

MM1/2 Rates and Differential Calculus Mini Test 2019

Name: ANSWERS Total Marks: _____ / 26

Notes or calculator NOT allowed

Time allowed: 40 minutes

Multiple Choice – Circle the correct response

Question 1

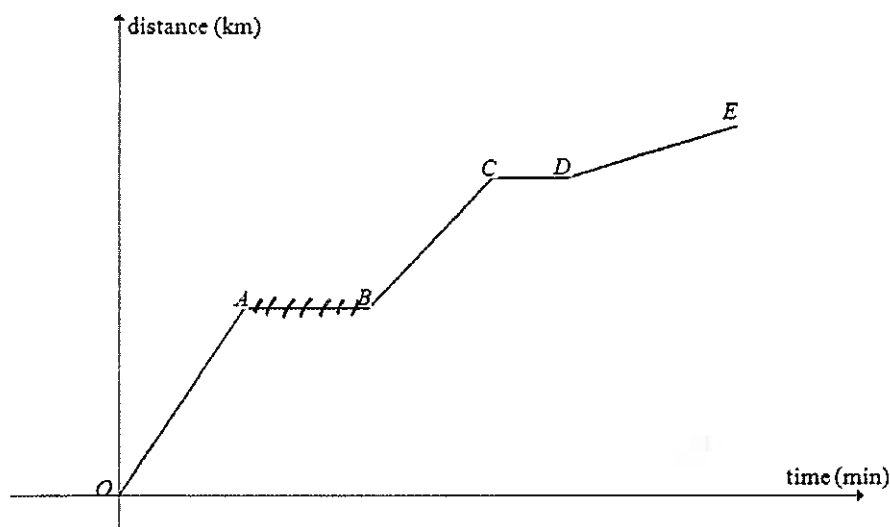
If $g(x) = 1 - 3x + x^2$, then $g(x+h) - g(x)$ is equal to:

- A. $h(h+1)$
- ☒ B. $h(h+2x-3)$
- C. $1-3h+h^2$
- D. $-3+2h$
- E. $h(2x-3)$

$$\begin{aligned} & g(x+h) - g(x) \\ &= 1 - 3(x+h) + (x+h)^2 - 1 + 3x - x^2 \\ &= 1 - 3x - 3h + x^2 + 2xh + h^2 - 1 + 3x - x^2 \\ &= -3h + 2xh + h^2 \\ &= h(h+2x-3) \end{aligned}$$

Question 2

The graph below shows the movement of a train over a period of time.



For the section of the graph between A and B, the train is:

- A. speeding up
- B. slowing down
- C. travelling east
- D. travelling at a constant speed greater than zero
- ☒ E. stationary

Question 3

If $y = x - 4(x + 4)(x + 1)$ then $\frac{dy}{dx}$ equals:

- ☒ A. $-19 - 8x$
- B. $3x^2 + 2x - 16$
- C. $4x - 32$
- D. $-2x - 4$
- E. $-8x - 35$

$$y = x - 4(x^2 + 5x + 4)$$

$$y = -4x^2 - 19x - 20$$

$$\frac{dy}{dx} = -8x - 19$$

Question 4

For the function with the rule $f(x) = x^3 - 3x + 3$, the gradient of the chord that connects the points $x = 2$ and $x = 3$ is:

- A. 21
- ☒ B. 16
- C. 5
- D. 8
- E. 4

$$x = 2 \quad y = 5$$

$$x = 3 \quad y = 21$$

$$\text{gradient} = \frac{21 - 5}{3 - 2} = 16$$

Question 5

Given that $f(x) = 2x^3 + 3x^2 - 12x$, the value(s) of x for which $f'(x) = 0$ are:

- A. 0 and 3
- B. 2
- C. 2 and -1
- ☒ D. -2 and 1
- E. 0, 1 and 2

$$f'(x) = 6x^2 + 6x - 12$$

$$f'(x) = 0$$

$$6x^2 + 6x - 12 = 0$$

$$x^2 + x - 2 = 0$$

$$(x + 2)(x - 1) = 0$$

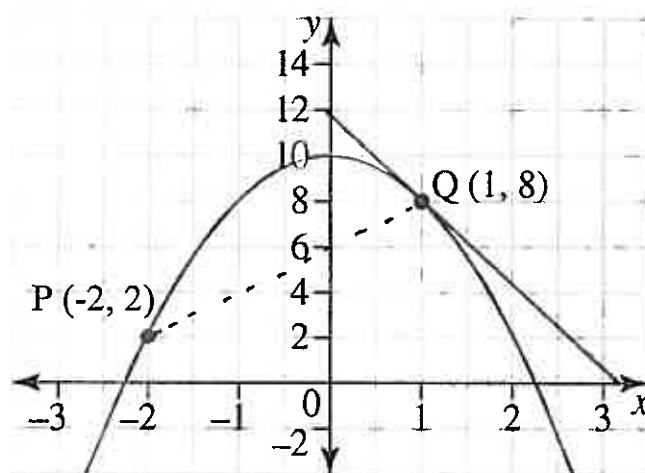
$$x = -2, 1$$

Short Answer Questions

Write your answers in the spaces provided.

Question 6

The graph shows a function $y = f(x)$ on which the point $Q(1, 8)$ lies.



- (a) Find the gradient of the secant connecting the points $P(-2, 2)$ and $Q(1, 8)$

$$m = \frac{8-2}{1-(-2)} \quad \textcircled{1} \text{ attempt } \frac{y_2 - y_1}{x_2 - x_1} \text{ or } \frac{\text{rise}}{\text{run}}$$
$$= 2 \quad \textcircled{1} A$$

- (b) Draw the tangent to the curve at the point $Q(1, 8)$ and find an estimate of the gradient of this function at the point Q .

$$m = -\frac{12}{3}$$

$$m = -4$$

① tangent drawn at Q

① 'reasonable' answer

(2 + 2 = 4 marks)

Question 7

Evaluate:

$$\lim_{h \rightarrow 0} \frac{(2+h)^2 - 4}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{4} + 4h + h^2 - \cancel{4}}{h} \quad \textcircled{1} \text{ A expand}$$

$$= \lim_{h \rightarrow 0} 4 + h$$

$$= 4 \quad \textcircled{1} \text{ A}$$

(2 marks)

Question 8

Given $f(x) = 2x^2 - 3x$, use **first principles** to show that $f'(x) = 4x - 3$

$$\begin{aligned} f(x+h) &= 2(x+h)^2 - 3(x+h) \\ &= 2x^2 + 4xh + 2h^2 - 3x - 3h \end{aligned} \quad \textcircled{1} \text{ A correct } f(x+h)$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{2x^2} + 4xh + 2h^2 - \cancel{3x} - 3h - \cancel{2x^2} + \cancel{3x}}{h} \quad \textcircled{1} \text{ M using } f(x+h) - f(x)$$

$$= \lim_{h \rightarrow 0} \frac{4xh + 2h^2 - 3h}{h} \quad \textcircled{1} \text{ M simplify their}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{h} (4x + 2h - 3)}{\cancel{h}} \quad \textcircled{1} \text{ A}$$

$$= \lim_{h \rightarrow 0} 4x + 2h - 3$$

$$= 4x - 3$$

(4 marks)

Question 9

Find the derivative of the following, giving positive powers only.

(a) $y = 5x^3 - x^2 + 2x - 4$

$$\frac{dy}{dx} = 15x^2 - 2x + 2 \quad (1) A$$

(b) $f(x) = x^4 + 6\sqrt{x} + \frac{3}{x^2}$

$$f(x) = x^4 + 6x^{1/2} + 3x^{-2} \quad (1) A$$

$$f'(x) = 4x^3 + 3x^{-1/2} - 6x^{-3}$$

$$= 4x^3 + \frac{3}{x^{1/2}} - \frac{6}{x^3} \quad (1) A$$

(1 + 2 = 3 marks)

Question 10

(a) Use the **product rule** to differentiate the function $f(x) = (x^2 - 4x)(2x^2 - 3)$. Expand and simplify your answer.

$$u = x^2 - 4x \quad v = 2x^2 - 3$$

$$u' = 2x - 4 \quad v' = 4x$$

$$f'(x) = (x^2 - 4x) \cdot 4x + (2x^2 - 3)(2x - 4) \quad (1) M \text{ attempt product rule}$$

$$= 4x^3 - 16x^2 + 4x^3 - 8x^2 - 6x + 12 \quad (1) M \text{ simplify}$$

$$= 8x^3 - 24x^2 - 6x + 12 \quad (1) A$$

(b) Use the **chain rule** to differentiate $y = (4x^3 - 5x)^5$. **Do not** expand your answer.

$$\frac{dy}{dx} = 5(12x^2 - 5)(4x^3 - 5x)^4$$

①
① answer

(3 + 2 = 5 marks)

Question 11

Given the function $f: (-\infty, \frac{1}{2}) \rightarrow \mathbb{R}$, $f(x) = \frac{2x-3}{3-6x}$.

Show that $f'(x) = \frac{-12}{(3-6x)^2}$

$$\begin{aligned} u &= 2x-3 & v &= 3-6x \\ u' &= 2 & v' &= -6 \end{aligned} \quad \text{① A}$$

$$f'(x) = \frac{(3-6x) \times 2 - (2x-3) \times -6}{(3-6x)^2} \quad \text{① M attempt quotient rule (or product for } (2x-3)(3-6x)$$

$$= \frac{6 - 12x + 12x - 18}{(3-6x)^2} \quad \text{① M their expansion done correctly.}$$

$$= \frac{-12}{(3-6x)^2}$$

(3 marks)