



Presbyterian Ladies' College

MELBOURNE

VCE
Year 11 + 10 Accelerated
2019 Semester 2
Mathematical Methods Unit 2 Examination

PAPER 1

Class/Teacher: **Lang** **Lewis** **Mendan** **Smith** **Taylor**

10MATHAC/Lewis

(Please circle)

Name: ANSWERS. **Form:** _____

READING TIME:

(15 minutes in total for Papers 1 + 2)

WRITING TIME:

(40 minutes)

Section	Number of Questions	Number of Questions to be answered	Number of Marks
A	9	9	40

No. of Pages: 9

Instructions

1. No calculators are permitted.
2. No notes are permitted.
3. You can refer to the formula sheet provided.

SECTION A: Short Answer: All questions should be answered in the spaces provided.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Section A: Short Answer: No notes and no calculators allowed.
Exact answers should be given unless instructed otherwise.

Question 1 (3 marks)

At a country show, Billy plays a game of chance that involves trying to knock a pineapple off a stand by throwing a ball. He has three throws and on each throw, the probability of success is $\frac{1}{5}$.

- a. What is the probability that Bill knocks the pineapple off twice?

2 marks

$${}^3C_2 \left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right) \\ = \frac{12}{125}$$

- b. What is the probability that he knocks the pineapple off with his last throw only?

1 mark

$$\frac{4}{5} \times \frac{4}{5} \times \frac{1}{5} \\ = \frac{16}{125}$$

Question 2 (3 marks)

A workplace has 6 female and 5 male employees.

- a. In how many ways can two employees fill the positions of manager and assistant manager?

1 mark

$$11 \times 10 = 110$$

- b. In how many ways can an advisory committee of 5 be chosen so that there are 3 women and 2 men?

2 marks

$${}^6C_3 \times {}^5C_2 \\ = 20 \times 10 \\ = 200$$

Question 3 (6 marks)

a. Let $g(x) = \frac{5}{x^2} + 2\sqrt{x} + 1$, $x > 0$.

Differentiate g with respect to x .

2 marks

$$g(x) = 5x^{-2} + 2x^{1/2} + 1$$

$$g'(x) = -10x^{-3} + x^{-1/2}$$

b. Let $h: \mathbb{R} \rightarrow \mathbb{R}$, $h(x) = 2x^2(x^2 - 1)$

Evaluate $h'(-1)$

2 marks

$$h(x) = 2x^4 - 2x^2$$

$$h'(x) = 8x^3 - 4x$$

$$h'(-1) = -4$$

c. If $y = (2x - x^3)^5$, find $\frac{dy}{dx}$

2 marks

$$\frac{dy}{dx} = 5(2 - 3x^2)(2x - x^3)^4$$

Question 4 (4 marks)

a. If $f'(x) = 3x^2 - 2$ and $f(1) = 2$, find $f(x)$

2 marks

$$f(x) = \int (3x^2 - 2) dx$$

$$f(x) = x^3 - 2x + C$$

$$\text{sub } (1, 2) \quad 2 = 1 - 2 + C$$

$$f(x) = x^3 - 2x + 3$$

b. Evaluate $\int_1^2 \frac{x^3 - 3x}{x} dx$

2 marks

$$\int_1^2 x^2 - 3 dx$$

$$= \left[\frac{x^3}{3} - 3x \right]_1^2$$

$$= \left(\frac{8}{3} - 6 \right) - \left(\frac{1}{3} - 3 \right)$$

$$= -\frac{2}{3}$$

Question 5 (3marks)

Solve the equation $2 \log_5(a) - \log_5 24 + \log_5 6 = 0$ for a .

$$\log_5 a^2 - \log_5 24 + \log_5 6 = 0$$

$$\log_5 \frac{a^2}{4} = 0$$

$$\frac{a^2}{4} = 1$$

$$a^2 = 4$$

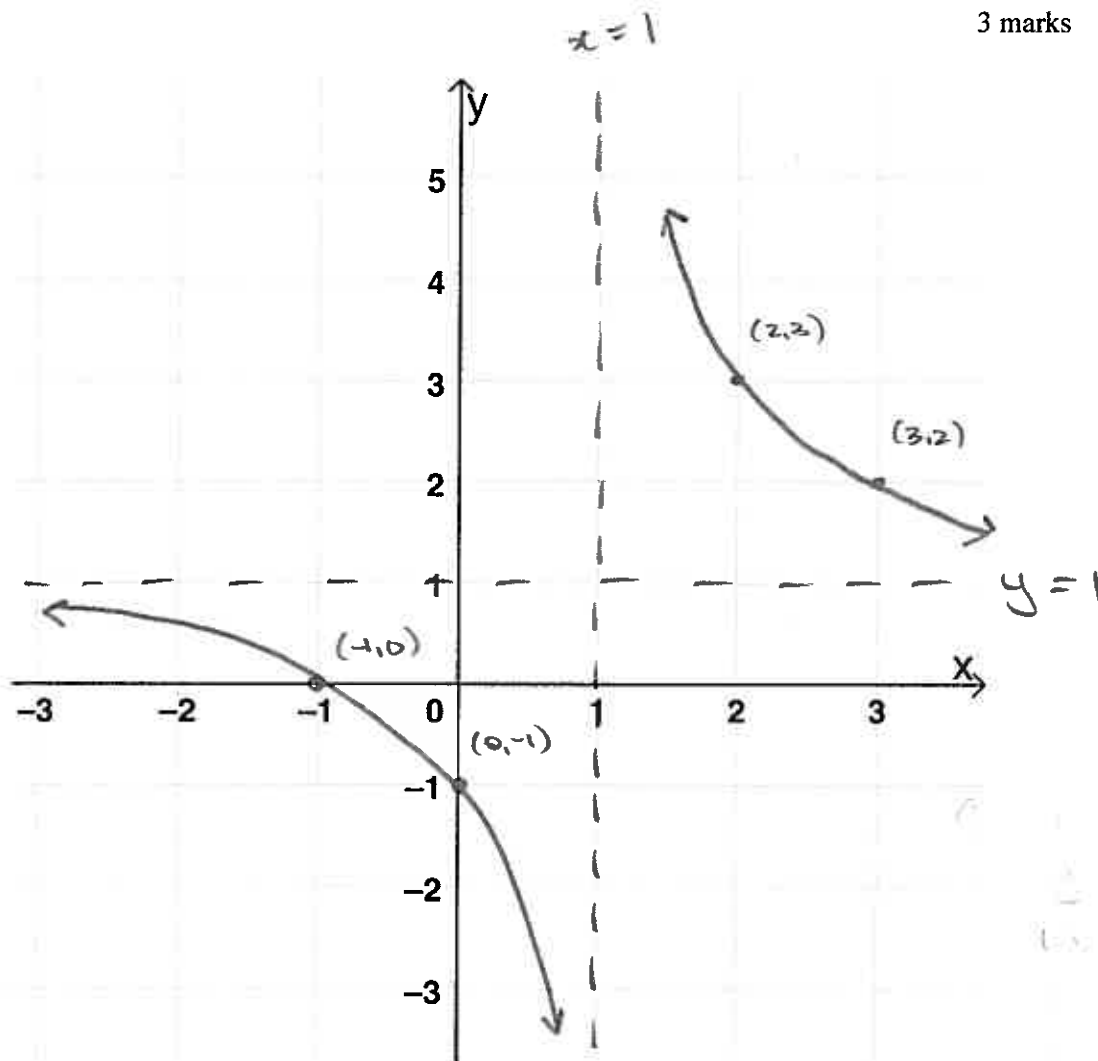
$$a = \pm 2$$

$$a > 0 \text{ so } a = 2$$

Question 6 (5 marks)

- a. Sketch the graph of $y = \frac{2}{x-1} + 1$ on the axes below. Label the asymptotes. The coordinates of the axial intercepts should also be labelled.

3 marks



- b. State the domain and range of the graph.

2 marks

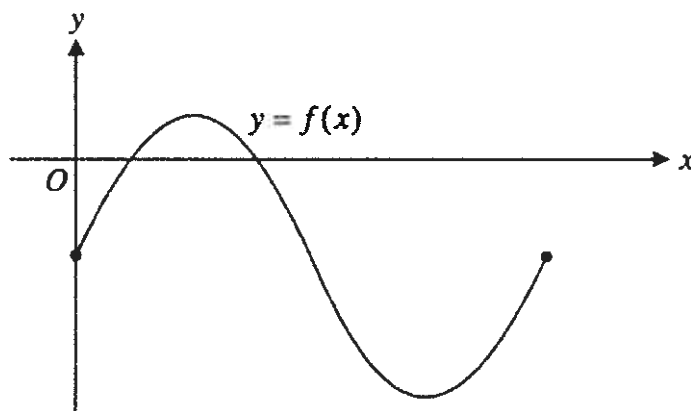
dom $x \in \mathbb{R} \setminus \{1\}$

ran $y \in \mathbb{R} \setminus \{1\}$

Question 7 (6 marks)

Let $f: [0, a] \rightarrow \mathbb{R}$, $f(x) = \sqrt{2} \sin(4x) - 1$

The graph of f is shown below.



- a. Find a , given that $f(0) = f(a)$

1 mark

$$\text{Period} = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$a = \frac{\pi}{2}$$

- b. Solve $f(x) = 0$, for x .

2 marks

$$\sqrt{2} \sin(4x) - 1 = 0$$

$$\sin(4x) = \frac{1}{\sqrt{2}}$$

$$4x = \frac{\pi}{4}, \frac{3\pi}{4}$$

$$x = \frac{\pi}{16}, \frac{3\pi}{16}$$

- c. Find the gradient of the line segment connecting the points where $x = 0$ and $x = \frac{\pi}{12}$

Express your answer in the form $\frac{m\sqrt{n}}{n}$ where m and n are constants.

3 marks

$$x = 0 \quad y = -1$$

$$x = \frac{\pi}{12} \quad y = \frac{\sqrt{2}}{2} - 1$$

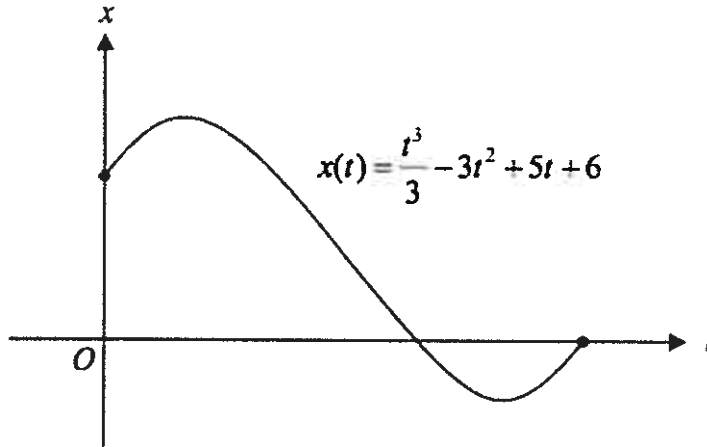
$$m = \frac{\frac{\sqrt{2}}{2} - 1 + 1}{\frac{\pi}{12} - 0}$$

$$m = \frac{6\sqrt{2}}{\pi}$$

Question 8 (4 marks)

A particle moves in a straight line such that its position, x centimetres from a fixed origin at O , at time t seconds, is given by $x(t) = \frac{t^3}{3} - 3t^2 + 5t + 6$, $t \in [0, 6]$.

A graph of this function is shown below.



- a. At what times did the particle change direction?

2 marks

$$v(t) = t^2 - 6t + 5$$

$$v(t) = 0 \quad t^2 - 6t + 5 = 0$$

$$(t-5)(t-1) = 0$$

$$t = 5 \text{ or } t = 1 \quad \text{changed direction}$$

- b. What is the total distance travelled by the particle when it **first** reaches the origin?

2 marks

$$x(0) = 6$$

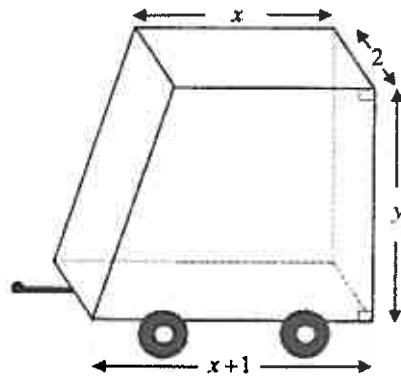
$$x(1) = 8\frac{1}{3}$$

$$\text{Distance} = 2\frac{1}{3} + 8\frac{1}{3}$$

$$= 10\frac{2}{3} \text{ cm}$$

Question 9 (6 marks)

An enclosed trailer in the shape of a trapezoidal prism is shown below.



The length of the prism along the top of the trailer is x metres and along the base it is $x + 1$ metres. The height of the trailer is y metres and the width is two metres.

The length of the top of the trailer together with the height, must add to equal 6 metres.

- a. Show that the volume of the trailer, V , in cubic metres, is given by $V = -2x^2 + 11x + 6$

3 marks

$$\begin{aligned}
 x + y &= 6 \Rightarrow y = 6 - x \\
 V &= \frac{1}{2}(x + x + 1)y \times 2 \\
 &= (2x + 1)(6 - x) \\
 &= -2x^2 + 12x + 6 - x \\
 &= -2x^2 + 11x + 6
 \end{aligned}$$

- b. Find the value of x for which the volume is a maximum.

2 marks

$$\begin{aligned}
 \frac{dV}{dx} &= -4x + 11 \\
 \text{let } -4x + 11 &= 0 \\
 x &= \frac{11}{4} \text{ m}
 \end{aligned}$$

- c. Find the maximum volume of the trailer.

1 mark

$$\begin{aligned}
 V &= -2\left(\frac{11}{4}\right)^2 + 11\left(\frac{11}{4}\right) + 6 \\
 &= \frac{169}{8}
 \end{aligned}$$

End of Section A

Name

Answers

Teacher

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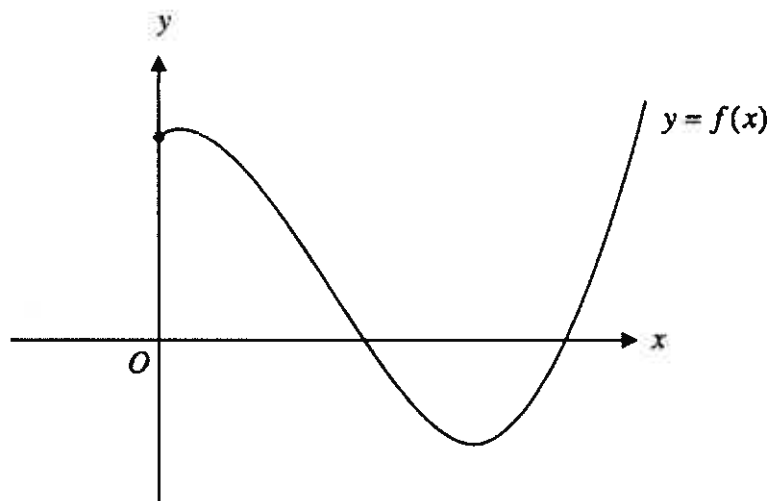
2019 MM EXAM 2 (0940)

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4 (A) (B) (C) (D) (E) 15 (A) (B) (C) (D) (E)
5 (A) (B) (C) (D) (E) 16 (A) (B) (C) (D) (E)
6 (A) (B) (C) (D) (E) 17 (A) (B) (C) (D) (E)
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8 (A) (B) (C) (D) (E) 19 (A) (B) (C) (D) (E)
9 (A) (B) (C) (D) (E) 20 (A) (B) (C) (D) (E)
10 (A) (B) (C) (D) (E)
11 (A) (B) (C) (D) (E)

Section C: Analysis: Notes and calculators allowed.
Exact answers should be given unless instructed otherwise.

Question 1 (15 marks)

Let $f: [0, \infty) \rightarrow \mathbb{R}, f(x) = x^3 - 5x^2 + 2x + 8$.
 Part of the graph of f is shown below.



- a. Find the **coordinates** of the y -intercept of the graph.

1 mark

$(0, 8)$

- b. Find the **coordinates** of the x -intercepts of the graph.

2 marks

$(2, 0)$ $(4, 0)$

- c. Find the derivative of f and hence find the exact value of the **x -coordinates** of the turning points

3 marks

$$f'(x) = 3x^2 - 10x + 2$$

$$\text{Let } f'(x) = 0 \quad 3x^2 - 10x + 2 = 0$$

$$x = \frac{5 \pm \sqrt{19}}{3}$$

- d. Find the range of f , expressing any endpoints of the interval in **exact** form.

2 marks

range $\left[\frac{56-38\sqrt{19}}{27}, \infty \right)$

- e. Find the gradient of the tangent to the graph of f , at the point where $x = 1$.

1 mark

-5

- f. Find the equation of the tangent to f at the point where $x = 1$.

1 mark

$y = 11 - 5x$

- g. The tangent to the curve at $x = 1$ crosses the curve of f at one other point. Find the coordinates of this point.

2 marks

$11 - 5x = f(x)$

$(3, -4)$

- h. Evaluate the integral $\int_0^2 f(x) dx$.

1 mark

$\frac{32}{3}$

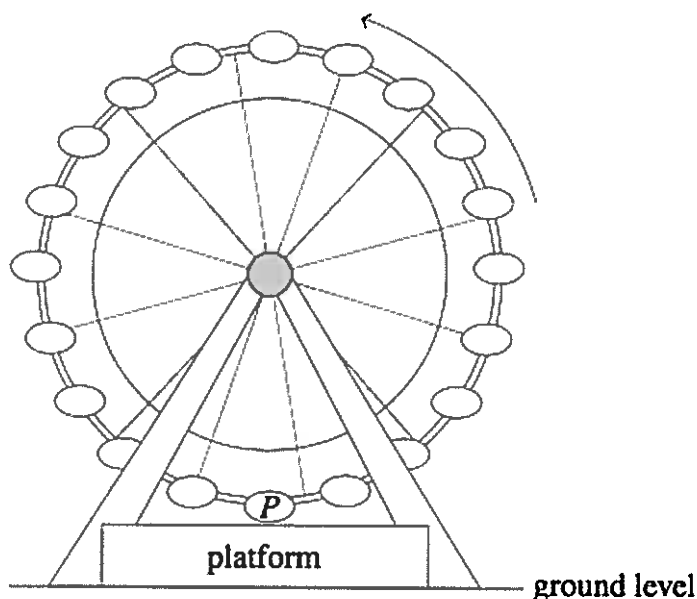
- i. Find the exact area bound by the curve $y = f(x)$, the x -axis, the y -axis and the line $x = 5$

2 marks

$A = \int_0^2 f(x) dx - \int_2^4 f(x) dx + \int_4^5 f(x) dx$
 $= \frac{283}{12} \text{ units}^2$

Question 2 (14 marks)

A large Ferris wheel is a tourist attraction that lets people experience views of the city. This Ferris wheel has 20 pods that are evenly spaced around the wheel and rotates at a constant rate in an anticlockwise direction. To ride the wheel, people enter the pods from a platform that is above ground level. The Ferris wheel does not stop at any time but moves slowly enough for people to enter and exit the pods safely.



A camera is attached to a point P on the side of a pod as shown in the diagram above. This happens when the point P is at its lowest point, 4 metres above the ground at 2.00 pm.

The height, h metres, of the point P above ground level at any time t hours after 2.00 pm is given by

$$h(t) = 64 - 60\cos\left(\frac{5\pi t}{2}\right)$$

- a. What is the amplitude of $h(t)$?

1 mark

60

- b. What is the maximum height reached by point P ?

1 mark

124 m

- c. Find the period of the graph of $h(t) = 64 - 60\cos\left(\frac{5\pi t}{2}\right)$ and explain, in a simple sentence, what is meant by the period in this context.

2 marks

$$\text{period} = \frac{4}{5}$$

1 revolution takes $\frac{4}{5}$ hours.

- d. At what time after 2.00 pm does the point P first return to its lowest point?

2 marks

$$t = 0.8$$

$$2.48 \text{ pm}$$

- e. What is the radius of the Ferris wheel?

1 mark

$$60 \text{ m}$$

- f. Find the distance travelled by the point P in one rotation of the Ferris wheel correct to the nearest metre.

2 marks

$$d = 2\pi \times 60$$

$$= 120\pi = 377 \text{ m}$$

- g. Find the height of the point P above the ground at 2.36 pm

2 marks

$$t = 0.6$$

$$h(0.6) = 64 \text{ m}$$

- h. The camera starts filming when the point P reaches 94 metres above the ground and continues filming as long as the point P remains 94 metres or more above the ground. When the point P is less than 94 metres the camera stops filming. For what fraction of one rotation is the camera filming?

3 marks

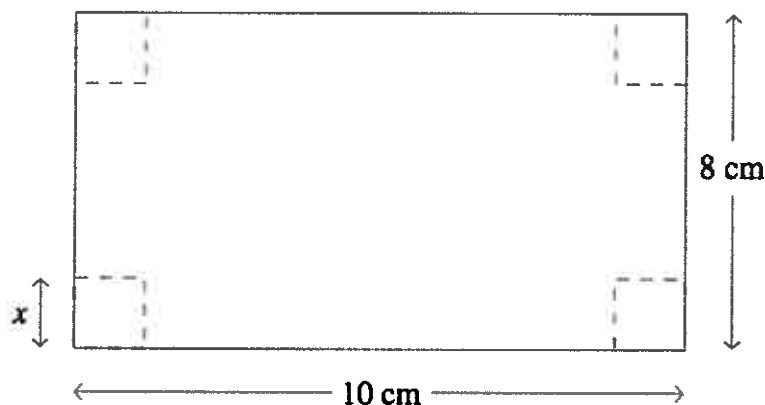
$$h(t) = 94$$

$$t = \frac{4}{15}, \frac{8}{15}$$

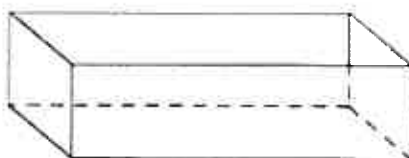
$$\text{above } 94\text{m} \quad \frac{\frac{4}{15}}{\frac{4}{15}} = \frac{1}{3} \text{ of rotation}$$

Question 3 (8 marks)

A rectangular sheet of cardboard is 10 cm long and 8 cm wide. A square of side length x cm is cut from each corner as shown in the diagram below



The remaining cardboard is then folded to make an open box as shown in the diagram below.



- a. Show that the volume of the box, $V \text{ cm}^3$, is given by $V(x) = 4x(5 - x)(4 - x)$

2 marks

$$V = x(8 - 2x)(10 - 2x)$$

$$= x \times 2(4 - x) \times 2(5 - x)$$

- b. The domain of V is $a < x < b$
State the values of a and b .

2 marks

$$a = 0$$

$$b = 4$$

- c. Find the value(s) of x for which the box has a volume of 24 cm^3 correct to one decimal place where appropriate.

2 marks

$$V(x) = 24$$

$$x = 0.4$$

$$3.0$$

- d. Find the maximum possible volume for this box and the value of x for which this occurs.
Give your answers correct to one decimal place.

2 marks

$$x = 1.5$$

$$V = 52.5$$

Question 4 (13 marks)

At a family gathering there are 8 cousins. Five are boys and three are girls. They are asked to stand in a line so that they can have their photo taken.

a. In how many ways can this be done if

i. There are no restrictions

1 mark

$$8! = 40320$$

ii. The tallest boy is first and the tallest girl is second in the line.

1 mark

$$1 \times 1 \times 6! = 720$$

b. (i) Three of the cousins are randomly chosen to serve afternoon tea. In how many ways could two boys be chosen?

2 marks

$$5C_2 \times 3C_1$$

$$= 30$$

(ii) What is the **probability** that two boys are chosen?

2 marks

$$\frac{30}{8C_3}$$

$$= \frac{30}{56}$$

$$= \frac{15}{28}$$

At the family gathering, the cousins play a board game that involves rolling a die. When a player rolls a 6 they win a small prize.

If a player has 5 rolls of the die, find the probability that they win

(3 dp).

c. Exactly two prizes

1 mark

$$\frac{652}{3888}$$

$$(\sim 0.168)$$

d. At least one prize.

1 mark

$$\frac{4651}{7776}$$

$$(\sim 0.598)$$

e. Three or four prizes, given that they win at least one prize.

2 marks

$$\frac{\Pr(3 \leq X \leq 4)}{\Pr(X \geq 1)}$$

$$= \frac{0.035365}{0.598122}$$

$$= 0.059$$

A new game is introduced that involves 6 trials. The probability of success is p .

Let $X \sim \text{Bi}(6, p)$

f. Show that $\Pr(3 \leq X \leq 4) = 5p^3(1-p)^2(4-p)$

3 marks

$$\Pr(3 \leq X \leq 4) = {}^6C_3 p^3(1-p)^3 + {}^6C_4 p^4(1-p)^2$$

$$= 20 p^3(1-p)^3 + 15 p^4(1-p)^2$$

$$= 5 p^3(1-p)^2 [4(1-p) + 3p]$$

$$= 5 p^3(1-p)^2 (4-p)$$

End of Section C

End of Examination