MM12 Probability and Combinatorics Test 2019

Name: ANSWERS.

Skills (Section A and B) ..... / 24 **Analysis (Section C) ..... / 23**

Section A: Short Answer **No calculators or notes allowed**

NOTE: Exact answers are required unless instructed otherwise within the question.

Time allowed: 20 minutes

1 Simplify and evaluate where possible:

(a) ${}^5P_2 = 20$ (b) ${}^6C_0 = 1$

(c) ${}^nC_1 = n$ (d) $4! = 24$

(4 marks)

2 If no digit can be used more than once, how many different numbers can be formed from the digits 4, 5, 6, 7 and 9 which are:

(a) 2 digit numbers $\boxed{5}\boxed{4} = 20$ (1)

(b) even 2 digit numbers $\overset{\text{even}}{\boxed{4}}\boxed{2} = 8$ (1)

(c) 3 or 4 digit numbers $\boxed{5}\boxed{4}\boxed{3} + \boxed{5}\boxed{4}\boxed{3}\boxed{2} = 180$ (1)A

(1) either correct

(d) greater than 600 and less than 6000

$\overset{\geq 6}{\boxed{3}}\boxed{4}\boxed{3} + \overset{\leq 6}{\boxed{2}}\boxed{4}\boxed{3}\boxed{2} = 84$ (1)A

(1) either correct

(1+1+2+2 = 6 marks)

3 Solve the following equation for n, given $n > 0$.

${}^nC_2 = 15$

$\frac{n!}{2!(n-2)!} = 15$ (1).

$\frac{n(n-1)(n-2)!}{2(n-2)!} = 15$

$n^2 - n = 30$

$n^2 - n - 30 = 0$ (1)

$(n-6)(n+5) = 0$

$n = 6$ or -5

since $n > 0$, $n = 6$. (1)

or trial + error 3/3

(3 marks)

4 Consider the probability distribution given in the table.

x	0	1	2	3	4
$\Pr(X = x)$	0.02	0.15	0.33	0.30	m

(a) Find the value of m.

$0.02 + 0.15 + 0.33 + 0.30 + m = 1$

$m = 0.2$ (1)

(b) $\Pr(X \leq 2)$

$= 0.5$ (1)

(c) $\Pr(2 < X \leq 4)$

$= 0.5$ (1)

(3 marks)

5. In a binomial probability distribution with 3 trials and the probability of success equal to 0.4, evaluate the probability of;

Let $X \sim Bi(3, 0.4)$

(a) 0 successes

$\Pr(X = 0) = (0.6)^3$ (1)

$= 0.216$ (1) or $\frac{27}{125}$

(b) 2 successes

$\Pr(X = 2) = {}^3C_2 (0.4)^2 (0.6)^1$ (1)

$= 3 \times 0.16 \times 0.6$

$= 0.288$ (1) or $\frac{36}{125}$

(2+2 = 4 marks)

Name: _____

Section B: Multiple Choice Questions — Calculators and notes are allowed

Circle Correct Response — **Time allowed: 25 minutes**

1. In Mathland, number plates consist of 3 digits from 0 -- 9 followed by 4 letters. The letters and numbers can be repeated. The total number of different number plates is equal to:
   - A $9^3 \times 26^4$
   - (B) $10^3 \times 26^4$
   - C $3^{10} \times 4^{26}$
   - D $10^3 + 26^4$
   - E $9^3 + 26^4$

2. In how many ways can six people line up in a queue at the bank?
   - (A) 6!
   - B 6
   - C 30
   - D ${}^6C_2$
   - E ${}^6C_1$

3. In how many ways can four students be chosen from a group of 12 students?
   - A 12
   - B 48
   - (C) 495
   - D 11880
   - E 40320

4. A multiple choice test has 10 questions, each with 5 possible answers. A student who has not prepared for the test guesses the answers to each question. The probability that the student gets either 4 or 5 correct answers is given by:
   - (A) ${}^{10}C_4(0.2)^4(0.8)^6 + {}^{10}C_5(0.2)^5(0.8)^5$
   - B ${}^{10}C_4(0.8)^4(0.2)^6 + {}^{10}C_5(0.8)^5(0.2)^5$
   - C ${}^{10}C_4(0.2)^4 + {}^{10}C_5(0.2)^5$
   - D ${}^{10}C_4(0.8)^4 + {}^{10}C_5(0.8)^5$
   - E $(0.2)^4(0.8)^6 + (0.2)^5(0.8)^5$

Section C: Analysis — Calculators and notes are allowed

Question 1

A Year 11 Latin class has nine students, five boys and four girls.

a. Find the number of ways in which they can be seated in a row

i if there are no restrictions

$9! = 362880$ ①

ii if a girl must sit at each end

$4 \times 3 \times 7! = 60480$ ①

(1+1 = 2 marks)

b. If the students are able to sit anywhere, find the probability that the boys and girls will alternate along the row.

B G B G B G B G B

5 | 4 | 4 | 3 | 3 | 2 | 2 | 1 | 1

$= 5! \times 4! = 2880$ ①

$\Pr(\text{alternate}) = \frac{2880}{362880} = \frac{1}{126}$ ① M using their a

(2 marks)

c. A team of five students is to be chosen to present a special project. How many of these teams would consist of:

i three boys

${}^5C_3 \times {}^4C_2 = 60$ ①

① for ${}^5C_3$

ii a majority of boys

3 or 4 or 5 boys ①

$60 + {}^5C_4 \times {}^4C_1 + {}^5C_5$ ① any two

$\therefore 81$ ① A

(2 + 3 = 5 marks)

Question 2

A darts player knows that his probability of hitting the bullseye with any throw is 0.65

Answers to questions below should be given correct to 3 decimal places.

a. If he has **five throws**, find the probability that he scores:

i three bullseyes

$Pr(3 \text{ bullseyes}) = {}^5C_3 (0.65)^3 (0.35)^2$

$= 0.336$ (1)

ii a bullseye with his first two throws only

$(0.65)^2 (0.35)^3$

$= 0.018$ (1) (1+1=2 marks)

b. In a major competition, he has a total of **10 throws**. Find (correct to 3 decimal places), the probability that:

i at least three of his throws are bullseyes

Let $X \sim Bi(10, 0.65)$

$Pr(X \geq 3) = 0.995$

(1) (1) (2 marks)

ii he scores three bullseyes given that he misses with his first three throws

Let $Y \sim Bi(7, 0.65)$ (1)M or other correct method

$Pr(Y = 3) = 0.144$ (1)

(2 marks)

c. What is the least number of throws required to ensure that the probability of at least one bullseye exceeds 0.999? (An inequation or other working must be shown)

$Pr(X \geq 1) > 0.999$ (1)

$1 - Pr(X = 0) > 0.999$

$1 - (0.35)^n > 0.999$ (1) (2 marks)

$n > 6.58$

7 throws (1)

or more

Question 3

A die is weighted so that $\Pr(6) = \frac{1}{2}$, $\Pr(2) = \Pr(4) = \frac{1}{6}$, and $\Pr(1) = \Pr(3) = \Pr(5) = x$.

If the die is rolled five times, find each of the following correct to 3 decimal places.

a. The probability of at least 3 sixes.

Let $X \sim Bi(5, \frac{1}{2})$

$Pr(X \geq 3) = 0.5$ (1)

b. The probability of at least 2 even numbers

Let $Y \sim Bi(5, \frac{5}{6})$

$Pr(Y \geq 2) = 0.997$ (1)

c. The probability of not more than 2 odd numbers

$Pr(Y \geq 3) = 0.965$ (1)

(3 marks)

Question 4

The table below represents a probability distribution.

1	2	3	4
d^2	$d + 0.4$	$0.3 - 0.1d$	$0.45 - 0.1d$

Find the possible value(s) of d. Justify your answer.

$d^2 + d + 0.4 + 0.3 - 0.1d + 0.45 - 0.1d = 1$ (1)

$d = -0.5$ or -0.3 (1)

$d = -0.3$ since $d + 0.4$ is negative if $d = -0.5$. (1) (3 marks)