

VCE METHODS EXAM 1 2019 (Nov)

Question 1

a) i) $f(x) = (3x-1)^{-1}$

$$f'(x) = -(3x-1)^{-2} \times 3$$

$$= \boxed{-\frac{3}{(3x-1)^2}}$$

ii) $\int \frac{1}{3x-1} dx = \frac{1}{3} \log_e |3x-1| + c$

as $x > \frac{1}{3}$

$$= \boxed{\frac{1}{3} \log_e (3x-1) + c}$$

b) $g(x) = \frac{\sin(\pi x)}{x+1}$

$$g'(x) = \frac{\pi \cos(\pi x) (x+1) - \sin(\pi x)}{(x+1)^2}$$

$$g'(1) = \frac{\pi \cos(\pi) \cdot 2 - \sin(\pi)}{2^2}$$

$$= \frac{-2\pi}{4}$$

$$= \boxed{-\frac{\pi}{2}}$$

QUESTION 2

$$a) f(x) = \frac{1}{3x-1}$$

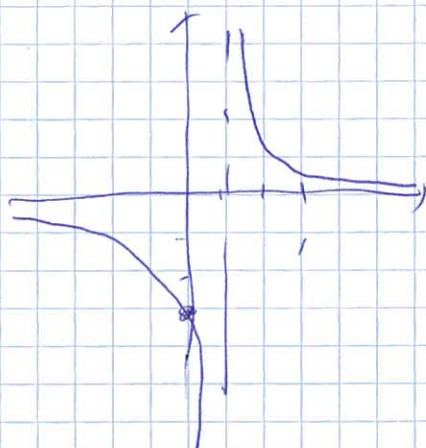
$$x = \frac{1}{3y-1}$$

$$x(3y-1) = 1$$

$$3xy - x = 1$$

$$3xy = x + 1$$

$$y = \frac{x+1}{3x} \Rightarrow f^{-1}(x) = \frac{x+1}{3x}$$



range $f(x) = \mathbb{R} \setminus \{0\}$

$$b) \text{ domain } f^{-1}(x) \subseteq \boxed{x \in (-\infty, 0) \cup (0, \infty)}$$

or:

$$\boxed{x \in \mathbb{R} \setminus \{0\}}$$

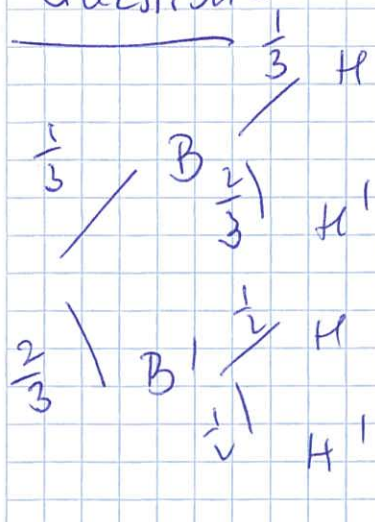
$$c) \frac{1}{3x-1} \rightarrow \frac{x+1}{3x}$$

$$\frac{1}{3(x-\frac{1}{3})} \rightarrow \frac{1}{3} + \frac{1}{3x}$$

$$c = -\frac{1}{3}$$

$$d = \frac{1}{3}$$

Question 3 a)



BH or $B'H$

$$\frac{1}{3} \times \frac{1}{3} + \frac{2}{3} \times \frac{1}{2}$$

$$\Pr(H) = \frac{4}{9}$$

$$b) \Pr(B' | H) = \frac{\Pr(B' \cap H)}{\Pr(H)}$$

$$= \frac{\frac{1}{3}}{\frac{4}{9}} = \frac{3}{4}$$

QUESTION 4

$$a) 1 - \cos\left(\frac{x}{2}\right) = \cos\left(\frac{x}{2}\right)$$

$$x \in [-2\pi, \pi]$$

$$2\cos\left(\frac{x}{2}\right) = 1$$

$$\cos\left(\frac{x}{2}\right) = \frac{1}{2}$$

$$\frac{x}{2} = \frac{\pi}{3}, \frac{5\pi}{3} \quad \text{period} = 4\pi$$

$$x = \frac{2\pi}{3}, \frac{10\pi}{3}$$

$$x = -\frac{2\pi}{3}, \frac{2\pi}{3}$$

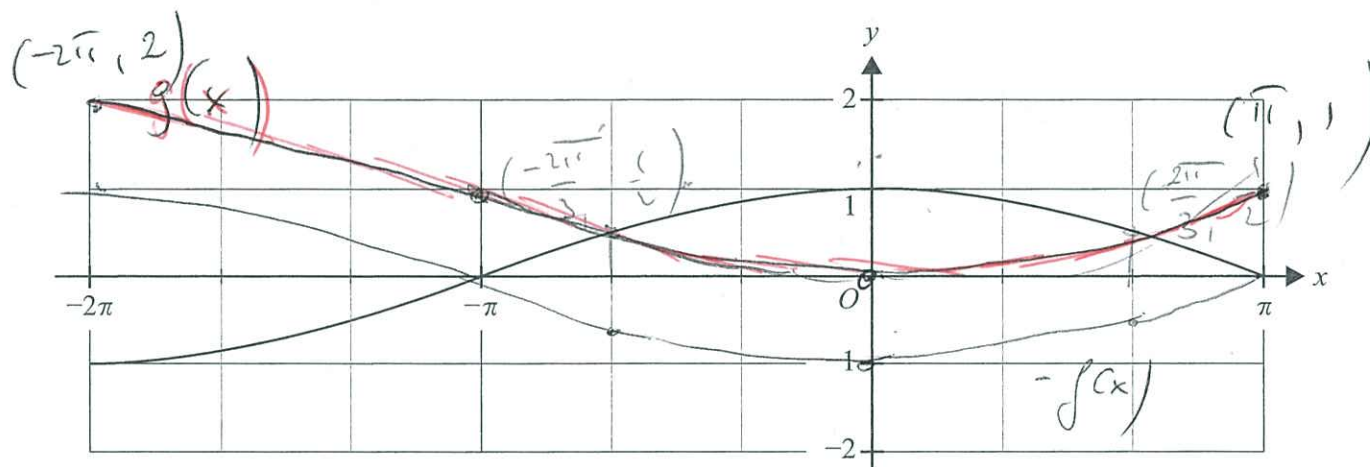
b) next page.

Question 4 (4 marks)

- a. Solve $1 - \cos\left(\frac{x}{2}\right) = \cos\left(\frac{x}{2}\right)$ for $x \in [-2\pi, \pi]$.

2 marks

- b. The function $f: [-2\pi, \pi] \rightarrow \mathbb{R}$, $f(x) = \cos\left(\frac{x}{2}\right)$ is shown on the axes below.



Let $g: [-2\pi, \pi] \rightarrow \mathbb{R}$, $g(x) = 1 - f(x)$.

Sketch the graph of g on the axes above. Label all points of intersection of the graphs of f and g , and the endpoints of g , with their coordinates.

2 marks

$$g(x) = 1 - \cos\left(\frac{x}{2}\right)$$



Question 5

$$a) i) f(x) = \frac{2}{(x-1)^2} + 1$$

$$f(-1) = \left[\frac{3}{2} \right]$$

ii) NEXT PAGE

$$b) \text{ Area} = \int_{-1}^0 \frac{2}{(x-1)^2} + 1 \, dx$$

$$= 2 \int_{-1}^0 (x-1)^{-2} + 1 \, dx$$

$$= \left[2 \frac{(x-1)^{-1}}{-1} + x \right]_{-1}^0$$

$$= \boxed{2}$$

Question 5 (5 marks)

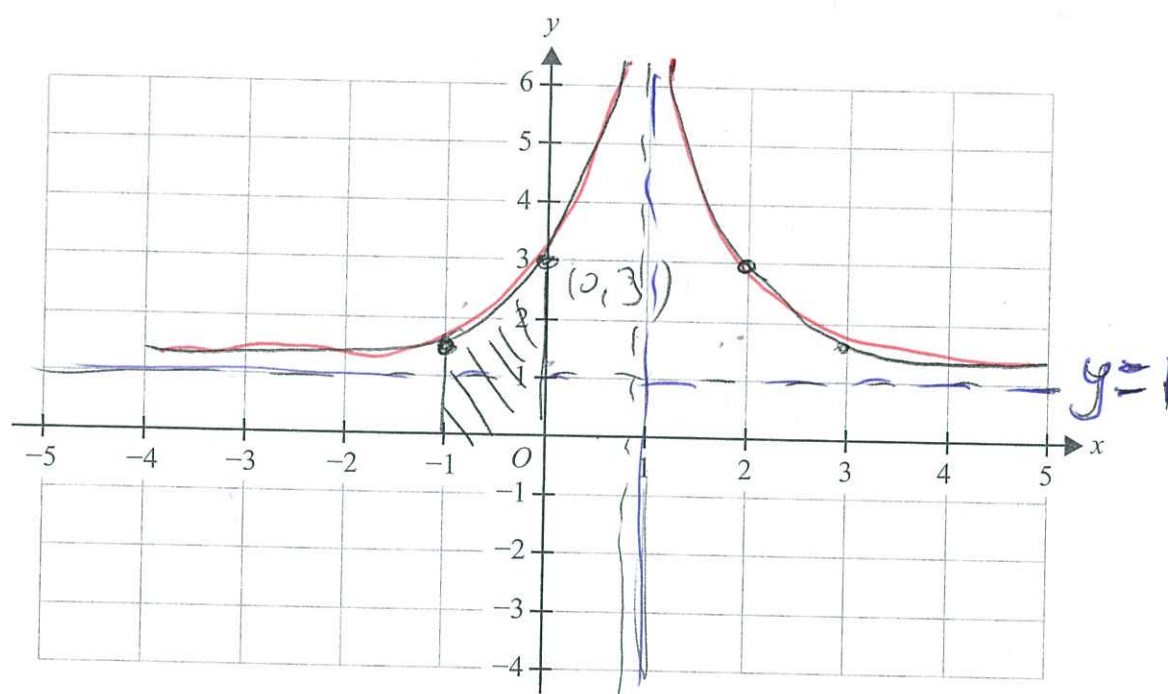
Let $f: \mathbb{R} \setminus \{1\} \rightarrow \mathbb{R}$, $f(x) = \frac{2}{(x-1)^2} + 1$.

- a. i. Evaluate $f(-1)$.

1 mark

- ii. Sketch the graph of f on the axes below, labelling all asymptotes with their equations.

2 marks



- b. Find the area bounded by the graph of f , the x -axis, the line $x = -1$ and the line $x = 0$.

2 marks

TURN OVER



QUESTION 6

$$n = 41$$

a)

$$p = \frac{9}{41}$$

b) $p = \frac{1}{6}$

$$n = 12$$

$$Pr\left(\hat{p} \leq \frac{1}{6}\right)$$

$$a = \frac{17}{6}$$

$$b = \frac{5}{6}$$

$$n = 11$$

Question 7

a) $P(x, \sqrt{1-x^2})$ $B(x, 0)$

$$PB = \sqrt{(x-x)^2 + (0-\sqrt{1-x^2})^2}$$

$$= \sqrt{1-x^2}$$

b) $A_\Delta = \frac{1}{2} (x+1) \sqrt{1-x^2}$

$$\frac{dA}{dx} = \frac{1}{2} \sqrt{1-x^2} + \frac{1}{2} (x+1) \cdot \frac{-2x}{2\sqrt{1-x^2}}$$

$$= \frac{1}{2} \sqrt{1-x^2} - \frac{x(x+1)}{2\sqrt{1-x^2}}$$

$$= \frac{1-x^2-x^2-x}{2\sqrt{1-x^2}}$$

$$= \frac{-2x^2-x+1}{2\sqrt{1-x^2}} = 0$$

$$2x^2+x-1=0 \quad (x+1)(2x-1)=0$$

$$x = \frac{1}{2}$$

$$A = \frac{1}{2} \left(\frac{1}{2} + 1 \right) \cdot \sqrt{1 - \frac{1}{4}}$$

$$= \frac{1}{2} \cdot \frac{3}{2} \cdot \sqrt{\frac{3}{4}} = \frac{3}{4} \cdot \frac{1}{2} \sqrt{3} = \frac{3\sqrt{3}}{8}$$

QUESTIONS

a) $f(x) = ax^2(x+1)(x-1)$

$$f(x) = ax^2(x^2-1)$$

$$1 = a \cdot \frac{1}{2} \left(\frac{1}{2} - 1 \right)$$

$$2 = a \left(-\frac{1}{2} \right)$$

$$a = -4$$

$$f(x) = -4x^2(x^2-1)$$

b) $g(x) = -4x^2(x^2-1)$
 $= 4x^2(1-x^2)$

$$h(x) = \log_e [4x^2(1-x)(1+x)] - \log_e [x^2(x+1)]$$

$x \neq 0 \quad 1-x^2 > 0 \quad x+1 > 0$

$$D = (-1, 0) \cup (0, 1)$$

c) range:

$$(-\infty, \log_e(4)) \cup (\log_e(4), 3 \log_e(2))$$

Question 9

$$f(x) = 3 + 2x - x^2$$

$$g(x) = e^x$$

$$a) \quad g(f(x)) = e^{3+2x-x^2}$$

$$b) \quad [g(f(x))]' = (-2x+2) e^{3+2x-x^2}$$

$$-2x+2 < 0$$

$$-2x < -2$$

$$x > 1$$

$$c) \quad f(g(x)) = 3 + 2e^x - e^{2x}$$

$$d) \quad -e^{2x} + 2e^x + 3 = 0$$

$$e^{2x} - 2e^x - 3 = 0$$

$$\text{let } a = e^x$$

$$a^2 - 2a - 3 = 0$$

$$(a-3)(a+1) = 0$$

$$a = 3$$

$$e^x = 3$$

$$x = \log_e 3$$

$$e) \quad f(g(x)) = 3 + 2e^x - e^{2x}$$

$$(f(g(x)))' = 2e^x - 2e^{2x} = 0$$

$$2e^x - 2e^x \cdot e^x = 0$$

$$2e^x(1 - e^x) = 0$$

$$e^x = 1$$

$$x = 0$$

$$f(g(x))(0) = 3 + 2 - 1 = 4$$

$$(0, 4)$$

$$f) \quad e^{3+2x-x^2} \neq 3 + 2e^x - e^{2x} = 0$$

$$e^{3+2x-x^2} = e^{2x} - 2e^x - 3$$

$$\frac{e^3 \cdot e^{2x}}{e^{x^2}} = e^{2x} - 2e^x - 3$$

$$e^3 \cdot e^{2x} = e^{x^2} (e^{2x} - 2e^x - 3)$$

one solution