

Trial Examination 2019

MATHEMATICAL METHODS

Trial Written Examination 1 - SOLUTIONS

Question 1

a. $\frac{d}{dx}\left(\frac{\cos(x)}{x}\right) = \frac{-x\sin(x) - \cos(x)}{x^2}$ 1M, 1A

b. $f(x) = 5x^2 \tan(3x)$

$f'(x) = 10x \tan(3x) + 15x^2 \sec^2(3x)$ 1M

$f'(\pi) = 10\pi \tan(3\pi) + 15\pi^2 \sec^2(3\pi)$
 $= 15\pi^2$ 1A

Question 2

$2e^x + 5 = 3e^{-x}$

$2e^x + 5 - 3e^{-x} = 0$

$2e^{2x} + 5e^x - 3 = 0$ 1M

Let $a = e^x$

$2a^2 + 5a - 3 = 0$

$(2a - 1)(a + 3) = 0$ 1M

$a = \frac{1}{2}, a = e^x \neq -3$

$e^x = \frac{1}{2}$

$x = \log_e\left(\frac{1}{2}\right) = -\log_e(2)$ either form 1A

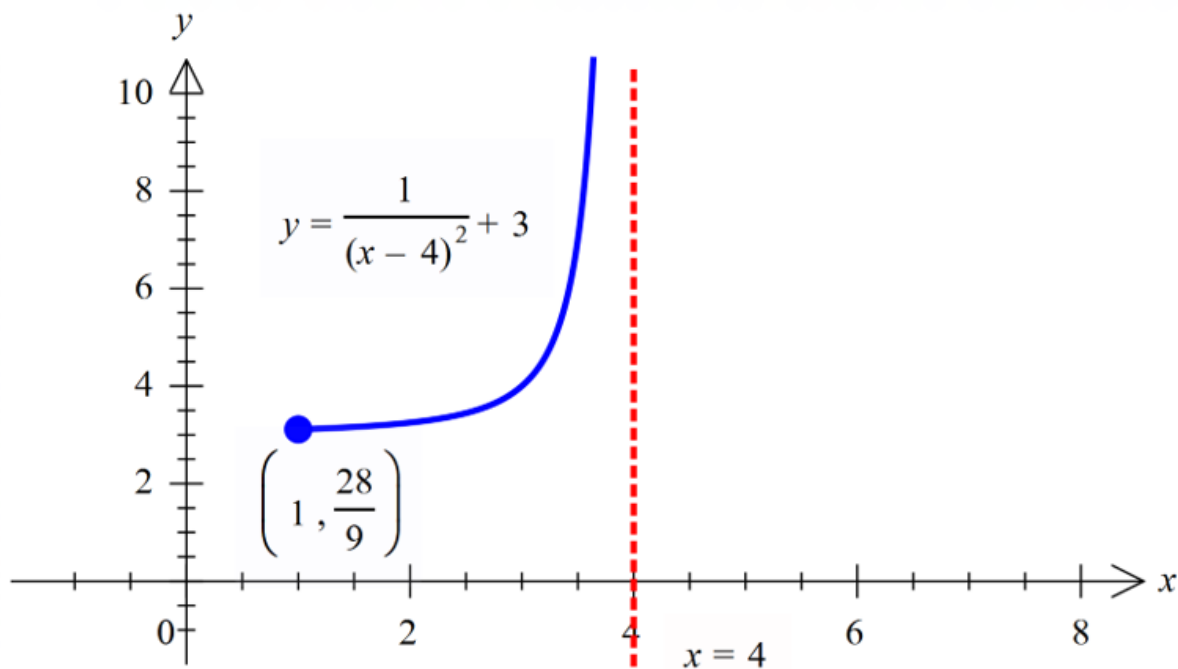
Question 3

a. The range of f , $(0, 9]$, is a subset of the domain of g , $R \setminus \{0\}$. 1A

b. $g(f(x)) = \frac{1}{(x-4)^2} + 3$ 1A

Domain $[1, 4)$ 1A

c. Shape, including correct domain 1A
Asymptote 1A

**Question 4**

a. $h: [-1, \infty) \rightarrow \mathbb{R}, h(x) = -\sqrt{x+1}$

Let $y = -\sqrt{x+1}$.

Inverse swap x and y

$$x = -\sqrt{y+1}$$

$$x^2 = y+1$$

$$h^{-1}(x) = x^2 - 1 \quad \mathbf{1A}$$

$$\text{Domain } (-\infty, 0] \quad \mathbf{1A}$$

OR

$$h^{-1}: (-\infty, 0] \rightarrow \mathbb{R}, h^{-1}(x) = x^2 - 1 \quad \mathbf{2A}$$

b. Solve $-\sqrt{x+1} = x$ for x .

$$x+1 = x^2$$

$$x^2 - x - 1 = 0 \quad \mathbf{1M}$$

$$x = \frac{1-\sqrt{5}}{2}, x \neq \frac{1+\sqrt{5}}{2}$$

$$\left(\frac{1-\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2} \right) \quad \mathbf{1A}$$

$$(-1, 0), (0, -1) \quad \mathbf{1A}$$

OR

Solve $-\sqrt{x+1} = x^2 - 1$ for x .

$$x+1 = (x^2 - 1)^2$$

$$0 = x^4 - 2x^2 - x \quad \mathbf{1M}$$

$$0 = x(x^3 - 2x - 1)$$

$$0 = x(x+1)(x^2 - x - 1)$$

$$x = 0, x = -1, x = \frac{1-\sqrt{5}}{2}, x \neq \frac{1+\sqrt{5}}{2}$$

$$\left(\frac{1-\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2} \right) \quad \mathbf{1A}$$

$$(-1, 0), (0, -1) \quad \mathbf{1A}$$

Question 5

a. $\frac{d}{dx}(x \log_e(x) - x)$

$$= \log_e(x) + 1 - 1$$

$$= \log_e(x) \quad \mathbf{1A}$$

b.i. $g(x) = 2 \log_e(x-1)$

A dilation by a factor of 2 from the x -axis. $\mathbf{1A}$

A translation of 1 unit to the right. $\mathbf{1A}$

b.ii. $\int_2^3 (2 \log_e(x-1)) dx \quad \mathbf{1A}$

$$= 2 \int_1^2 (\log_e(x)) dx$$

$$= 2[x \log_e(x) - x]_1^2 \quad \mathbf{1M}$$

$$= 2((2 \log_e(2) - 2) - (-1))$$

$$= 4 \log_e(2) - 2 \quad \mathbf{1A}$$

OR

$$\int_2^3 (2 \log_e(x-1)) dx \quad \mathbf{1A}$$

$$= 2[(x-1) \log_e(x-1) - (x-1)]_2^3 \quad \mathbf{1M}$$

$$= 2((2 \log_e(2) - 2) - (-1))$$

$$= 4 \log_e(2) - 2 \quad \mathbf{1A}$$

Question 6

a. $\frac{1}{2} \times \frac{6}{16} \times \frac{5}{15} + \frac{1}{2} \times \frac{4}{9} \times \frac{3}{8} \quad \mathbf{1M}$

$$= \frac{1}{16} + \frac{1}{12} = \frac{7}{48} \quad \mathbf{1A}$$

b. $\Pr(B_A | 2W) = \frac{\Pr(B_A \cap 2W)}{\Pr(2W)} = \frac{\frac{1}{16}}{\frac{7}{48}} = \frac{3}{7} \quad \mathbf{1A}$

Question 7

$$\int_{-1}^0 \left((x+1)^{\frac{1}{2}} \right) dx$$

$$= \left[\frac{2}{3} (x+1)^{\frac{3}{2}} \right]_{-1}^0 \quad \mathbf{1M}$$

$$= \frac{2}{3} \quad \mathbf{1A}$$

Solve $\int_0^a \left(\frac{2}{x+2} \right) dx = \frac{1}{3}$ for a .

$$2[\log_e(x+2)]_0^a = \frac{1}{3} \quad \mathbf{1M}$$

$$\log_e(a+2) - \log_e(2) = \frac{1}{6}$$

$$\log_e\left(\frac{a+2}{2}\right) = \frac{1}{6}$$

$$e^{\frac{1}{6}} = \frac{a+2}{2}$$

$$a = 2e^{\frac{1}{6}} - 2 \quad \mathbf{1A}$$

Question 8

a. $\sqrt{3} \tan\left(2x - \frac{\pi}{2}\right) + 2 = 1$

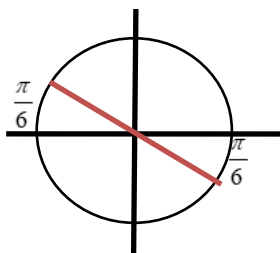
$$\tan\left(2x - \frac{\pi}{2}\right) = -\frac{1}{\sqrt{3}}$$

$$2x - \frac{\pi}{2} = \frac{5\pi}{6} \dots \quad \mathbf{1A}$$

$$2x = \frac{8\pi}{6} \dots$$

$$x = \frac{8\pi}{12} \dots \text{ (add and subtract the period, } \frac{\pi}{2} \text{)} \quad \mathbf{1A}$$

$$x = -\frac{\pi}{3}, \frac{\pi}{6} \quad \mathbf{1A}$$

ORUnit circle location 2nd/4th quadrant with basic angle

and/or adjustment of domain i.e. $2x - \frac{\pi}{2} \in \left(-\frac{3\pi}{2}, \frac{\pi}{2}\right) \quad \mathbf{1A}$

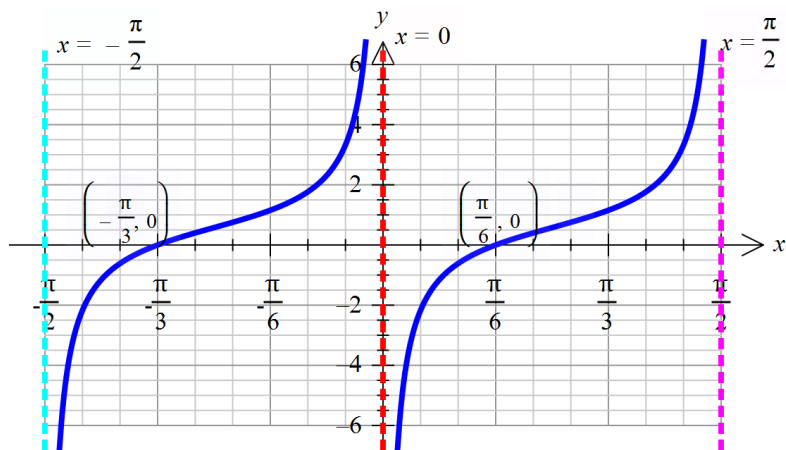
$$2x - \frac{\pi}{2} = -\frac{7\pi}{6}, -\frac{\pi}{6} \quad \mathbf{1A}$$

$$x = -\frac{\pi}{3}, \frac{\pi}{6} \quad \mathbf{1A}$$

b. Shape $\mathbf{1A}$

Asymptotes $\mathbf{1A}$

Intercepts $\mathbf{1A}$



Question 9

a. $f(x) = x^3 + 2x$

$$f'(x) = 3x^2 + 2$$

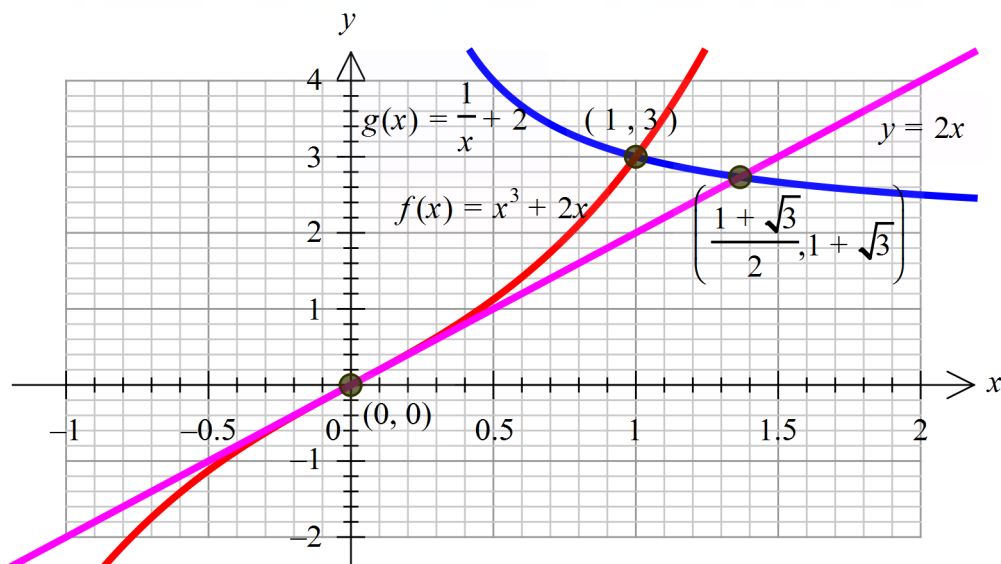
$$m = f'(0) = 2$$

$$f(0) = 0$$

$$y = 2x$$

b.

1M Show that



$$x^3 + 2x = \frac{1}{x} + 2$$

$$x^4 + 2x^2 - 2x - 1 = 0$$

$$x = 1 \text{ is a solution}$$

1A

As there is only one positive solution (curvature of graphs) there is no need to investigate further solutions.

$$2x = \frac{1}{x} + 2$$

$$2x^2 - 2x - 1 = 0$$

$$x = \frac{2 \pm \sqrt{12}}{4} = \frac{1 \pm \sqrt{3}}{2}$$

$$x = \frac{1 + \sqrt{3}}{2}, x > 0 \quad \mathbf{1A}$$

$$\int_0^1 (f(x) - 2x) dx + \int_1^{\frac{1+\sqrt{3}}{2}} (g(x) - 2x) dx \quad \mathbf{1A}$$

END OF SOLUTIONS