

MM12 Assignment 3 Cubics and Functions

Name: ANSWERS.

Skills (Section A and B) / 33

Analysis (Section C) / 24

Section A: Short Answer

No Calculators Allowed

NOTE: Exact answers are required unless instructed otherwise within the question.

1. (a) Fully factorise $P(x) = x^3 + x^2 - 17x + 15$

$$P(1) = 1 + 1 - 17 + 15 = 0$$

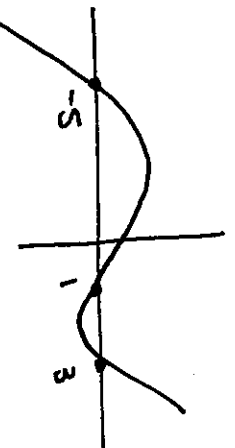
$x-1$ is a factor

$$\begin{array}{r} x-1 \overline{) \begin{array}{l} x^2 + 2x - 15 \\ x^3 + x^2 - 17x + 15 \end{array}} \\ \underline{x^2 - x^2} \\ 2x^2 - 17x \\ \underline{2x^2 - 2x} \\ -15x + 15 \\ \underline{-15x + 15} \\ 0 \end{array}$$

$$= (x-1)(x^2 + 2x - 15) \\ = (x-1)(x+5)(x-3)$$

- (b) Hence solve the equation $x^3 + x^2 - 17x + 15 = 0$

(3 marks)

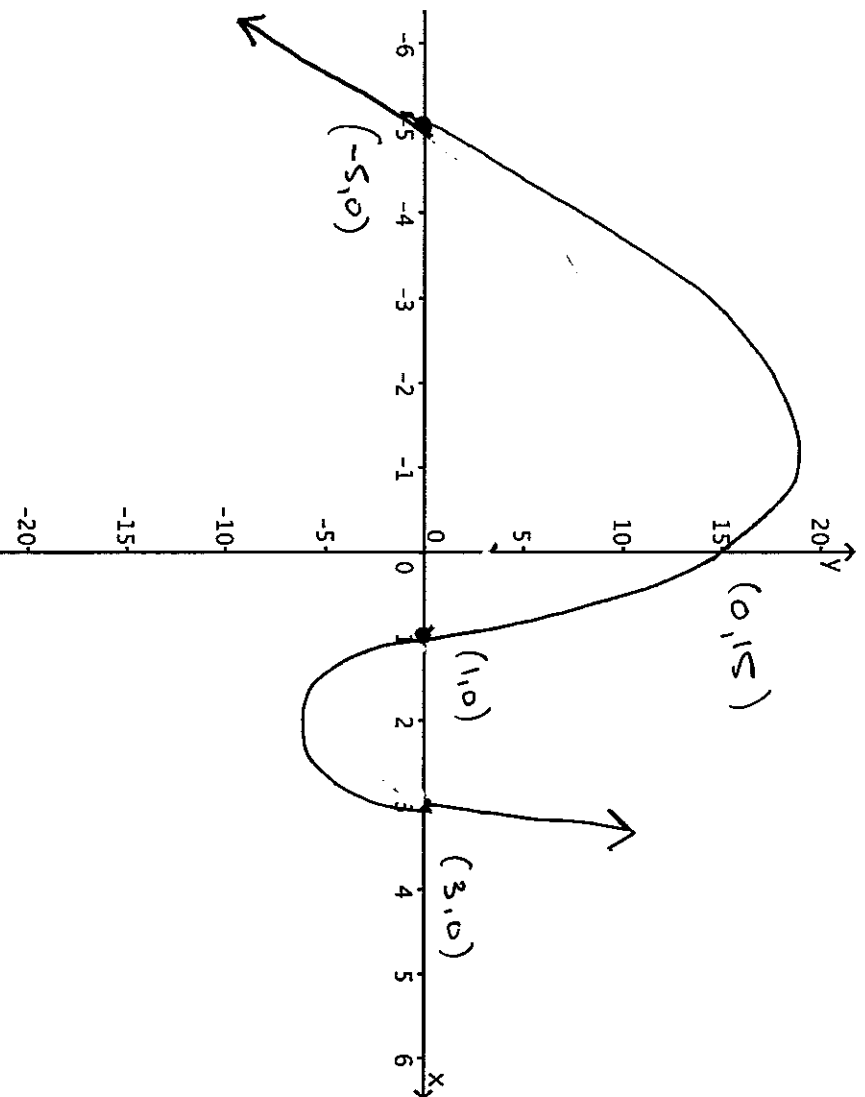


(2 marks)

$$(x-1)(x+5)(x-3) = 0$$

$$x = -5, -1, 3$$

- (c) Sketch the graph of $y = x^3 + x^2 - 17x + 15$ showing the coordinates of the x and y intercepts. You do not need to show the coordinates of the turning points.

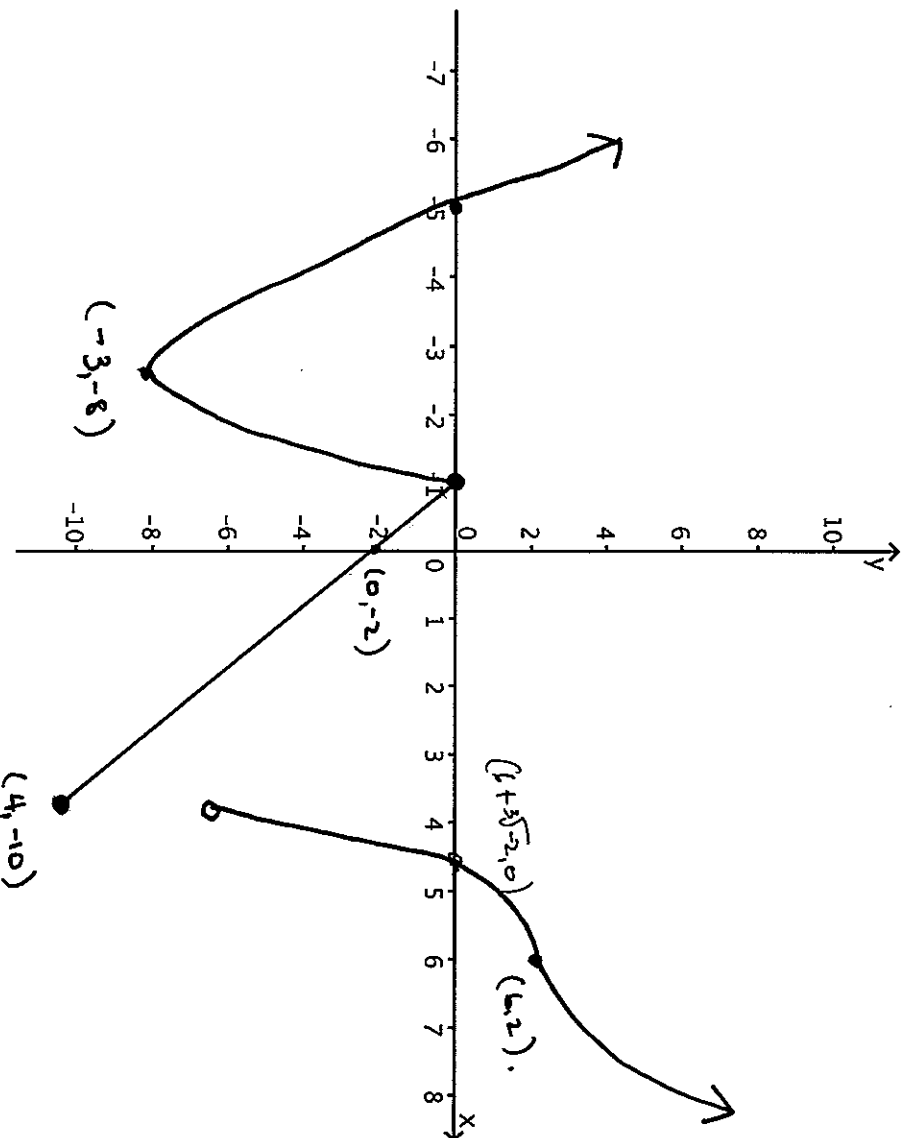


- (d) Hence find the interval for which $x^3 + x^2 - 17x + 15 < 0$ (3 marks)

$$x \in (-\infty, -5) \cup (1, 3)$$

(1 mark)

2. For the given hybrid function $g(x) = \begin{cases} 2(x+5)(x+1), & x < -1 \\ -2(x+1), & -1 \leq x \leq 4 \\ (x-6)^3 + 2, & x > 4 \end{cases}$ calculate all key points that will enable you to sketch a detailed graph of $g(x)$ on the set of axes provided below.



(7 marks)

Work Space:

parabola T.P. $x = -3$
 $y = -8$

line $x = -1$ $y = 0$
 $x = 4$ $y = -10$
 $x = 0$ $y = -2$

cubic $x = 4$ $y = -6$

SPOT $(6, 2)$

x int $(6 + \sqrt[3]{2}, 0)$

3. A circle has rule $x^2 + 2x + y^2 - 6y + 1 = 0$.

- (a) Show that this rule can be written in the form $(x + 1)^2 + (y - 3)^2 = 9$.

$$(x^2 + 2x + 1) + (y^2 - 6y + 9) - 1 - 9 + 1 = 0$$

$$(x+1)^2 + (y-3)^2 - 9 = 0$$

$$(x+1)^2 + (y-3)^2 = 9$$

(3 marks)

- (b) State the coordinates of:

- (i) the centre of the circle and

$$(-1, 3)$$

- (ii) its radius

$$3$$

(2 marks)

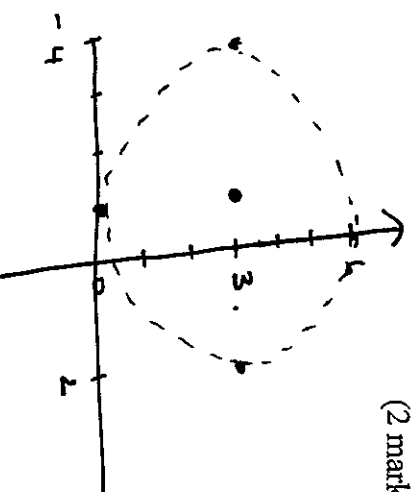
- (c) How many x-intercepts does the circle have? Why? Draw a diagram to assist.

$$x \text{ int}, y = 0$$

$$(x+1)^2 + (-3)^2 = 9$$

$$(x+1)^2 = 0$$

$$x = -1 \text{ is the only solution}$$



(3 marks)

- (d) State:

- (i) the domain and

$$x \in [-4, 2]$$

- (ii) the range of this relation.

$$y \in [0, 6]$$

(2 marks)

End of Section A

Total: /26

Full Name: _____

Section B: Multiple Choice Questions

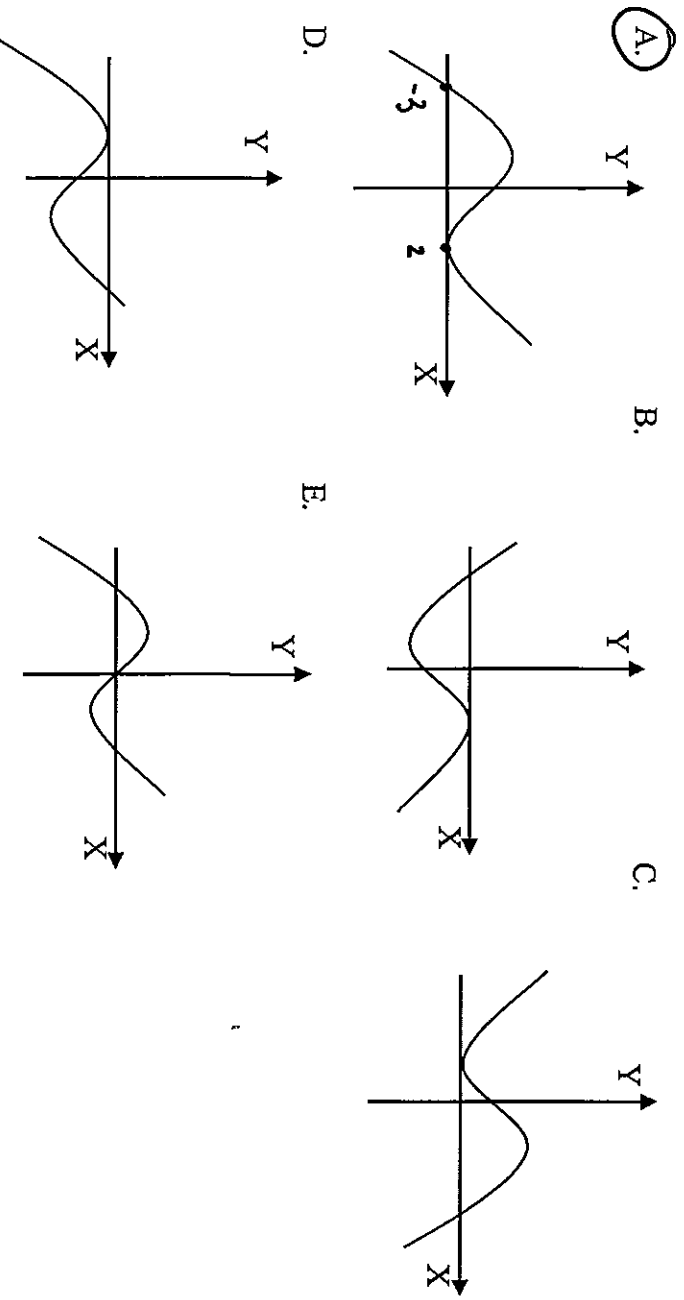
Calculators Are Allowed

Circle Correct Response

1. The graph with equation $y = -3(x + 1)^2(2x - 3)$

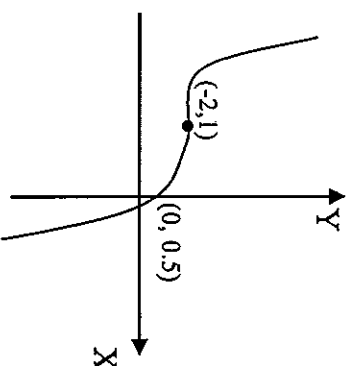
☐ A. has 3 distinct x-intercepts ✗
☒ B. has a turning point at $x = -1$ ✓
☐ C. has a turning point at $x = 1.5$ ✗
☐ D. is the graph of $y = x^3$ translated 1 unit to the left and 1.5 units down.
☐ E. all of the above

2. The graph of $y = (x - 2)^2(x + 3)$ could be:



3. The graph shown has rule

☐ A. $y = (x - 2)^3 + 1$ ✗
☐ B. $y = (x + 2)^3 + 0.5$ ✗
☒ C. $y = -\frac{1}{16}(x + 2)^3 + 1$ ✓
☐ D. $y = (x - 1)^3 + 0.5$ ✗
☐ E. $y = (x + 2)^3 + 1$ ✗

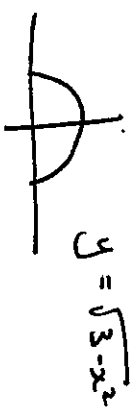


4. If $f(x) = 5(x + 2)^2 + 7$, then $f(-1)$ equals

☐ A. 52
☐ B. 17
☐ C. 27
☒ D. 12 ✓
☐ E. 54

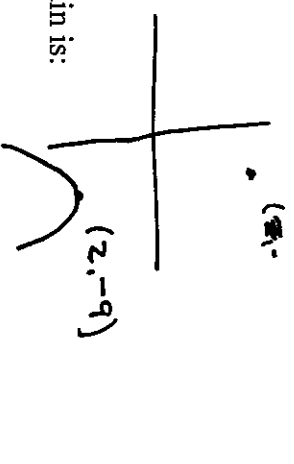
5. Which of the following functions is **not** a one-to-one function?

- A. $y = 3x^2 - 3x, x > 3$
- ☒ B. $y = \sqrt{3 - x^2}$
- C. $y = 3x^2, x > 0$ ✓
- D. $y = 3\sqrt{x}$ ✓
- E. $y = 3x$ ✓



6. The function $f: R \rightarrow R$, where $f(x) = -(x - 2)^2 + 9$, has range

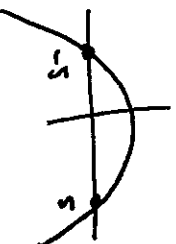
- A. $(-\infty, 9)$
- B. $(-\infty, 2)$
- ☒ C. $(-\infty, -9]$
- D. $[9, \infty)$
- E. $[-9, \infty)$



7. For the function with rule $f(x) = \sqrt{25 - x^2}$, the maximal domain is:

- A. $[0, 25]$
- ☒ B. $[-5, 5]$
- C. $(-\infty, 5]$
- D. $(-5, 5)$
- E. $[-5, \infty)$

$$25 - x^2 \geq 0$$



End of Section B

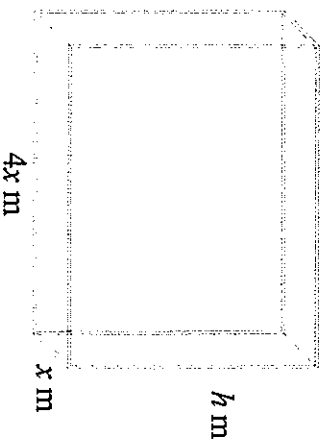
Total: /7

Section C: Analysis:

Calculator Allowed

Exact answers are required unless instructed otherwise within the question

1. A cuboid (rectangular prism) has dimensions x metres, h metres and $4x$ metres, as shown on the diagram. The edges of the cuboid are made from 640 m of wire.



- (a) Find h in terms of x .

$$20x + 4h = 640$$

$$h = 160 - 5x$$

(2 marks)

- (b) Show that the volume, $V \text{ m}^3$, of the cuboid is equal to $20x^2(32 - x)$.

$$V = x \times 4x \times (160 - 5x)$$

$$= 4x^2 (160 - 5x)$$

$$= 4x^2 \times 5(32 - x)$$

$$= 20x^2(32 - x)$$

(2 marks)

- (c) Find V when $x = 11$.

$$V = 50820 \text{ m}^3.$$

(2 marks)

- (d) Find the possible values of x for the cuboid to exist.

$$0 < x < 32$$

$$\text{Since } x > 0 \text{ and } 160 - 5x > 0$$

(2 marks)

- (e) Find the possible value(s) of x , correct to 2 decimal places, when $V = 60\,000$.

$$60,000 = 20x^2(32-x)$$

$$x = 12.35 \quad \text{or} \quad 28.24 \quad \text{or} \quad -8.60$$

$$x = 12.35 \quad \text{or} \quad 28.24 \quad \text{since} \quad 0 < x < 32 \quad (2 \text{ marks})$$

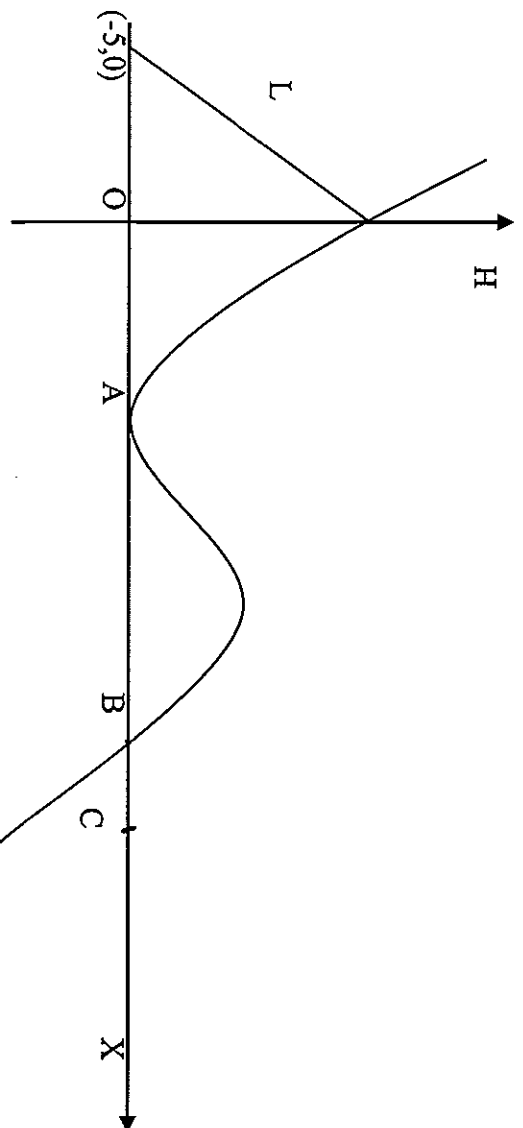
- (f) Find the maximum volume of the cuboid and the value of x for which it occurs.
Give your answer correct to 1 decimal place.

$$x = 21.3 \text{ m} \quad V = 970897 \text{ m}^3$$

(2 marks)

2.

It is proposed to have a slippery dip at the new aquatic complex. The diagram below shows that the slippery dip is to be based on a cubic function with rule $H = -0.2(x^3 - 9x^2 + 24x - 20)$, where H metres is the height of the slippery dip above the water level at a distance of x metres from the edge of the pool at the origin. L represents a ladder that goes to the top of the slippery dip.



- (a) What is the height of the slippery dip at the edge of the pool?

4 m

(2 marks)

- (b) (i) Show algebraically that $H = -0.2(x-2)^2(x-5)$.

$$\text{Let } P(x) = x^3 - 9x^2 + 24x - 20$$

$$P(2) = 8 - 36 + 48 - 20 = 0$$

$$x-2 \text{ is a factor}$$

$$\begin{array}{r} x-2 \quad \overline{) \quad x^3 - 9x^2 + 24x - 20} \\ \underline{x^3 - 2x^2 + 4x - 4} \\ -7x^2 + 24x - 16 \\ \underline{-7x^2 + 14x - 14} \\ 10x - 20 \\ \underline{10x - 20} \\ 0 \end{array}$$

$$\begin{array}{r} x^3 - 9x^2 + 24x - 20 \\ \underline{-7x^2 + 24x - 14} \\ 10x - 20 \\ \underline{10x - 20} \\ 0 \end{array}$$

(3 marks)

- (ii) Find the coordinates of the points marked A and B where the slippery dip is level with the water.

$$A(2,0) \quad B(5,0)$$

(2 marks)

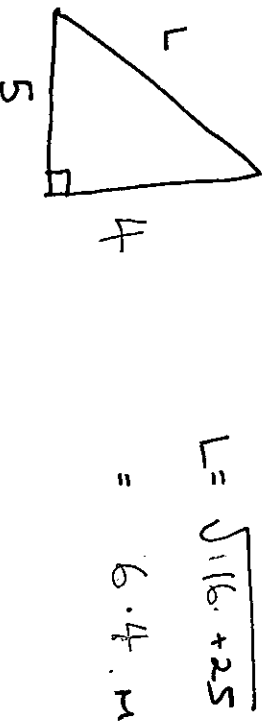
- (c) After the slippery dip enters the water at B , it continues for 1 metre horizontally. What is the depth of the water at the end of the slippery dip at C ?

$$C(6, -3.2)$$

$$\text{Depth is } 3.2 \text{ m.}$$

(1 marks)

- (d) Find the length, L metres, of the ladder used to climb onto the slippery dip at the edge of the pool, where $x=0$. Give your answer correct to 1 decimal point.



(3 marks)

End of Section C

Total: /24