

MM12 Cubics and Functions Test 2019

Skills - Part A /22
- Part B /3

Name: ANSWERS

Total Skills: /25

Analysis: /23

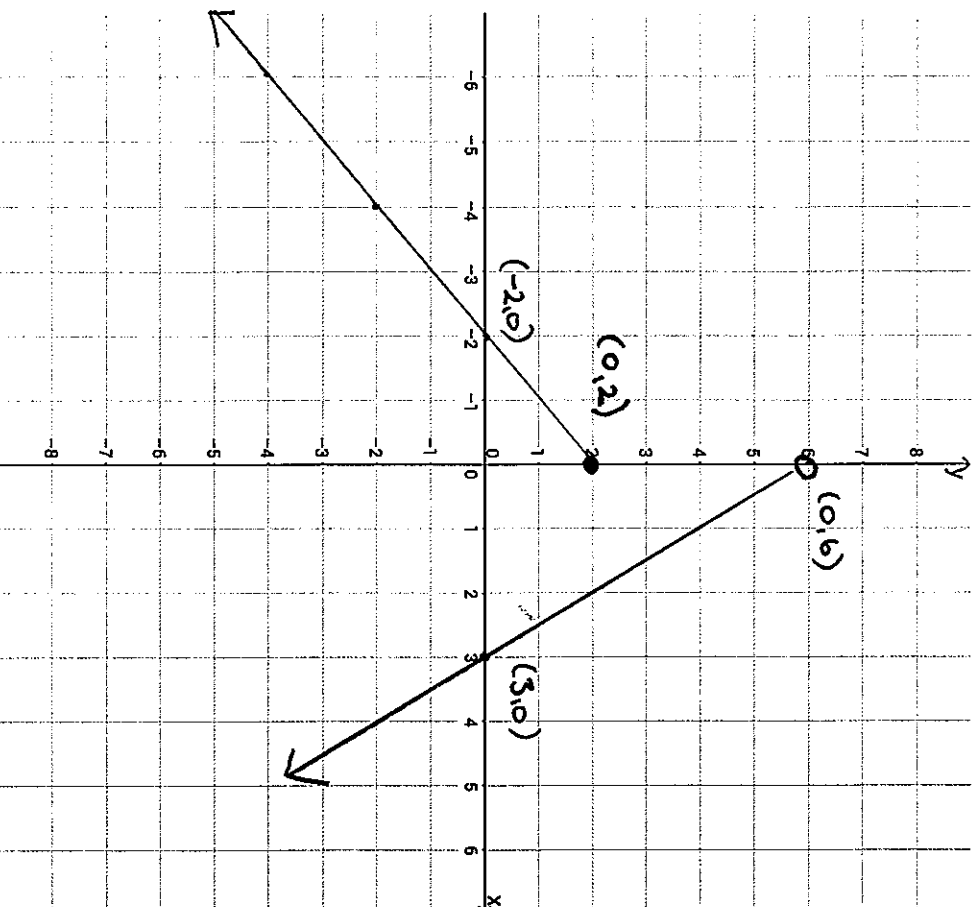
Part A: Short Answer

No Calculator

Time Allowed: 20 minutes

1. (a) Sketch the graph of the following function. Clearly label any axial intercepts and end points with their coordinates.

$$f(x) = \begin{cases} x + 2 & x \leq 0 \\ -2x + 6 & x > 0 \end{cases}$$



- (b) State the range of the function.

range $y \in (-\infty, 6)$

(4+1 = 5 Marks)

2. For the function with the rule $f(x) = x^2 - 4x + 3$, find:

$$(a) f(2) = 4 - 8 + 3 = -1$$

$$(b) f(a) + f(-2a) = a^2 - 4a + 3 + (-2a)^2 - 4(-2a) + 3 \\ = 5a^2 + 4a + 6$$

$$(c) f(a+2) + f(-a-2) = (a+2)^2 - 4(a+2) + 3 + (-a-2)^2 - 4(-a-2) + 3 \\ = a^2 + 4a + 4 - 4a - 8 + 3 + a^2 + 4a + 4 + 4a + 8 + 3 \\ = 2a^2 + 8a + 14$$

(1+2+3 = 6 marks)

3. (a) Show that the circle with the equation $x^2 + y^2 - 2x - 4y - 20 = 0$ can be written in the form $(x-1)^2 + (y-2)^2 = 25$

$$(x^2 - 2x + 1) + (y^2 - 4y + 4) - 20 - 1 - 4 = 0 \\ (x-1)^2 + (y-2)^2 = 25$$

(b) State the radius of the circle and the coordinates of its centre.

$$\text{radius} = 5 \quad \text{centre } (1, 2)$$

(c) Find the exact horizontal distance between the x-intercepts of the circle

$$\text{x-intercepts, } y = 0$$

$$(x-1)^2 + (-2)^2 = 25$$

$$(x-1)^2 = 21$$

$$x-1 = \pm\sqrt{21}$$

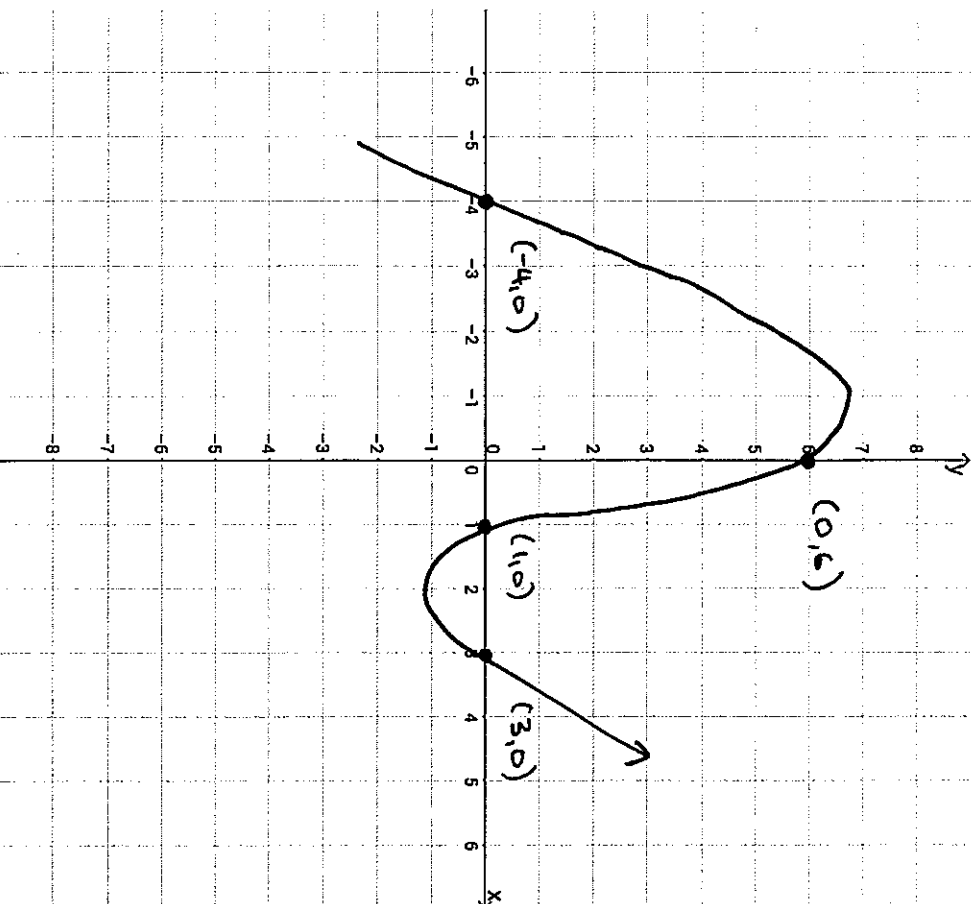
(2+2+2 = 6 marks)

$$x = 1 \pm \sqrt{21}$$

$$\text{distance} = 2\sqrt{21}$$

2

4. (a) Sketch the graph of $y = \frac{1}{2}(x+4)(x-1)(x-3)$. Label the coordinates of all axial intercepts. Coordinates of turning points do not need to be shown.



- (b) Find $\{x: \frac{1}{2}(x+4)(x-1)(x-3) \geq 0\}$.

$$x \in [-4, 1] \cup [3, \infty)$$

①

correct
answer

- (c) How many solutions will there be to the equation $\frac{1}{2}(x+4)(x-1)(x-3) = 1$

3

(2+2+1 = 5 marks)

End of Section A

Name: _____

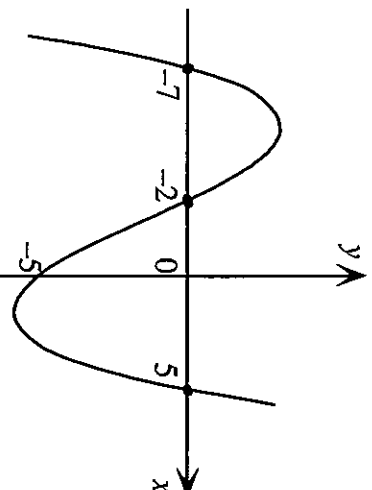
Section B: Multiple Choice Questions Calculators and notes are allowed

Circle the correct response Time allowed: 25 minutes

Question 1

A possible equation for the curve shown is

- A. $y = (x + 7)(x + 2)(x - 5)$
- B. $y = (x - 7)(x - 2)(x + 5)$
- ☒ C. $y = \frac{1}{14}(x + 7)(x + 2)(x - 5)$
- D. $y = \frac{1}{14}(x - 7)(x - 2)(x + 5)$
- E. $y = -\frac{1}{14}(x + 7)(x + 2)(x - 5)$



Question 2

The graph of the equation $y = x^3$ is reflected in the x -axis and then translated 2 units to the right and

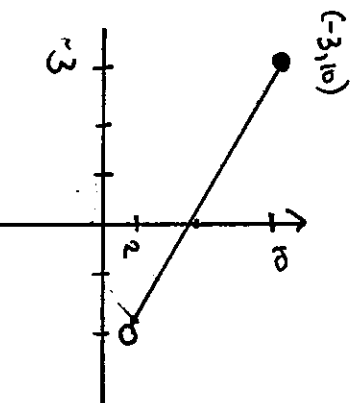
1 unit down. The equation of the new graph is

- ☒ A. $y = -(x - 2)^3 - 1$
- B. $y = -(x - 2)^3 + 1$
- C. $y = (-x)^3 - 3$
- D. $y = -(x)^3 - 1$
- E. $y = -(x)^3 - 2 - 1$

Question 3

The range of the function $f: [-3, 5] \rightarrow \mathbb{R}, f(x) = 7 - x$ is

- ☒ A. $(2, 10]$
- B. $[2, 10)$
- C. $(2, \infty)$
- D. $(2, 10)$
- E. $\mathbb{R}$

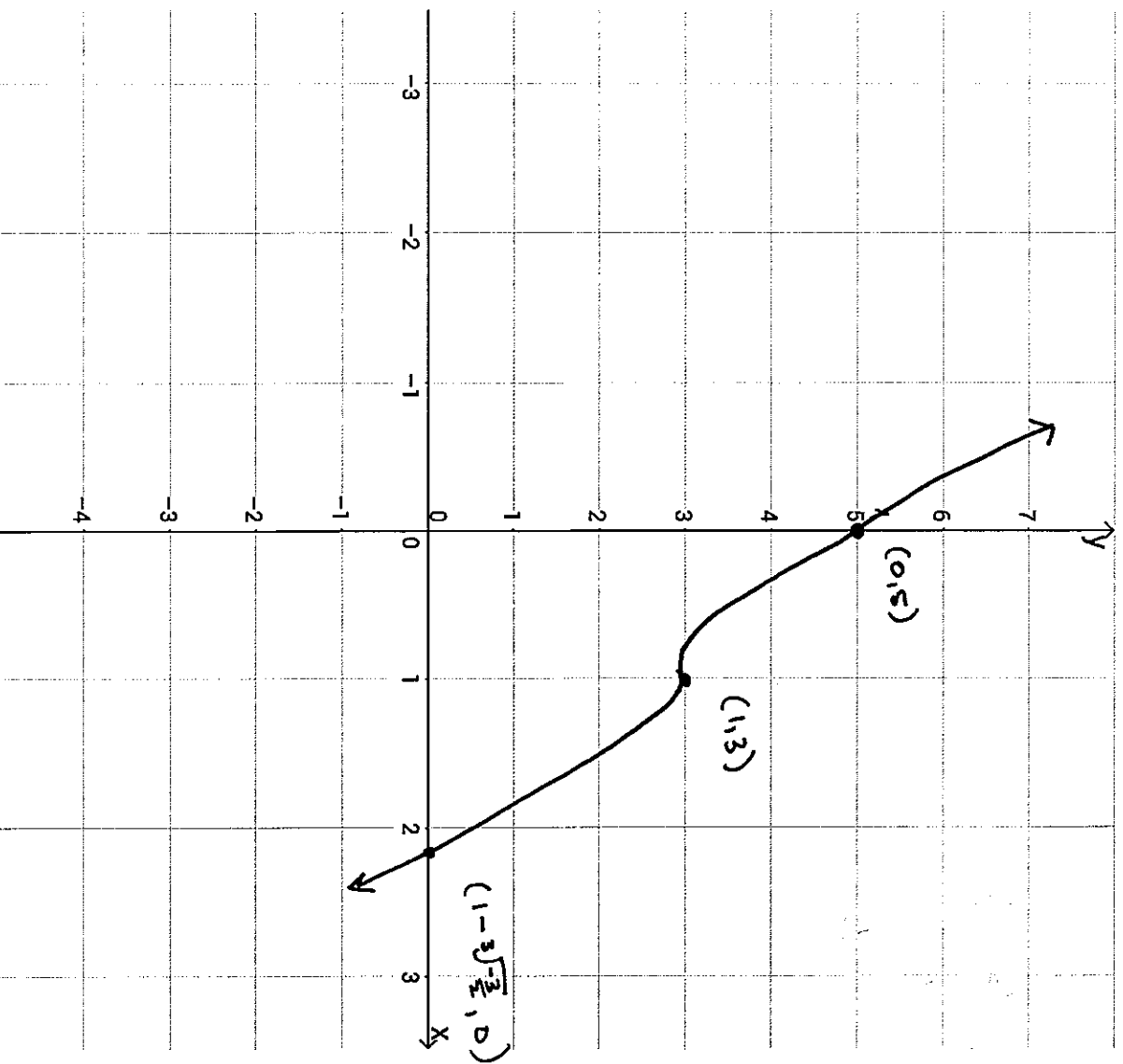


Section C: Extended Response

Exact answers should be given unless otherwise specified

Question 1

- a. Sketch the graph of $y = 2(1 - x)^3 + 3$ labelling any intercepts with the coordinate axes and the stationary point of inflexion with their **exact** coordinates.



- b. If the domain was restricted to $x \in [0, 3]$ what would be the range?

$$[-13, 5]$$

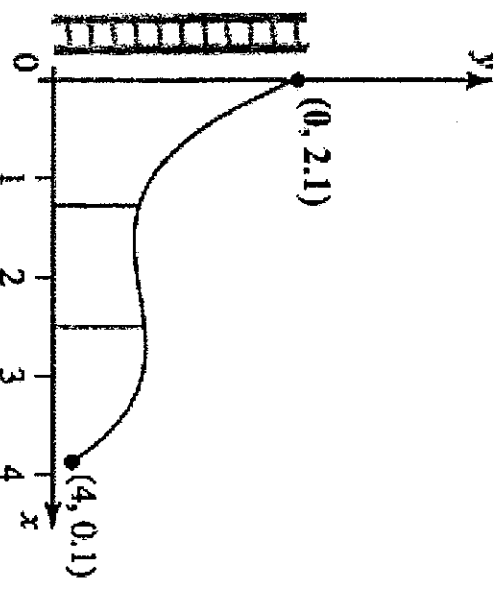
Question 2

A new playground slide is to be constructed at a local park. At the foot of the slide is a vertical ladder. The slide starts at a vertical height of 2.1 m above the ground.

The end point is 0.1 m above the ground and is 4 m horizontally from the foot of the slide.

A model for the slide is $h(x) = ax^3 + bx^2 + cx + d$ where h metres is the height of the slide above ground level at a horizontal distance, x m from the foot. The foot is at the origin.

The slide has two vertical supports, one is 1 m long and is 1.25 m horizontally from the foot and the other is 1.1 m long and 2.5 m horizontally from the foot.



- (a) Give the coordinates of 4 points that lie on the cubic graph of the slide

$$(0, 2.1) \quad (4, 0.1) \quad (1.25, 1) \quad (2.5, 1.1)$$

①

①

- (b) State the value of d in the equation of the slide

$$d = 2.1$$

①

- (c) Write three simultaneous equations that could be used to find the coefficients a , b , and c .

$$0.1 = 64a + 16b + 4c + 2.1$$

①

$$1 = (1.25)^3 a + (1.25)^2 b + 1.25c + 2.1$$

①

$$1.1 = (2.5)^3 a + (2.5)^2 b + 2.5c + 2.1$$

①

- (d) The equation of the slide can be shown to be

$$h(x) = -0.164x^3 + x^2 - 1.872x + 2.1$$

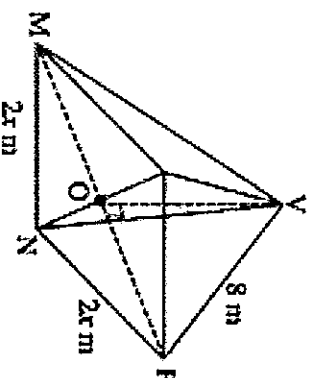
Use this equation to find the length of a third vertical support at $x = 3.5$ m. Give your answer correct to 2 decimal places.

$$h = 0.77 \text{ m}$$

Question 3

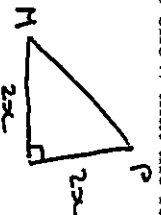
A tent used by a group of campers is in the shape of a square-based right pyramid with a slant edge, VP, of 8 metres.

For the figure shown, let OV, the height of the tent be h metres and the edge of the square base be $2x$ metres.



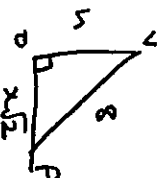
- (a) Use Pythagoras' theorem and triangle MPN

to show that the length of the diagonal (MP) of the square base is $2x\sqrt{2}$ metres



$$\begin{aligned} d(MP) &= \sqrt{(2x)^2 + (2x)^2} \\ &= \sqrt{8x^2} \\ &= 2x\sqrt{2} \end{aligned}$$

- (b) Use Pythagoras' theorem and triangle VOP to show that $2x^2 = 64 - h^2$



$$\begin{aligned} 8^2 &= h^2 + (x\sqrt{2})^2 \\ 64 &= h^2 + 2x^2 \\ 2x^2 &= 64 - h^2 \end{aligned}$$

- (c) The volume, V , of a pyramid is found using the formula $V = \frac{1}{3}Ah$, where A is the area of the base of the pyramid. Use this formula to show that the volume of the tent is given by $V = \frac{1}{3}(128h - 2h^3)$

$$\begin{aligned} V &= \frac{1}{3} \times 4x^2 \times h \\ &= \frac{1}{3} \times 2(64 - h^2) \times h \\ &= \frac{1}{3}(128h - 2h^3) \end{aligned}$$

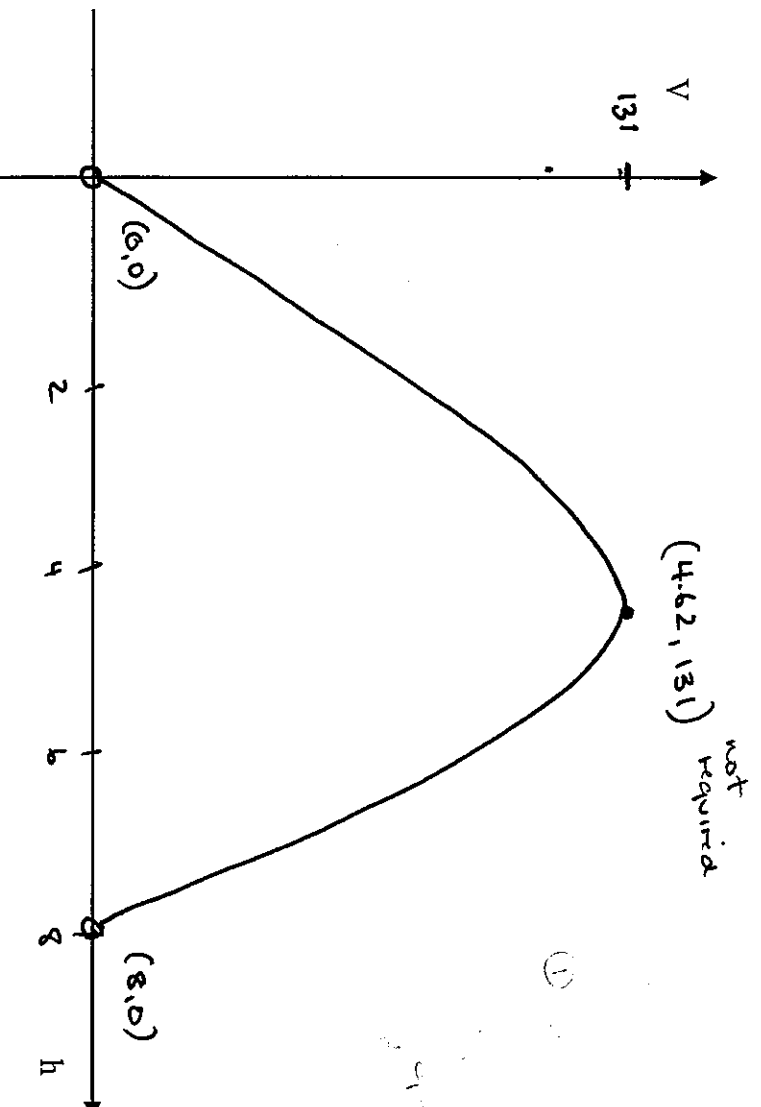
- (d) What is the domain of the volume function

$$0 < h < 8$$

- (e) If the height of the tent was 3 metres, what is the volume?

$$V = 110 \text{ m}^3$$

(f) Sketch a graph of $V = \frac{1}{3}(128h - 2h^3)$, showing the coordinates of any end points.



(g) What is the **maximum volume** of the tent (to the nearest whole number) and what **height** (correct to 1 d.p.) gives the maximum volume?

$$V = 131 \text{ m}^3$$

$$h = 4.6 \text{ m}$$

(2+2+2+1+1+3+2 = 13 marks)