

Name: Answers.

MM12 Assignment 5 – Applications of Calculus and Antidifferentiation

Section A: Short Answer

Total 21 marks

Unless otherwise stated, exact answers should be given

1. Find the gradient function $f'(x)$, given $f(x) = (3x^2 + 2)(x^2 + 3x)$

$$f(x) = 3x^4 + 9x^3 + 2x^2 + 6x$$

$$f'(x) = 12x^3 + 27x^2 + 4x + 6$$

2. Use the quotient rule to show that the derivative of $f(x) = \frac{3-2x}{3x+1}$ is $f'(x) = \frac{-11}{(3x+1)^2}$

(2 marks)

$$\text{Let } u = 3 - 2x \quad v = 3x + 1$$

$$u' = -2 \quad v' = 3$$

$$f'(x) = \frac{v u' - u v'}{v^2}$$

$$= \frac{-2(3x+1) - 3(3-2x)}{(3x+1)^2}$$

$$= \frac{-6x - 2 - 9 + 6x}{(3x+1)^2}$$

$$= \frac{-11}{(3x+1)^2}$$

(3 marks)

3. Find the equation of the tangent to the curve $y = 2x^3 - x + 4$ at the point $(-1, 3)$.

$$\frac{dy}{dx} = 6x^2 - 1$$

$$\text{when } x = -1 \quad \frac{dy}{dx} = 5$$

$$m = 5$$

Equation of tangent

$$y - 3 = 5(x + 1)$$

$$y = 5x + 8$$

(3 marks)

4. The graph of $y = 2x^3 + ax^2 + b$ has a stationary point at $(1, -7)$. Find the values of a and b .

$$\frac{dy}{dx} = 6x^2 + 2ax$$

$$\text{when } x = 1 \quad \frac{dy}{dx} = 0$$

$$6(1)^2 + 2a(1) = 0$$

$$a = -3$$

$$y = 2x^3 + ax^2 + b$$

$$y = 2x^3 - 3x^2 + b$$

$$\text{s.t. } (1, -7)$$

$$-7 = 2(1)^3 - 3(1)^2 + b$$

$$b = -6$$

(3 marks)

5. Find the following antiderivatives

(a) $\int (6x^2 + 4x) dx$

$$= \frac{6x^3}{3} + \frac{4x^2}{2} + C$$

$$= 2x^3 + 2x^2 + C$$

(b) $\int \frac{6}{x^2} + 3 dx$

$$= \int 6x^{-2} + 3 dx$$

$$= \frac{6x^{-1}}{-1} + 3x + C$$

$$= -\frac{6}{x} + 3x + C$$

(4 marks)

6. Find the value of the definite integral $\int_{-1}^3 (2x^2 - x - 3) dx$

$$\int_{-1}^3 (2x^2 - x - 3) dx$$

$$= \left[\frac{2x^3}{3} - \frac{x^2}{2} - 3x \right]_{-1}^3$$

$$= \left(\frac{2 \times 27}{3} - \frac{9}{2} - 9 \right) - \left(-\frac{2}{3} - \frac{1}{2} + 3 \right)$$

$$= \frac{8}{3}$$

(3 marks)

7. It is known that $f'(x) = x^3 - 2x^2$ and $f(0) = 6$. Find $f(x)$.

$$f(x) = \int (x^3 - 2x^2) dx$$

$$f(x) = \frac{x^4}{4} - \frac{2x^3}{3} + C$$

when $x=0$ $y=6 \Rightarrow C=6$

$$f(x) = \frac{x^4}{4} - \frac{2x^3}{3} + 6$$

(3 marks)

Name: _____

Section B: Multiple Choice Calculators are allowed

Time allowed: 25 minutes

Circle Correct Response

1. The x -coordinates of the stationary points of the curve $f(x) = \frac{1}{3}x^3 + 4x^2 + 7x - 66$ are:

- A $x = 3$
B $x = 1, x = 6$
C $x = 1, x = 7$
D $x = 7, x = 8$
☒ E $x = -7, x = -1$
- $f'(x) = x^2 + 8x + 7$
 $f'(x) = 0 \Rightarrow x^2 + 8x + 7 = 0$
 $x = -7, -1$

2. The curve with equation $y = \frac{7}{2x^2} + 5x$ has gradient -51 when x equals

- A -4
☒ B $\frac{1}{2}$
C $-\frac{1}{4}$
D 2
E $\frac{1}{4}$
- $y = \frac{7}{2}x^{-2} + 5x$
 $\frac{dy}{dx} = -7x^{-3} + 5$
 $-7x^{-3} + 5 = -51$
 $x =$

3. Given that $g'(x) = 2x^2 + 3$ and $g(0) = 2$, then $g(x)$ equals

- A $4x$
B $2x^2 - 1$
C $2x^3 + 3x + 2$
☒ D $\frac{2x^3}{3} + 3x + 2$
E $4x + 2$
- $g(x) = \int 2x^2 + 3 \, dx$
 $g(x) = \frac{2x^3}{3} + 3x + c$
sub (0,2)
 $c = 2$

4. A particle moves in a straight line so that its position x cm from a fixed point O at time t seconds ($t \geq 0$)

is given by $x = -t^3 + 6t^2 - 24t$. The particle's acceleration at $t = 3$ is

- ☒ A -6 cm/s^2
- ☐ B -26 cm/s^2
- ☐ C -18 cm/s^2
- ☐ D 54 cm/s^2
- ☐ E 0 cm/s^2

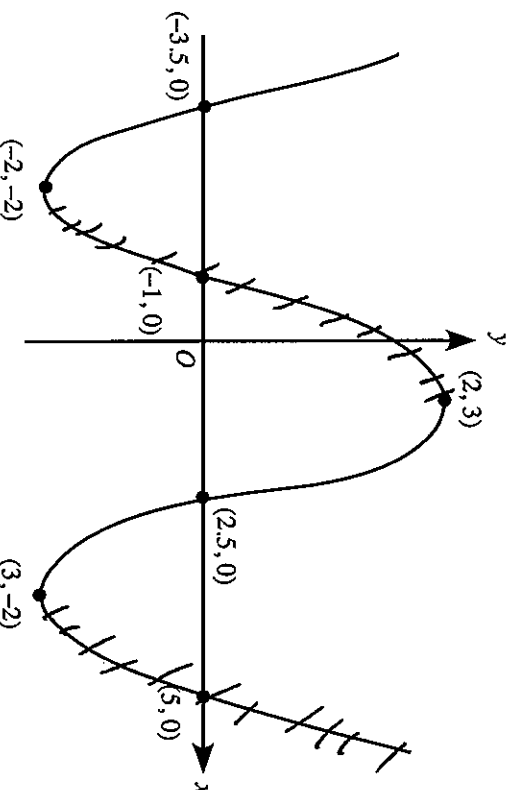
$$v = -3t^2 + 12t - 24$$

$$a = -6t + 12$$

$$\text{when } t = 3 \quad a = -6$$

5. Which of the following sets describes the conditions for which the gradient of the graph of $y = f(x)$ is positive? (Coordinates given are those of intercepts and turning points.)

- ☐ A $(-\infty, -3.5) \cup (-2, 2) \cup (5, \infty)$
- ☒ B $(-2, 2) \cup (3, \infty)$
- ☐ C $(-\infty, 3.5) \cup (5, 0)$
- ☐ D $(-3.5, -1) \cup (2, \infty)$
- ☐ E $(0, 2) \cup (5, \infty)$



SECTION C: ANALYSIS

Calculators allowed

Unless otherwise stated, exact answers should be given

1.

Wire is used to make the frame of a container in the shape of a cuboid as shown in the diagram. The total length of wire used to make the frame was 18 m.

a Show that $L = \frac{9-6x}{2}$.

$$18 = 4L + 12x$$

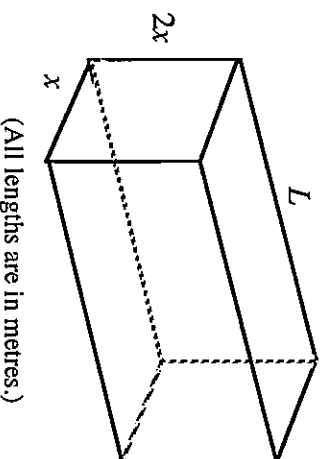
$$4L = 18 - 12x$$

$$L = \frac{18-12x}{4}$$

$$L = \frac{9-6x}{2}$$

b Show that the volume can be expressed as $V = 9x^2 - 6x^3$

(3 marks)



$$V = L \times 2x \times x$$

$$= 2x^2 \left(\frac{9-6x}{2} \right)$$

$$\Rightarrow 9x^2 - 6x^3$$

c Find the maximum volume of the container and the corresponding dimensions. You are not required to justify it is a maximum.

(1 mark)

$$V = 9x^2 - 6x^3$$

$$\frac{dV}{dx} = 18x - 18x^2$$

$$\text{Max/min when } \frac{dV}{dx} = 0$$

$$18x - 18x^2 = 0$$

$$18x(1-x) = 0$$

$$x = 0 \quad \text{or} \quad x = 1$$

(min) (max)

$$\begin{aligned} \nearrow V &= 9(1)^2 - 6(1)^3 \\ &= 3 \text{ m}^3 \end{aligned}$$

$$\begin{aligned} L &= \frac{3}{2} \text{ length} \\ x &= 1 \text{ width} \\ 2x &= 2 \text{ height} \end{aligned}$$

(3 marks)

2. A particle moves in a straight line and starts from a position one metre left of a fixed point O on the line.

Its velocity $v \text{ m/s}^2$ is given by $v = 15t - 9t^2$ for any time t seconds, $t \geq 0$.

a At what times is its velocity equal to zero?

$$15t - 9t^2 = 0$$

$$t = 0 \text{ and } t = \frac{5}{3} \text{ seconds.}$$

(2 marks)

b What is the acceleration of the particle when $t = 3$?

$$a = 15 - 18t$$

$$\text{when } t = 3 \quad a = -39 \text{ m/s}^2$$

(2 marks)

c What is its displacement from O at any time t seconds?

$$x = \int 15t - 9t^2 dt$$

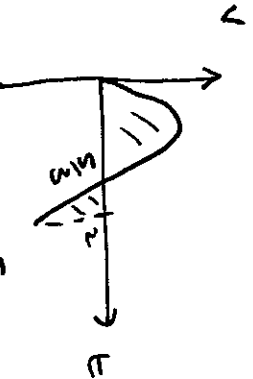
$$= \frac{15t^2}{2} - 3t^3 + c$$

$$t = 0, x = -1 \Rightarrow c = -1$$

$$x = \frac{15t^2}{2} - 3t^3 - 1$$

(3 marks)

d Find the total distance covered by the particle in the first 2 seconds.



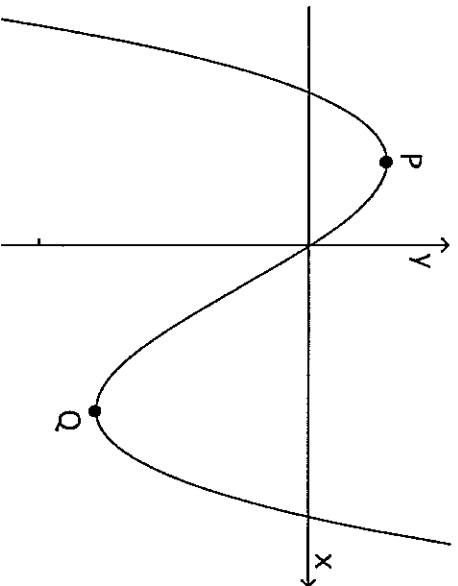
Distance travelled = area under v/t graph

$$d = \int_0^{5/3} 15t - 9t^2 dt - \int_{5/3}^2 15t - 9t^2 dt$$

(2 marks)

$$= \frac{71}{9} \text{ m.}$$

3. The graph of $f(x) = x^3 - 2x^2 - 3x$ is shown below. The tangent lines to the curve at the points P and Q are parallel to the x-axis.



- a Find the derivative $f'(x)$

$$f'(x) = 3x^2 - 4x - 3$$

- b Find the x values of the coordinates of points P and Q.

(1 mark)

$$3x^2 - 4x - 3 = 0$$

$$x = \frac{\sqrt{3+2}}{3}, -\frac{\sqrt{3+2}}{3}$$

Q

P

(2 marks)

- c Find the equation of the **tangent** line to the curve at point (1, -4).

$$y = -4x$$

* Use CAS.

Mean, 4, 9

(1 mark)

- d Find the equation of the **normal** to the curve at point (-1, 0).

$$y = -\frac{x}{4} - \frac{1}{4}$$

* Use CAS.

Mean 4, 1

(1 mark)

- e Write the integral expression that could be used to find the area bound by the curve and the x axis

$$f(x) = x^3 - 2x^2 - 3x$$

$$x \text{ intercepts } x = -1, 0, 3$$

$$\text{Area} = \int_{-1}^0 f(x) dx - \int_0^3 f(x) dx$$

(2 marks)

- f Hence, find the area bound by the curve and the x-axis

$$\text{Area} = \frac{71}{6} \text{ sq. units.}$$

(1 mark)

End of Section C
Analysis Mark =

/24