

Section A: Short Answer

Time allowed: 20 minutes

Total 19 marks

Name: Answers

1. Find the equation of the tangent to the curve
- $y = 3x^2 - 4x + 1$
- at the point
- $(2, 5)$

$$\frac{dy}{dx} = 6x - 4 \quad (1)$$

$$x = 2 \quad \frac{dy}{dx} = 8$$

$$M_T = 8 \quad (1)$$

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 5 &= 8(x - 2) \\ y &= 8x - 11 \quad (1) \end{aligned}$$

(3 marks)

2. For the graph of
- $f: [-1, 2] \rightarrow \mathbb{R}$
- ,
- $f(x) = x^2(1-x)$

- a. Find the exact coordinates of the turning points.

$$f(x) = x^2 - x^3$$

$$f'(x) = 2x - 3x^2 \quad (1)$$

$$\text{Let } 2x - 3x^2 = 0$$

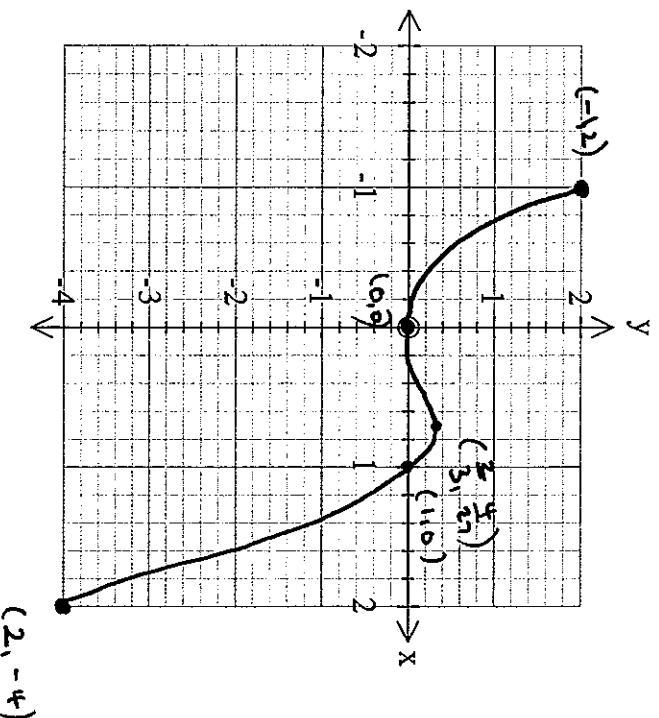
$$x(2 - 3x) = 0$$

$$x = 0 \quad x = \frac{2}{3} \quad (1)$$

(3 marks)

$$\begin{aligned} &\text{Turning points} \\ &(0, 0) \quad \left(\frac{2}{3}, \frac{4}{27}\right) \quad (1) \end{aligned}$$

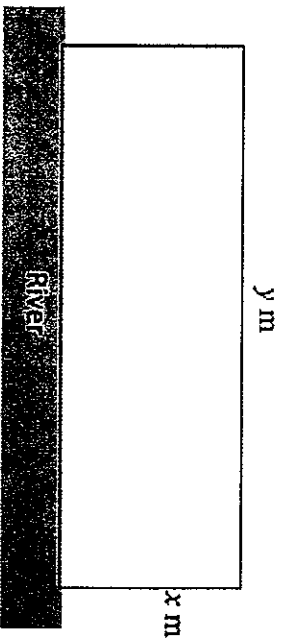
- b. Sketch the graph of
- $y = f(x)$
- , clearly labelling coordinates of intercepts, turning points and endpoints.



(1) for shape
 (1) for each point
 (round down)

(3 marks)

3. A farmer wishes to build a fence around a rectangular field. He has 100m of fencing wire and will use a river running through his land as a boundary.



- a. Find an expression for y in terms of x .

$$2x + y = 100$$

$$y = 100 - 2x \quad (1)$$

(1 mark)

- b. Show that the area, A of the field is given by the function $A = 100x - 2x^2$.

$$A = x \times y$$

$$= x(100 - 2x) \quad (1)$$

$$= 100x - 2x^2$$

(1 mark)

- c. Find $\frac{dA}{dx}$

$$\frac{dA}{dx} = 100 - 4x \quad (1)$$

(1 mark)

- d. Hence, find the maximum area of the field.

$$\text{Let } 100 - 4x = 0 \quad (1)$$

$$x = 25 \quad (1)$$

$$\text{Area} = 25 \times 50$$

$$= 1250 \text{ m}^2 \quad (1)$$

(3 marks)

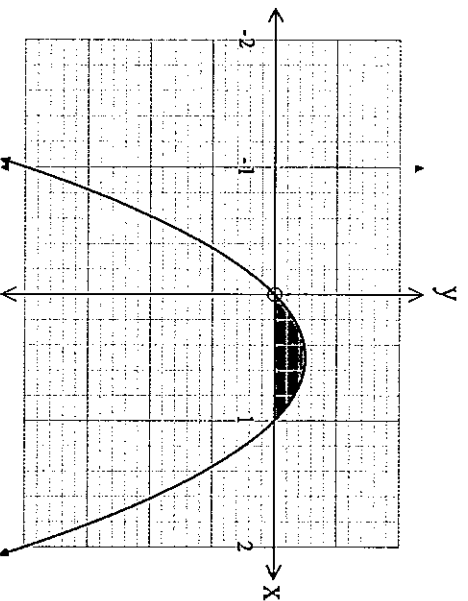
4 a. Find $\int (x - x^2) dx$

$$= \frac{x^2}{2} - \frac{x^3}{3} + C$$

①

(2 marks)

b. Hence evaluate the area of the shaded region of the graph of $y = x(1-x)$



$$\text{Area} = \int_0^1 (x - x^2) dx$$

$$= \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_0^1$$

①

$$= \frac{1}{2} - \frac{1}{3}$$

(2 marks)

$$= \frac{1}{6} \text{ sq units.}$$

①

Name: Answers

Section B: Multiple Choice Calculators are allowed

Time allowed: 25 minutes

7+19 = 26 marks

Circle Correct Response

1. If $f(x) = x^2(x - 2)$ then $f'(2)$ equals

- A. -1
- B. 0
- ☒ C. 4
- D. -4
- E. 10

2. The equation of the tangent to the curve $f(x) = 2x^2 - 3x + 1$ at the y-intercept is

- ☒ A. $y = -3x + 1$
- B. $y = 3x + 1$
- C. $y = -3x - 1$
- D. $y = -3x$
- E. $y = 3$

$$x = 0$$

3. A car is driving in a straight line. Its position, x (in metres), from the origin, is given by the following equation.

$$x(t) = 2t^2 + 5t + 10 \text{ for } t \geq 0, \text{ where } t \text{ is the time in seconds.}$$

The velocity of the car is 25 m/s when the time is

- A. 10 seconds
- B. -10 seconds
- C. -5 seconds
- ☒ D. 5 seconds
- E. 0 seconds

$$v(t) = 4t + 5$$

$$4t + 5 = 25$$

$$t = 5$$

4. If the curve with the equation $y = ax^2 + x + 1$ has a stationary point at $x = -2$, then a equals

- ☒ A. $\frac{-1}{4}$
- ☒ B. $\frac{1}{4}$
- C. $\frac{1}{2}$
- D. $\frac{-1}{2}$
- E. -1

$$\frac{dy}{dx} = 2ax + 1$$

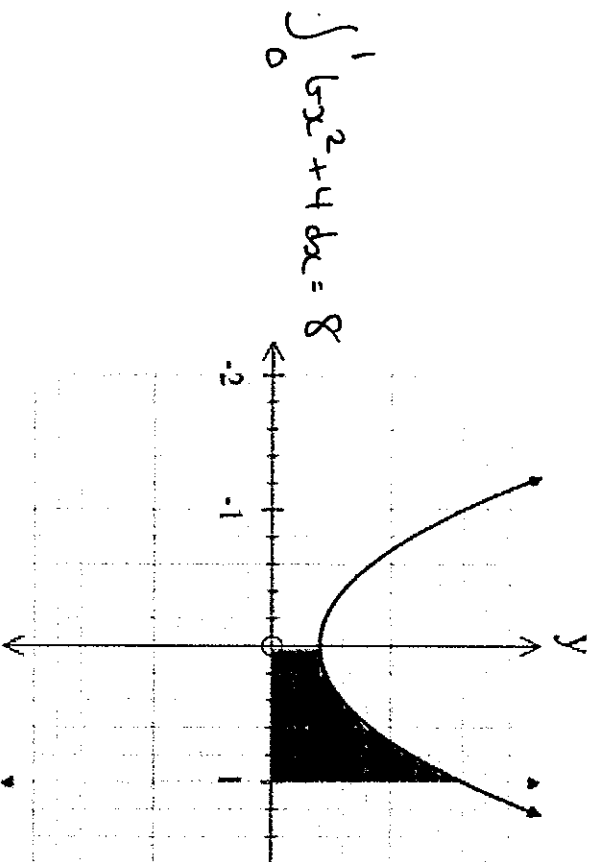
$$2a(-2) + 1 = 0$$

$$-4a + 1 = 0$$

$$a = \frac{1}{4}$$

5. The graph with equation $y = bx^2 + 4$ is shown. The area shaded is 8 square units. The value of b is;

- A. -12
- B. -15
- ☒ C. 12
- D. 15
- E. 30



$$\int_{-2}^2 bx^2 + 4 dx = 8$$

6. If $\frac{dy}{dx} = 2x + 4$ and $(0, 1)$ is a point on the curve $y = f(x)$, then an expression for y is

- A. $y = x^2 + 4x$
- B. $y = x^2 + 4x - 1$
- ☒ C. $y = x + 4$
- ☒ D. $y = x^2 + 4x + 1$
- E. $y = 2x^2 + 4x - 1$

$$y = \int 2x + 4 dx$$

$$y = x^2 + 4x + C$$

$$y = x^2 + 4x + 1$$

$$\begin{aligned} x &= 0 \\ y &= 1 \\ C &= 1 \end{aligned}$$

7. The value of $\int_1^2 (6x^2 - 5) dx$ is

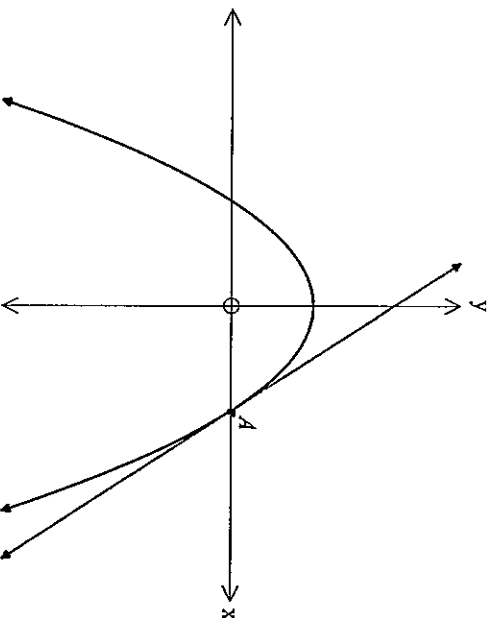
- A. 0
- B. 3
- ☒ C. 9
- D. 20
- E. 37

SECTION C: ANALYSIS **Calculators allowed**
Unless otherwise stated, exact answers should be given

Answer all questions in the space allocated.

Question 1 (7 marks)

The graph of $y = 4 - x^2$ is shown below. A tangent to the graph is drawn at point A, one of the X-intercepts of the graph.



- a. Find the coordinates of the point A.

$(2, 0)$

(1 mark)

- b. Show the equation of the tangent is $y = -4x + 8$.

(2 marks)

$$\frac{dy}{dx} = -2x$$

$$x = 2 \quad \frac{dy}{dx} = -4$$

$$m_T = -4$$

$$y - y_1 = m(x - x_1)$$

$$y = -4(x - 2)$$

$$y = -4x + 8$$

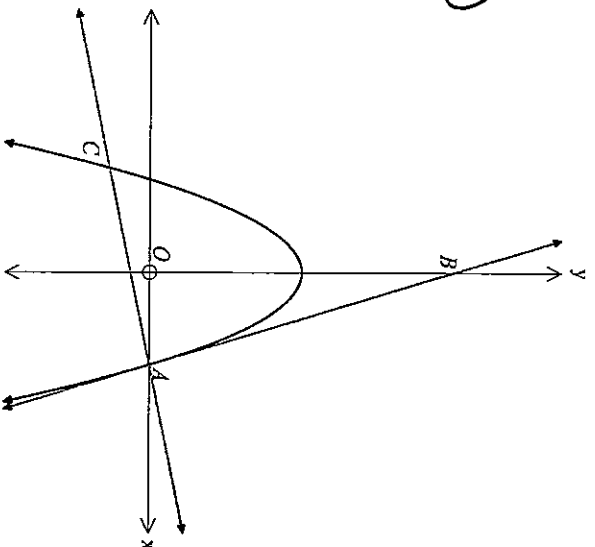
c. Find the equation of the normal to the curve at point A.

(1 mark)

$$m_n = \frac{1}{4}$$

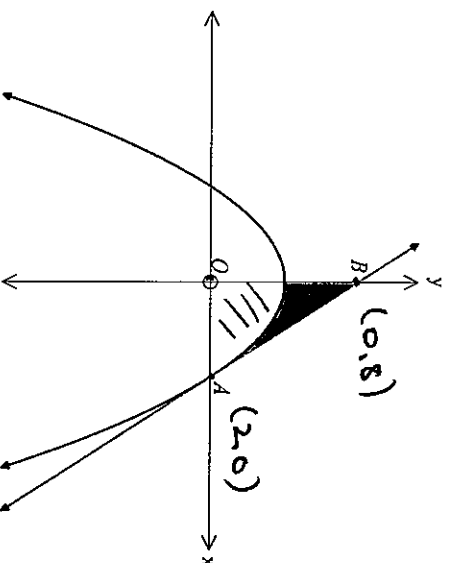
$$y - 0 = \frac{1}{4}(x - 2)$$

$$y = \frac{x}{4} - \frac{1}{2}$$



d. Find the area of triangle OAB and hence find the exact value of the shaded area. Showing all relevant working.

(3 marks)



$$\begin{aligned} \text{Area} &= \frac{1}{2}bh \\ &= \frac{1}{2} \times 2 \times 8 \\ &= 8 \text{ sq units.} \end{aligned}$$

$$\begin{aligned} \text{Shaded area} &= \int_0^2 (4 - x^2) dx \\ &= \frac{16}{3} \end{aligned}$$

$$\begin{aligned} \text{Required area} &= 8 - \frac{16}{3} \\ &= \frac{8}{3} \text{ sq. units} \end{aligned}$$

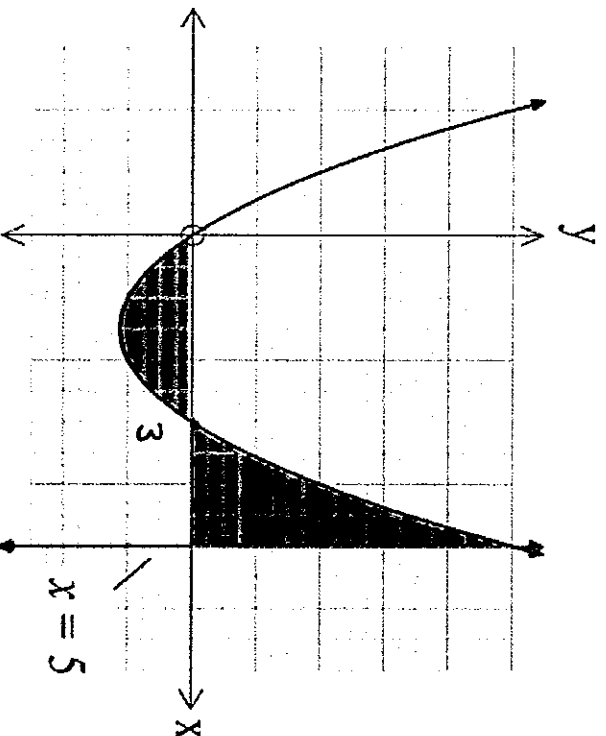
Question 2 (5 marks)

a. Find $\int (x^2 - 3x) dx$

$$= \frac{x^3}{3} - \frac{3x^2}{2} + C$$

(2 marks)

Part of the graph of $y = x^2 - 3x$ is shown.



b. Write the integral expression that could be used to find the shaded area.

$$A = -\int_0^3 (x^2 - 3x) dx + \int_3^5 (x^2 - 3x) dx$$

(2 marks)

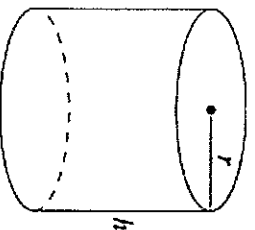
c. Hence, find the area of the shaded region.

(1 mark)

$$A = \frac{19}{6} \text{ sq. units.}$$

Question 3 (7 marks)

A closed cylindrical tank of height h metres and radius r metres has a volume of 64π cubic metres.



- a. Given that $V = \pi r^2 h$, find and expression for h in terms of r .

(1 mark)

$$64\pi = \pi r^2 h$$

$$h = \frac{64}{r^2}$$

- b. Given that surface area, A , is given by $A = 2\pi r^2 + 2\pi r h$, write an expression for A in terms of r .

(1 mark)

$$\begin{aligned} A &= 2\pi r^2 + 2\pi r \times \frac{64}{r^2} \\ &= 2\pi r^2 + \frac{128\pi}{r} \end{aligned}$$

- c. Find $\frac{dA}{dr}$

(1 mark)

$$\frac{dA}{dr} = 4\pi r - \frac{128\pi}{r^2}$$

- d. Hence find the values of r and h , (correct to 2 decimal places) that would give a minimum surface area. Justify that your values give a minimum.

(4 marks)

$$\text{let } 4\pi r - \frac{128\pi}{r^2} = 0$$

$$r = 3.17 \quad (1)$$

$$h = 6.35 \quad (1)$$

r	3	3.17	4
$\frac{dA}{dr}$	-6.98	0	25.12

local minimum. (1)