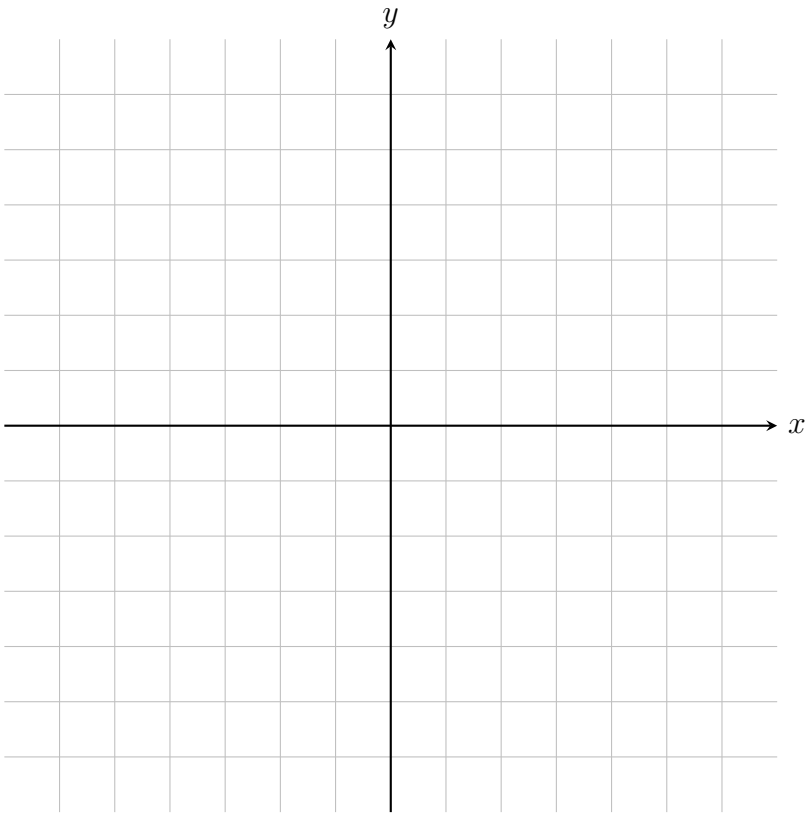
2 marks

Question 2 (3 marks)

The line $0 = y - bx - 4 + \dfrac{\pi}{2\sqrt{3}}$ is a tangent to the curve $y = 2\sin\left(\dfrac{x}{2}\right) + a$ at the point $\left(\dfrac{\pi}{3},\ c\right)$, where a, b and c are real constants. Find the values of a, b and c.

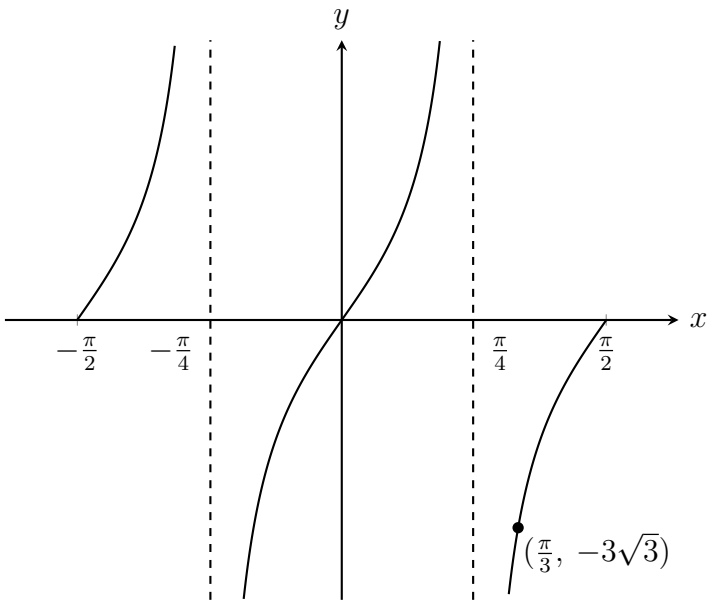
Question 3 (3 marks)

On the axes below, sketch the graph of $f(x) = 4x^2 - 2x^3,\ x \in [-1, 2]$.
Label all endpoints and axis intercepts.



Question 4 (2 marks)

A section of the graph of f is shown below.



If the function f is defined by $f(x) = a\tan(bx)$, find the values of a and b.

Question 5 (3 marks)

Each night, Jack gets takeaway for dinner. He can choose from a healthy dinner or an unhealthy dinner. If he has a healthy meal one night, the probability that he has a healthy meal the next night is 0.7. If he has an unhealthy meal one night, the probability that he has an unhealthy meal the following night is 0.2. Jack has a healthy meal on a Sunday.

What is the probability that he has a healthy meal on the following Tuesday night?

Question 6 (5 marks)

The function

$$f(x) = \begin{cases} a\cos\left(\dfrac{\pi x}{2}\right), & 0 \leq x \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

is a probability density function for the continuous random variable X.

a. Show that $a = \dfrac{\pi}{2}$.

2 marks

b. Find $\Pr\left(X \geq \dfrac{1}{2} \mid X \geq \dfrac{1}{3}\right)$.

3 marks

Question 7 (3 marks)

Let $f : R^- \to R,\ f(x) = \dfrac{2}{x}$.

a. Find h, where $h(x) = f(f(2x))$, and state the maximal domain for which h is defined.

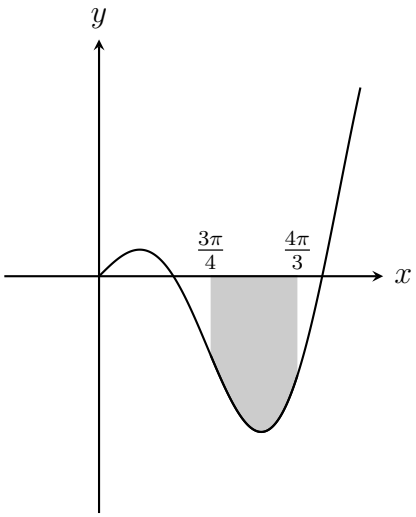
2 marks

b. Evaluate $h^{-1}(24)$, where h^{-1} is the inverse function of h.

1 mark

Question 8 (4 marks)

Part of the graph of the function $f : R \to R,\ f(x) = x\cos(x)$ is shown below.



a. Find the derivative of $2x\sin(x)$.

1 mark

b. Hence, find the area of the shaded region in the diagram above.

3 marks

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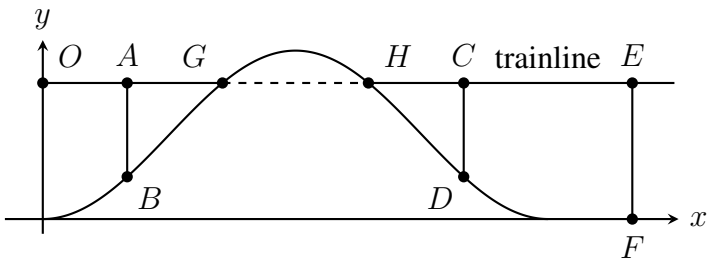
Question 9 (7 marks)

A trainline runs along a bridge, through a tunnel in a mountain, and emerges onto another bridge on the other side of the mountain as shown in the diagram below.



The shape of the mountain is modelled by the function

$$y = 40 \sin\left(\frac{\pi}{60}(x - 30)\right) + 40$$

The length of tunnel GH is 20 metres and the sum of the lengths of bridge supports AB and CD is $(40\sqrt{30} + 40)$ metres. The distance from A to G is equal to the distance from H to C.

a. How high above ground level is the peak of the mountain?

1 mark

b. What is the length of bridge support EF?

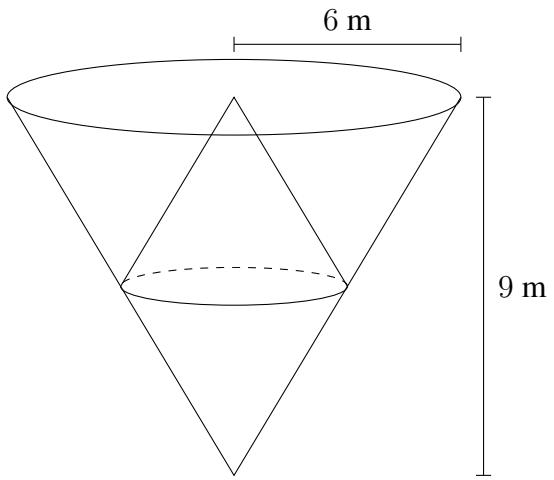
3 marks

c. If a train travels at 50 metres per second, how long does it take to travel from A to C?

3 marks

Question 10 (6 marks)

A big inverted right circular cone holds a smaller right circular cone inside it as shown in the diagram below.



The radius of the bigger cone is 6 m and the height is 9 m. The radius of the smaller cone is r m and the height is h m.

a. Find h in terms of r.

2 marks

b. Find V, the volume of the smaller cone, in terms of r.

1 mark

c. Find the value of r for which V is at a maximum and state this maximum volume.

3 marks

END OF QUESTION AND ANSWER BOOK

TRIUMPH PUBLICATIONS

MATHEMATICAL METHODS

Written examination 1

FORMULA SHEET

Instructions

This formula sheet is provided for your reference.
A questions and answer book is provided with this formula sheet.

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Formula Sheet

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$	$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$
$\frac{d}{dx}\left((ax+b)^n\right) = an(ax+b)^{n-1}$	$\int (ax+b)^n\,dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c,\ n \neq -1$
$\frac{d}{dx}\left(\mathrm{e}^{ax}\right) = a\mathrm{e}^{ax}$	$\int \mathrm{e}^{ax}\,dx = \frac{1}{a}\mathrm{e}^{ax} + c$
$\frac{d}{dx}(\log_{\mathrm{e}}(x)) = \frac{1}{x}$	$\int \frac{1}{x}\,dx = \log_{\mathrm{e}}(x) + c,\ x > 0$
$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$	$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$	$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$	

product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$		

Formula Sheet

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

Probability distribution		**Mean**	**Variance**
discrete	$\Pr(X = x) = p(x)$	$\mu = \Sigma\, x\, p(x)$	$\sigma^2 = \Sigma\, (x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \displaystyle\int_a^b f(x)\, dx$	$\mu = \displaystyle\int_{-\infty}^{\infty} x\, f(x)\, dx$	$\sigma^2 = \displaystyle\int_{-\infty}^{\infty} (x-\mu)^2\, f(x)\, dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{p(1-p)}{n}},\ \hat{p} + z\sqrt{\dfrac{p(1-p)}{n}}\right)$

END OF FORMULA SHEET