

MM12 Assignment 2 - Polynomials and Quadratics

Name: _____

Skills (Section A and B)

Analysis (Section C)

Section A: Short Answer

No Calculators Allowed

NOTE: Exact answers are required unless instructed otherwise within the question.

1. Given $P(x) = x^3 - 2x^2 + 1$ and $Q(x) = 3x^2 + x - 5$

a) State the degree of $P(x)$

3

b) Find each of the following, simplifying where appropriate.

(1 mark)

i) $P(\frac{1}{2})$

ii) $Q(-1)$

$$= \frac{5}{2}$$

$$= -3$$

iii) $Q(2x+1)$

$$= 3(2x+1)^2 + (2x+1) - 5$$

$$= 3(4x^2 + 4x + 1) + 2x - 4$$

$$= 12x^2 + 14x - 1$$

(1+1+2 = 4 marks)

2. i) Write down the first 5 rows of Pascal's triangle.

$$\begin{array}{ccccccc} & & & & & & 1 \\ & & & & & 1 & \\ & & & & 1 & 1 & \\ & & 1 & 2 & 1 & & \\ & 1 & 3 & 3 & 1 & & \\ 1 & 4 & 6 & 4 & 1 & & \end{array}$$

ii) Hence expand $(2x - 1)^4$.

$$(2x)^4 - 4(2x)^3 + 6(2x)^2 - 4(2x) + 1$$

$$= 16x^4 - 32x^3 + 24x^2 - 8x + 1$$

(1+2 = 3 marks)

3. Factorise over $\mathbb{Q}$ the expression $1 + 27x^3$

$$1 + (3x)^3 \\ = (1 + 3x)(1 - 3x + 9x^2)$$

4. a) Show without division that $x - 2$ is a factor of $x^3 - 2x^2 - 25x + 50$ (2 marks)

$$\begin{aligned} P(2) &= (2)^3 - 2(2)^2 - 25(2) + 50 \\ &= 8 - 8 - 50 + 50 \\ &= 0 \end{aligned}$$

Since $P(2) = 0$, $x - 2$ is a factor

- b) Hence fully factorise $x^3 - 2x^2 - 25x + 50$. (1 mark)

$$\begin{array}{r} x-2 \overline{) x^3 - 2x^2 - 25x + 50} \\ \underline{x^3 - 2x^2} \\ -25x + 50 \\ \underline{-25x + 50} \\ 0 \end{array}$$

Factors $(x-2)(x+5)(x-5)$

5. Given $P(x) = x^3 - kx^2 - 1$ has a remainder of -1 when divided by $x - 3$ find k . (3 marks)

$$P(3) = -1$$

$$27 - 9k - 1 = -1$$

$$9k = 27$$

$$k = 3$$

(2 marks)

6. Let $y = 3x^2 - 12x + 6$

- a) Find the coordinates of the y-intercept of the graph of $y = 3x^2 - 12x + 6$

y intercept, $x = 0$

$$(0, 6)$$

(1 mark)

- b) Use completing the square to write the equation in the form $y = a(x + b)^2 + c$

$$\begin{aligned} y &= 3(x^2 - 4x + 2) \\ &= 3[(x^2 - 4x + 4) - 4 + 2] \\ &= 3[(x-2)^2 - 2] \\ y &= 3(x-2)^2 - 6 \end{aligned}$$

(3 marks)

- c) Hence find the exact coordinates of:

- i) the turning point $y = 3x^2 - 12x + 6$

$$(2, -6)$$

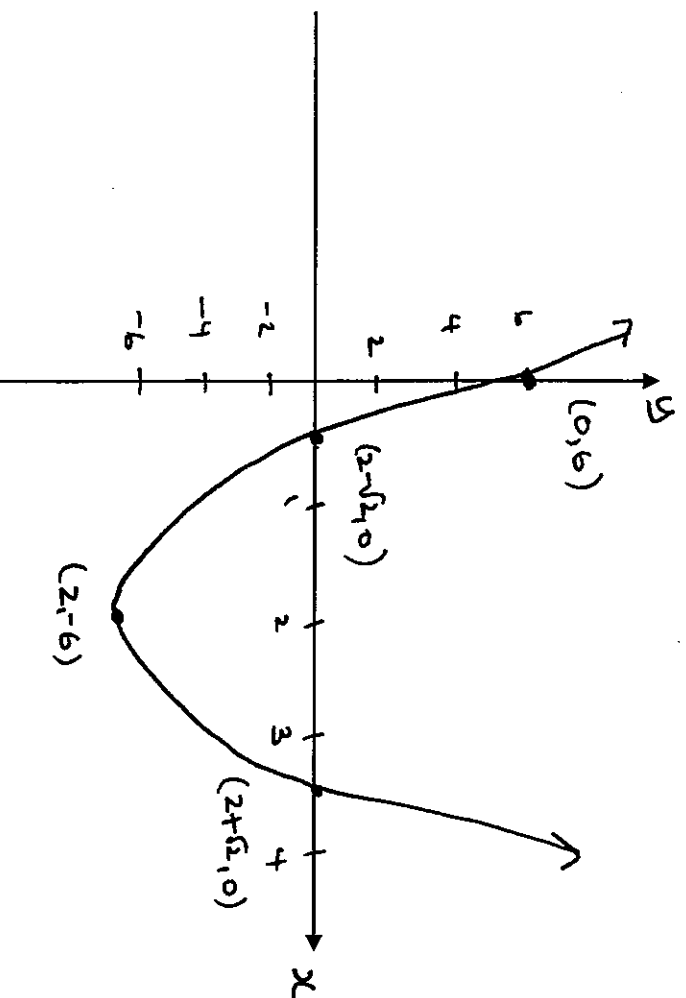
- ii) the x-intercepts of $y = 3x^2 - 12x + 6$

$$\begin{aligned} 3(x-2)^2 - 6 &= 0 \\ (x-2)^2 - 2 &= 0 \\ (x-2+\sqrt{2})(x-2-\sqrt{2}) &= 0 \end{aligned}$$

x intercepts. $(2+\sqrt{2}, 0) (2-\sqrt{2}, 0)$.

(1 + 3 = 4 marks).

- d) Sketch the graph of $y = 3x^2 - 12x + 6$, labelling the coordinates of all key points.



(3 marks)

End of Short Answer

Section B: Multiple Choice Questions

Calculators Are Allowed

Circle Correct Response

1. Which one of the following functions is a polynomial.

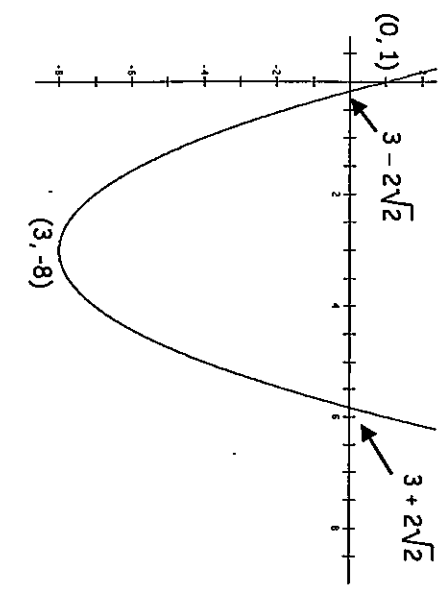
- A $y = 3x^4 - \frac{1}{x}$
- B $y = \sqrt{x}$
- ☒ C $y = \frac{1}{2}x^3 - 1$
- D $y = x^{-2} + 3x + 2$
- E $y = \frac{1}{x^2}$

2. Correct to 2 decimal places the graph of $y = x^2 - 38x + 260$ has x-intercept(s) at:

- A only (8.95, 0)
- B only (-8.95, 0)
- C (8.95, 0) and (29.04, 0)
- ☒ D (8.95, 0) and (29.05, 0)
- E (8.95, 0) and (-8.95, 0)

3. Given the graph of $y = x^2 - 6x + 1$ shown below the solution for $x^2 - 6x + 1 > 0$ is:

- A $x \in (-\infty, 3 - 2\sqrt{2}] \cup [3 + 2\sqrt{2}, \infty)$
- ☒ B $x \in (-\infty, 3 - 2\sqrt{2}) \cup (3 + 2\sqrt{2}, \infty)$
- C $x \in (3 - 2\sqrt{2}, 3 + 2\sqrt{2})$
- D $x \in [3 - 2\sqrt{2}, 3 + 2\sqrt{2}]$
- E $x \in [-8, \infty)$



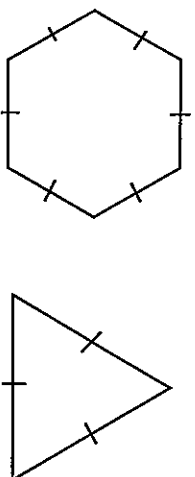
*end points are
not included*

4. Which of the following linear expressions could not possibly be a factor of $x^3 + kx^2 - x + 12$

- A $x - 1$
- B $x + 2$
- C $x - 3$
- D $x + 4$
- ☒ E $x - 5$

*5 is not a factor
of 12*

1. A regular polygon is a closed shape with straight edges all of the same length. A couple of examples are shown beside.



If a regular polygon has n sides then the size of each interior angle, in degrees, is given by the formula $\frac{180(n-2)}{n}$.

- a) Write in terms of n , the size of each interior angle of a regular polygon with $n + 5$ sides.

$$\frac{180(n+5-2)}{n+5} = \frac{180(n+3)}{n+5}$$

(1 mark)

- b) Consider two regular polygons, named polygon A and polygon B. Polygon A has n sides and polygon B has $n + 5$ sides. If each interior angle of polygon A is 1° less than each interior angle of polygon B, show that n satisfies the equation $n^2 + 5n - 1800 = 0$.

$$B - A = 1$$

$$\frac{180(n+3)}{n+5} - \frac{180(n-2)}{n} = 1$$

$$180n(n+3) - 180(n-2)(n+5) = n(n+5)$$

$$180n^2 + 540n - 180(n^2 + 3n - 10) = n^2 + 5n$$

$$180n^2 + 540n - 180n^2 - 540n + 1800 = n^2 + 5n$$

$$n^2 + 5n - 1800 = 0$$

(3 marks)

- c) Solve $n^2 + 5n - 1800 = 0$ and hence find the number of sides of polygon B.

$$n^2 + 5n - 1800 = 0$$

$$n = -45 \text{ or } 40$$

$$\text{Since } n > 0, \quad n = 40$$

Polygon B has 45 sides

(3 marks)

2. Consider the following two functions, where k is an unknown constant.

$$y = kx^2 - 3x + 1$$

$$y = kx + 2$$

- a) Show that at the point(s) where the graphs of these functions intersect x must satisfy the equation $kx^2 - (3+k)x - 1 = 0$.

$$kx^2 - 3x + 1 = kx + 2$$

$$kx^2 - 3x - kx + 1 - 2 = 0$$

$$kx^2 - (3+k)x - 1 = 0$$

(2 marks)

- b) Show that the discriminant of the quadratic $kx^2 - (3+k)x - 1 = 0$ is equal to $k^2 + 10k + 9$.

$$\Delta = b^2 - 4ac$$

$$= [-(3+k)]^2 - 4 \times k \times -1$$

$$= k^2 + 6k + 9 + 4k$$

$$= k^2 + 10k + 9$$

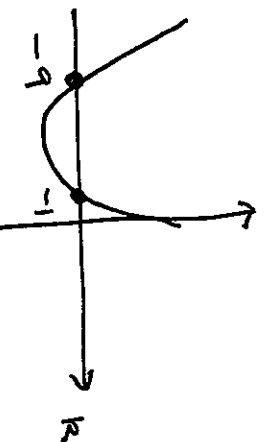
(2 marks)

- c) Hence, find the values of k for which the graphs of the two functions given at the start of this problem intersect at least once.

$$\Delta \geq 0$$

$$k^2 + 10k + 9 \geq 0$$

$$(-\infty, -9] \cup [-1, \infty)$$



(4 marks)

3. Delta works as a stunt double on television shows. For one stunt she has to jump from the top of a tall building. Her height, h metres, above the ground after t seconds from when she jumps is given by $h = 21 + t - 5t^2$.

a) Find the height of the building.

$$t = 0 \quad h = 21$$

Building is 21m high

(1 mark)

b) Find Delta's height above the ground after

i) 1 second

$$t = 1$$

$$h = 17 \text{ m}$$

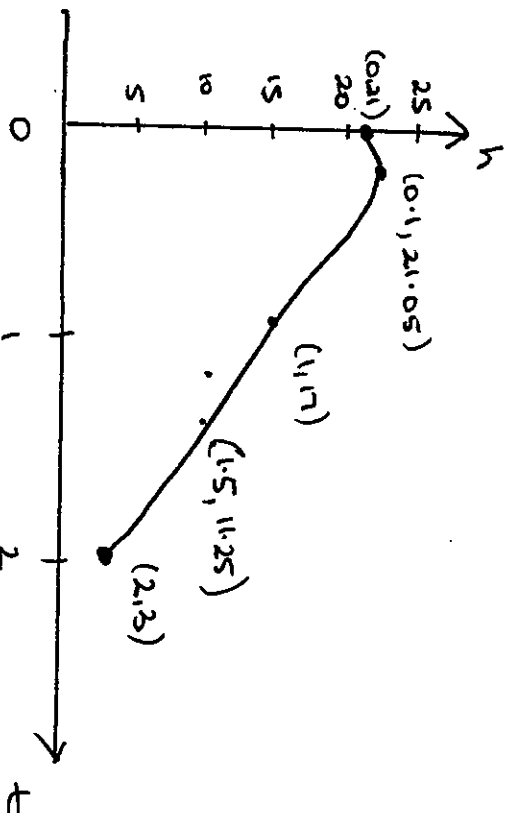
ii) 1.5 seconds

$$t = 1.5$$

$$h = 11.25$$

(2 marks)

c) Sketch the graph of h against t for $0 \leq t \leq 2$. Label coordinates of the end points and a turning point, giving approximate values correct to 1 decimal place.



d) Find the maximum height above the ground that Delta reached during this stunt. Give your answer in metres correct to 2 decimal places. (3 marks)

$$21.05 \text{ m}$$

(1 mark)