

2019 VCE Mathematical Methods Trial Examination 2 Detailed Answers



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SECTION A

ANSWERS

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E

SECTION A

Question 1 Answer E

The graphs of $\tan$ do not have an amplitude. **A.** and **B.** are incorrect.
All of **C.** **D.** and **E.** have an amplitude of b .

Only **E.** $T = \frac{2\pi}{\frac{2\pi}{b}} = b$ has a period of b .

Question 2 Answer B

$$p(x) = x^3 + bx^2 + cx + 5$$

$$p(1) = 6 \Rightarrow 6 = 1 + b + c + 5 = 6$$

$$(1) \quad b + c = 0$$

$$p(-2) = 0 \Rightarrow -8 + 4b - 2c + 5 = 0$$

$$(2) \quad 4b - 2c = 3$$

$$(2) + 2 \times (1) \quad 6b = 3$$

$$b = \frac{1}{2}, c = -\frac{1}{2}$$

Define $p(x) = x^3 + b \cdot x^2 + c \cdot x + 5$ Done

$p(1) = 6$ $b + c + 6 = 6$

$p(-2) = 0$ $4 \cdot b - 2 \cdot c - 3 = 0$

solve($p(1)=6$ and $p(-2)=0$, $\{b, c\}$)

$b = \frac{1}{2}$ and $c = -\frac{1}{2}$

Question 3 Answer D

$$f(x) = \log_e(x - a)$$

has a maximal domain $x > a = (a, \infty)$

domain($\ln(x-a), x$) $a < x < \infty$

Question 4 Answer A

$$g(x) = f^{-1}(x), f(1) = 2, g(2) = f^{-1}(2) = 1$$

$$f(g(x)) = f^{-1}(f(x)) = x \text{ differentiating using the chain rule}$$

$$g'(x) f'(g(x)) = 1$$

$$g'(x) = \frac{1}{f'(g(x))}$$

$$g'(2) = \frac{1}{f'(g(2))} = \frac{1}{f'(1)} = \frac{-1}{5}$$

Question 5 Answer D

$$f(x) = e^x + e^{-x}, f'(x) = e^x - e^{-x} = g(x) \quad \text{A. is true}$$

$$g(x) = e^x - e^{-x}, g'(x) = e^x + e^{-x} = f(x) \quad \text{B. is true}$$

$$[f(x)]^2 = (e^x + e^{-x})^2 = e^{2x} + 2 + e^{-2x} = f(2x) + 2 \quad \text{C. is true}$$

$$[g(x)]^2 = (e^x - e^{-x})^2 = e^{2x} - 2 + e^{-2x} = f(2x) - 2 \quad \text{D. is false}$$

$$f(x)g(x) = (e^x + e^{-x})(e^x - e^{-x}) = e^{2x} - 2 + 2 + e^{-2x} = e^{2x} + e^{-2x} = g(2x) \quad \text{E. is true}$$

Question 6

Answer D

$$f(x) = x^3 - bx$$

$$f'(x) = 3x^2 - b = 0 \text{ for stationary points } x = \pm\sqrt{\frac{b}{3}} = \pm\frac{\sqrt{3b}}{3}$$

$$f'(\pm 2) = 0 = 12 - b \Rightarrow b = 12. \text{ A. is correct}$$

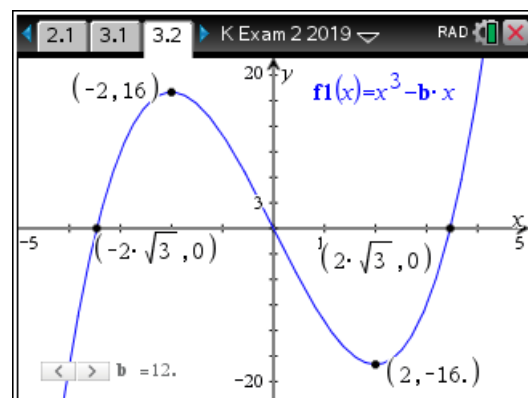
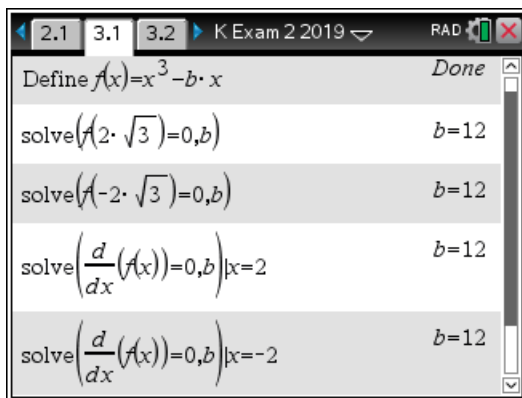
$$f(2\sqrt{3}) = (2\sqrt{3})^3 - 2\sqrt{3}b = 0 \Rightarrow b = 12.$$

$$f(-2\sqrt{3}) = (-2\sqrt{3})^3 + 2\sqrt{3}b = 0 \Rightarrow b = 12. \text{ B. is correct}$$

There is a stationary point of inflexion at $x = 0$. E. is correct

For $x \in \left(-\infty, -\frac{\sqrt{3b}}{3}\right)$ the function is one-one. C. is correct

When $x \in \left(-\frac{\sqrt{3b}}{3}, \frac{\sqrt{3b}}{3}\right)$ the function is decreasing. D. is false



Question 7

Answer E

$$r'(6) = \frac{r(8) - r(4)}{8 - 4} = \frac{86 - 78}{4} = 2$$

Question 8

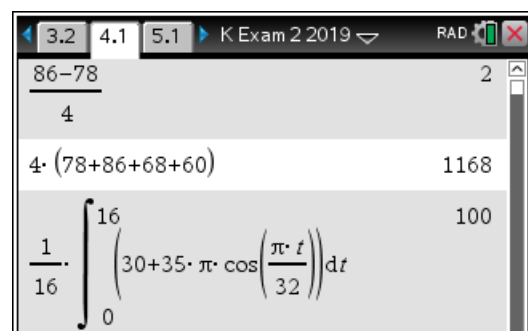
Answer B

$$\int_0^{16} r(t) dt = 4[78 + 86 + 68 + 60] = 1168$$

Question 9

Answer A

$$\bar{r} = \frac{1}{16 - 0} \int_0^{16} \left(30 + 35\pi \cos\left(\frac{\pi t}{32}\right) \right) dt = 100$$



Question 10

Answer C

$$y = \log_e(\cos(2x))$$

$$\frac{dy}{dx} = \frac{-2\sin(2x)}{\cos(2x)} = -2\tan(2x) = 2$$

$$\tan(2x) = -1$$

$$2x = n\pi - \frac{\pi}{4}$$

$$x = \frac{n\pi}{2} - \frac{\pi}{8}$$

$$x = \frac{\pi}{8}(4n-1)$$

Calculator screenshot showing the derivative of $\ln(\cos(2x))$ as $-2 \cdot \tan(2x)$ and solving $-2 \cdot \tan(2x) = 2$ for x , resulting in $x = \frac{(4n-1) \cdot \pi}{8}$.

Question 11

Answer E

$$f(x) = 2 - 3g(x) \text{ and } \int_4^{-1} g(x) dx = 3 \text{ then}$$

$$\begin{aligned} & \int_{-1}^4 (f(x) + g(x)) dx \\ &= \int_{-1}^4 (2 - 3g(x) + g(x)) dx = \int_{-1}^4 (2 - 2g(x)) dx \\ &= [2x]_{-1}^4 - 2 \int_{-1}^4 g(x) dx = (8 - (-2)) + 2 \int_4^{-1} g(x) dx \\ &= 10 + 2 \times 3 = 16 \end{aligned}$$

Question 12

Answer C

$$f(x) = \begin{cases} x^2 & 0 \leq x \leq 2 \\ 5-x & 2 < x \leq 5 \end{cases}$$

$$\lim_{x \rightarrow 2^-} f(x) = 4, \quad \lim_{x \rightarrow 2^+} f(x) = 3$$

The function is not continuous at $x = 2$.

The average value does exist.

$$\begin{aligned} \bar{f} &= \frac{1}{5-0} \left[\int_0^5 f(x) dx \right] \\ &= \frac{1}{5} \left[\int_0^2 x^2 dx + \int_2^5 (5-x) dx \right] \\ &= \frac{1}{5} \left(\frac{8}{3} + \frac{9}{2} \right) = \frac{43}{30} \end{aligned}$$

Calculator screenshot showing the definition of a piecewise function $f1(x) = \begin{cases} x^2, & 0 \leq x \leq 2 \\ 5-x, & 2 < x \leq 5 \end{cases}$ and the calculation of its average value $\frac{1}{5} \int_0^5 f1(x) dx = \frac{43}{30}$.

Question 13 **Answer A**

$$f(x) = \log_e \left(\frac{1}{\sqrt{g(x)}} \right) = -\frac{1}{2} \log_e (g(x))$$

$$f'(x) = -\frac{g'(x)}{2g(x)}$$

$$g'(x) = -2g(x) f'(x)$$

$$g'(4) = -2g(4) f'(4)$$

$$\text{Now } f(4) = 2 = -\frac{1}{2} \log_e (g(4)), \quad g(4) = e^{-4}$$

$$g'(4) = -2e^{-4} \times -3 = 6e^{-4} = \frac{6}{e^4}$$

Question 14 **Answer D**

$$f(x) = \sqrt{x-a} \text{ and } g(x) = x^2 + b$$

	$f(x)$	$g(x)$
domain	$[a, \infty)$	R
range	$[0, \infty)$	$[b, \infty)$

For the function $f(g(x))$ to exist $\text{range } g \subseteq \text{domain } f \Rightarrow [b, \infty) \subseteq [a, \infty)$

So that $b \geq a$ or $b - a \geq 0$

Question 15 **Answer E**

The function, $y = \log_e (x+1)$, when $y = 2$, $2 = \log_e (x+1) \Rightarrow x = e^2 - 1$

The inverse function $f^{-1}: x = \log_e (y+1) \Rightarrow y = f^{-1}(x) = e^x - 1$

The area bounded by the graph of $y = \log_e (x+1)$

the y-axis and the line $y = 2$, is equal to $\int_0^2 (e^x - 1) dx$.

The area bounded by the graph of $y = \log_e (x+1)$

the x-axis and the vertical line, is equal to

$$\int_0^{e^2-1} \log_e (x+1) dx.$$

The total shaded area is the area of the rectangle and is equal to $2(e^2 - 1)$.

Amanda, Breeanna and Chloe are all correct.

The screenshot shows a calculator interface with three problems and their solutions:

- Problem 1: $a1 := \int_0^{e^2-1} \ln(x+1) dx$ with solution $e^2 + 1$
- Problem 2: $a2 := \int_0^2 (e^x - 1) dx$ with solution $e^2 - 3$
- Problem 3: $a1 + a2$ with solution $2 \cdot e^2 - 2$

Question 16

Answer B

$$n = 81, p = 0.1, 95\% \quad z = 1.96$$

$$p \pm z \sqrt{\frac{p(1-p)}{n}}$$

$$0.1 \pm 1.96 \sqrt{\frac{0.1 \times 0.9}{81}}$$

$$(0.035, 0.165)$$

6.1	7.1	8.1	K Exam 2 2019	RAD
invNorm(0.975,0,1)				1.95996
$0.1 + 1.96 \cdot \sqrt{\frac{0.1 \cdot (1-0.1)}{81}}$				0.165333
$0.1 - 1.96 \cdot \sqrt{\frac{0.1 \cdot (1-0.1)}{81}}$				0.034667

Question 17

Answer C

$y = \sqrt{x}$ is transformed into

$y = 2\sqrt{x}$ by a dilation by a scale factor of 2 parallel to the y-axis,

$y = -2\sqrt{x}$ by a reflection in the x-axis,

$y = -2\sqrt{-x}$ by a reflection in the y-axis,

$y = -2\sqrt{-(x-2)} = -2\sqrt{2-x}$ by translation of 2 units to the right parallel to the x-axis.

Question 18

Answer A

The original curve $y = \tan(x)$ is transformed into the image curve with equation

$$y' = 4 - \frac{3}{\tan\left(\frac{x'}{2}\right)} = 4 + 3 \tan\left(\frac{x'}{2} - \frac{\pi}{2}\right)$$

$$\frac{y' - 4}{3} = \tan\left(\frac{x'}{2} - \frac{\pi}{2}\right)$$

$$y = \frac{y' - 4}{3}, \quad x = \frac{x'}{2} - \frac{\pi}{2}$$

$$y' = 3y + 4 \quad \therefore \quad x' = 2x + \pi$$

in matrix form this becomes $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \pi \\ 4 \end{bmatrix}$

Question 19

Answer B

$$X \stackrel{d}{=} \text{Bi}(n, p) \quad , \quad \Pr(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$\text{and } \Pr(X = 0) = (1-p)^n \quad , \quad \Pr(X = 1) = np(1-p)^{n-1}$$

$$\begin{aligned} \Pr(X = 1 | X > 0) &= \frac{\Pr(X = 1 \cap X > 0)}{\Pr(X > 0)} = \frac{\Pr(X = 1)}{\Pr(X \geq 1)} \\ &= \frac{\Pr(X = 1)}{1 - \Pr(X = 0)} = \frac{np(1-p)^{n-1}}{1 - (1-p)^n} \end{aligned}$$

Question 20

Answer C

$$\sum \Pr(X = x) = 1$$

$$(1) \quad a + b + c = 1$$

$$E(X) = \sum x \Pr(X = x)$$

$$(2) \quad -a + c = \frac{1}{4}$$

$$E(X^2) = \sum x^2 \Pr(X = x)$$

$$E(X^2) = a + c$$

$$\text{var}(X) = E(X^2) - (E(X))^2$$

$$a + c - \left(\frac{1}{4}\right)^2 = \frac{3}{16}$$

$$(3) \quad a + c = \frac{1}{4}$$

$$(2) + (3) \Rightarrow 2c = \frac{1}{2}$$

$$c = \frac{1}{4} \quad , \quad a = 0 \quad , \quad b = \frac{3}{4}$$

7.1 8.1 9.1 K Exam 2 2019 RAD

eq1:=a+b+c=1 a+b+c=1

eq2:=c-a=1/4 c-a=1/4

eq3:=a+c-1/16=3/16 a+c-1/16=3/16

linSolve({eq1,eq2,eq3},{a,b,c}) {0,3/4,1/4}

END OF SECTION A SUGGESTED ANSWERS

SECTION B

Question 1

a. $f(x) = x^3 + bx^2 + cx + d$

$f'(x) = 3x^2 + 2bx + c = 0$ for stationary points

$\Delta = (2b)^2 - 4 \times 3 \times c = 4b^2 - 12c = 4(b^2 - 3c)$

$x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-2b \pm 2\sqrt{b^2 - 3c}}{6}$ M1

$x = \frac{-b \pm \sqrt{b^2 - 3c}}{3}$ since $q > p$, $q < 0$ so

$q = \frac{-b + \sqrt{b^2 - 3c}}{3}$, $p = \frac{-b - \sqrt{b^2 - 3c}}{3}$ and $b^2 > 3c$ A1

b. $q = -2 = \frac{-b + \sqrt{b^2 - 3c}}{3}$, $p = -6 = \frac{-b - \sqrt{b^2 - 3c}}{3}$

(1) $-6 = -b + \sqrt{b^2 - 3c}$ A1

(2) $-18 = -b - \sqrt{b^2 - 3c}$

(1) + (2) $-24 = -2b$, $b = 12$

$\sqrt{b^2 - 3c} = -6 + b = 6$

$144 - 3c = 36$

$3c = 144 - 36 = 108$

$c = \frac{108}{3} = 36$ A1

c. $f(x) = x^3 + 12x^2 + 36x + d$

$f(-2) = d - 32$, $f(-6) = d$

$P(-6, d)$, $Q(-2, d - 32)$

Since the graph crosses the x -axis three times, $d > 0$ and $d - 32 < 0$ so that
 $0 < d < 32$ A1

Define $f1(x) = x^3 + 12 \cdot x^2 + 36 \cdot x + d$	Done
$\frac{d}{dx}(f1(x))$	$3 \cdot x^2 + 24 \cdot x + 36$
$\text{solve}\left(\frac{d}{dx}(f1(x)) = 0, x\right)$	$x = -6 \text{ or } x = -2$
$f1(-6)$	d
$f1(-2)$	$d - 32$

d. $m(PQ) = \frac{d - (d - 32)}{-6 - 2} = \frac{32}{-4} = -8$

$L(x): y - d = -8(x + 6)$

$L(x): y = -8x + d - 48$

A1

e. $R\left(-\frac{6+2}{2}, \frac{d+d-32}{2}\right), R(-4, d-16)$

A1

f. $f(-4) = (-4)^3 + 12 \times (-4)^2 + 36 \times -4 + d = -64 + 192 - 144 + d$

$f(-4) = d - 16$, so R lies on the graph of f .

A1

$f(-4)$

$d-16$

Define $f2(x) = -8 \cdot x + d - 48$

Done

solve($f1(x) = f2(x)$, x)

$x = -6$ or $x = -4$ or $x = -2$

g. The point R is the point where a tangent crosses the curve.

The line joining PQ is not the tangent to the curve at R, but the minimum gradient occurs at R.

Let $m = f'(x) = 3x^2 + 24x + 36$, $\frac{dm}{dx} = 6x + 24 = 0$ for maximum or minimum gradient

$\Rightarrow x = -4$, $f'(-4) = 3 \times (-4)^2 + 24 \times -4 + 36 = 48 - 96 + 36 = -12$

The minimum gradient is -12

A1

h.i. $A_1 = \int_{xp}^{xr} (f(x) - L(x)) dx$

$A_1 = \int_{-6}^{-4} (x^3 + 12x^2 + 44x + 48) dx$

$x_1 = -6$, $x_2 = -4$, $m = 44$, $n = 48$

A1

h.ii. $A_1 = 4$

A1

j. $A_2 = \int_{xr}^{xp} (L(x) - f(x)) dx$

$A_2 = -\int_{-4}^{-2} (x^3 + 12x^2 + 44x + 48) dx = 4$

or by symmetry

A1

$f1(x) - f2(x)$

$x^3 + 12 \cdot x^2 + 44 \cdot x + 48$

$\int_{-6}^{-4} (x^3 + 12 \cdot x^2 + 44 \cdot x + 48) dx$

4

$-\int_{-4}^{-2} (x^3 + 12 \cdot x^2 + 44 \cdot x + 48) dx$

4

Question 2

a. $f(x) = a(x-h)^2 + k$ for $0 \leq x \leq 200$,
 $A(0,120)$, $f(0) = 120 \Rightarrow 120 = ah^2 + k$
 $B(100,140)$ since it is the maximum turning point,
 $f(100) = 140 \Rightarrow h = 100$, $k = 140$ A1

$C(200,120)$, $f(200) = 120 \Rightarrow 120 = a(100)^2 + 140$
 $a(100)^2 = -20$

$a = \frac{-20}{10,000} = -\frac{1}{500}$ A1

b. $\frac{1}{4}$ of a period $\frac{1}{4} \times \frac{2\pi}{\frac{\pi}{n}} = \frac{n}{2} = 160 \Rightarrow n = 320$ A1

the gradient function $g'(x) = \frac{p\pi}{n} \cos\left(\frac{\pi}{n}(x-\alpha)\right)$ has a maximum at $D(360,185)$

$g'(360) = 0 \Rightarrow \cos\left(\frac{\pi}{320}(360-\alpha)\right) = 0 = \cos\left(\frac{\pi}{2}\right)$

$360 - \alpha = 160$
 $\alpha = 200$ A1

$g(x) = p \sin\left(\frac{\pi}{320}(x-200)\right) + r$

$C(200,120)$, $g(200) = 120 \Rightarrow 120 = p \sin(0) + r = r$ so $r = 120$

$D(360,185)$, $g(360) = 185$, $185 = p \sin\left(\frac{\pi}{2}\right) + 120 = p + 120$

$p = 185 - 120$
 $p = 65$ A1

c. $D(360,185)$, $h(360) = 185 \Rightarrow 185 = s\sqrt{c+360d-360^2}$
 $E(500,0)$, $h(500) = 0 \Rightarrow 0 = s\sqrt{c+500d-500^2}$
 since it is smoothly joined at D, $h'(360) = g'(360) = 0$ M1

now $h'(x) = \frac{s(d-2x)}{2\sqrt{c+dx-x^2}}$, $h'(360) = 0 \Rightarrow d - 2 \times 360 = 0$

$d = 720$ A1

substitute $d = 720$ into $s\sqrt{c+500d-500^2} = 0 \Rightarrow c = 500^2 - 500 \times 720 = -110,000$

substitute $d = 720$ and $c = -110,000$ into $185 = s\sqrt{c+360d-360^2}$ gives

$s = \frac{185}{\sqrt{-110,000 + 360 \times 720 - 360^2}} = \frac{37}{28}$ A1

d. $(x-100)^2 + y^2 = 50^2$ A1

e.i. $f(x) = -\frac{1}{500}(x-100)^2 + 140$, $g(x) = 65 \sin\left(\frac{\pi}{320}(x-200)\right) + 120$

$$h(x) = \frac{37}{28} \sqrt{720x - 110,000 - x^2}$$

the neck is a rectangle length 400 by 100, the area of the circle is $\pi \times (50)^2$

the total amount of wood required is

$$A = 400 \times 100 + 2 \left[\int_0^{200} f(x) dx + \int_{200}^{360} g(x) dx + \int_{360}^{500} h(x) dx \right] - \pi \times 50^2$$
 A1

$$= 2 \left[\int_0^{200} \left(-\frac{1}{500}(x-100)^2 + 140 \right) dx + \int_{200}^{360} \left(65 \sin\left(\frac{\pi}{320}(x-200)\right) + 120 \right) dx + \int_{360}^{500} \left(\frac{37}{28} \sqrt{720x - 110,000 - x^2} \right) dx \right] + 40,000 - 2,500\pi$$

ii. $177,804.67 \text{ cm}^2$ A1

$f2(x)$	$\left\{ \frac{-x^2}{500} + \frac{2 \cdot x}{5} + 120, 0 \leq x \leq 200 \right\}$	
$f1(x)$	$\left\{ 120 - 65 \cdot \sin\left(\frac{\pi \cdot x}{320} + \frac{3 \cdot \pi}{8}\right), 200 \leq x \leq 360 \right\}$	
$f3(x)$	$\left\{ \frac{37 \cdot \sqrt{-x^2 + 720 \cdot x - 110000}}{28}, 360 \leq x \leq 500 \right\}$	
$2 \cdot \left(\int_0^{200} f2(x) dx + \int_{200}^{360} f1(x) dx + \int_{360}^{500} f3(x) dx \right) - \pi \cdot 50^2 + 400 \cdot 100$		177804.67

f. $x(t) = Re^{kt} \cos(m\pi t)$

$$x(0) = R = -\frac{1}{100} = -0.01$$

$$v(t) = \frac{dx}{dt} = Re^{kt} (k \cos(m\pi t) - m\pi \sin(m\pi t))$$

$$v(0) = Rk = 400 \Rightarrow k = \frac{400}{-0.01} = -40,000$$
 A1

$$T = \frac{2\pi}{m\pi} = \frac{2}{m} = \frac{1}{f} = \frac{1}{250}$$

$$R = \frac{1}{100} = -0.01 \text{ , } m = 500$$
 A1

Question 3

- a.i.** $P \stackrel{d}{=} N(\mu = 106, \sigma^2 = 2.5^2)$
 $\Pr(P > 103) = 0.8849$ A1
- ii.** $P_{10} \stackrel{d}{=} \text{Bi}(n = 10, p = 0.8849)$
 $\Pr(P_{10} \geq 8) = 0.9015$ A1
- iii.** $P_n \stackrel{d}{=} \text{Bi}(n = ?, p = 0.8849)$
 $\Pr(P_n \geq 5) \geq 0.95$ to find the value of n , using trial and error
 $n = 7$ A1

normCdf(103,∞,106,2.5)	0.8849
$p:=0.88493$	0.8849
binomCdf(10,0.88493,8,10)	0.9015
binomCdf($n,0.88493,5,n$) $n=6$	0.8549
binomCdf($n,0.88493,5,n$) $n=7$	0.9627

- b.i.** $P \stackrel{d}{=} N(\mu = ?, \sigma^2 = ?^2)$
 large or extra large, $\Pr(P > 230) = 0.21 \Rightarrow \Pr(P \leq 230) = 0.79$ M1
 (1) $\frac{230 - \mu}{\sigma} = 0.8064$
 small, $\Pr(P < 138) = 0.11$ M1
 (2) $\frac{138 - \mu}{\sigma} = -1.2265$
 solving (1) and (2) $\mu = 193.5$, $\sigma = 45.3$ A1

invNorm(0.79,0,1)	0.8064
$\frac{230-m}{s}=0.8064$	$\frac{230-m}{s}=0.8064$
invNorm(0.11,0,1)	-1.2265
solve($\frac{230-m}{s}=0.806421$ and $\frac{138-m}{s}=-1.226528, \{m,s\}$)	$s=45.2545$ and $m=193.5059$

b.ii. $\Pr(P \geq L) = 0.08$, $\Pr(P \leq L) = 0.92$

$$\frac{L - 193.5}{45.25} = 1.4051 \text{ solving } L = 257.1$$

minimum weight for extra large potato $L = 257.1$

A1

invNorm(0.92,0,1)	1.4051
invNorm(0.92,193.5059,45.25)	257.0854
solve($\frac{l-193.5}{45.25}=1.4051, l$)	$l=257.0808$

iii. completing the table

	small	medium	large	extra large
Probability	0.11	0.68	0.13	0.08
potato weight, w grams	$w < 138$	$138 \leq w \leq 230$	$230 < w < 257.1$	$w \geq 257.1$
revenue cents	-0.03	0.05	0.10	0.15

expected revenue $E(R) = 0.08 \times 0.15 + 0.13 \times 0.10 + 0.68 \times 0.05 - 0.11 \times 0.03 = 0.06$
= 6 cents

A1

iv. $\Pr(138 \leq P \leq 230 | P \geq 138) = \frac{\Pr(138 \leq P \leq 230)}{\Pr(P \geq 138)} = \frac{0.68}{1 - 0.11} = \frac{0.68}{0.89}$
= 0.764

A1

c.i. $b \left[\int_0^5 \frac{x}{5} dx + \int_5^{10} \cos\left(\frac{\pi(x-5)}{10}\right) dx \right] = 1$ total area under the curve is one.

$$b \left[\left[\frac{x^2}{10} \right]_0^5 + \left[\frac{10}{\pi} \sin\left(\frac{\pi(x-5)}{10}\right) \right]_5^{10} \right] = 1$$

M1

$$b \left[\left(\frac{25}{10} - 0 \right) + \frac{10}{\pi} \left(\sin\left(\frac{\pi}{2}\right) - \sin(0) \right) \right] = 1$$

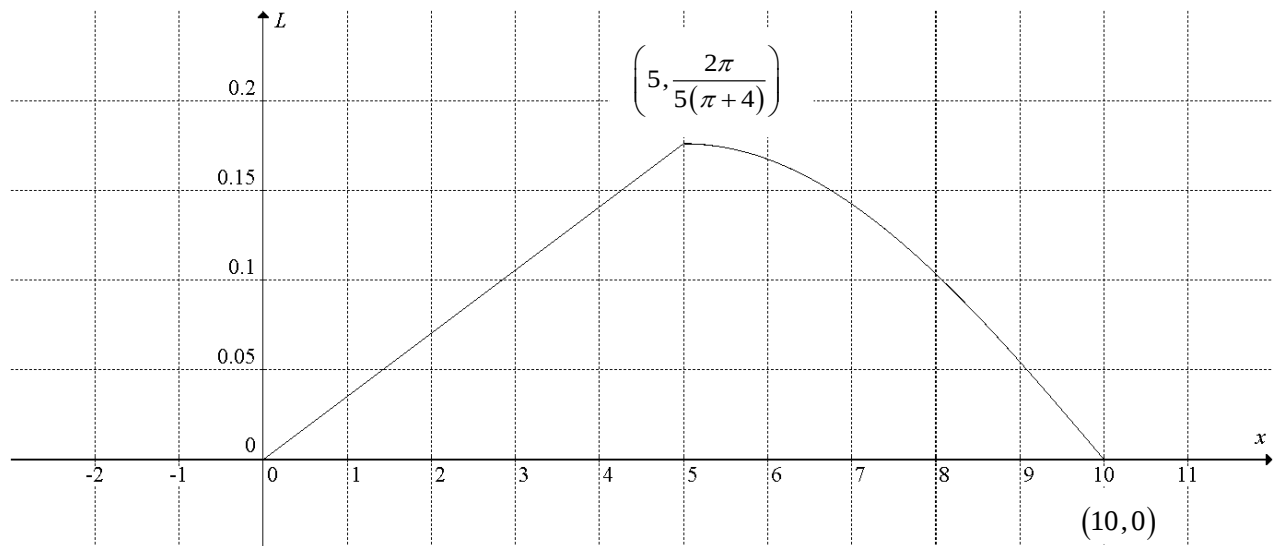
$$b \left(\frac{5}{2} + \frac{10}{\pi} \right) = b \left(\frac{5\pi + 20}{2\pi} \right) = 1$$

A1

$$b = \frac{2\pi}{5(\pi + 4)}$$

ii. The function is continuous at $x = 5$ and $\frac{2\pi}{5(\pi + 4)} \approx 0.176$. correct scaling, shape,
zero elsewhere.

G1



iii. $E(L) = \frac{2\pi}{5(\pi+4)} \left[\int_0^5 \frac{x^2}{5} dx + \int_5^{10} x \cos\left(\frac{\pi(x-5)}{10}\right) dx \right] = 5.284$ A1

$$E(L^2) = \frac{2\pi}{5(\pi+4)} \left[\int_0^5 \frac{x^3}{5} dx + \int_5^{10} x^2 \cos\left(\frac{\pi(x-5)}{10}\right) dx \right] = 32.330$$

$$\text{Var}(L) = E(L^2) - (E(L))^2 = 32.33018 - 5.28448^2 = 4.404$$
 A1

$b := \frac{2 \cdot \pi}{5 \cdot (\pi + 4)}$	$\frac{2 \cdot \pi}{5 \cdot (\pi + 4)}$
Define $f1(x) = \frac{b \cdot x}{5}$	Done
Define $f2(x) = b \cdot \cos\left(\frac{\pi \cdot (x - 5)}{10}\right)$	Done
Define $f3(x) = \begin{cases} f1(x), & 0 \leq x \leq 5 \\ f2(x), & 5 \leq x \leq 10 \end{cases}$	Done
$\int_0^{10} f3(x) dx$	1
$\int_0^{10} (x \cdot f3(x)) dx$	5.284
$\int_0^{10} (x^2 \cdot f3(x)) dx$	32.330
$32.330184313499 - (5.2844767120181)^2$	4.404

iv. Since $\frac{2\pi}{5(\pi+4)} \left[\int_0^5 \frac{x}{5} dx \right] = 0.44$ the median satisfies

$$\frac{2\pi}{5(\pi+4)} \left[\int_5^m \cos \left(\frac{\pi(x-5)}{10} \right) dx \right] = 0.5 - 0.440 = 0.06$$

solving gives $m = 5.342$

A1

$\int_0^5 f1(x) dx$	0.440
$\text{solve} \left(\int_5^m f2(x) dx = 0.5 - 0.44, m \right) 5 < m < 10$	$m = 5.342$
$\text{solve} \left(\int_0^m f3(x) dx = 0.5, m \right)$	$m = 5.342$

d.i. $E(\hat{P}) = p = 0.64$

$$\text{sd}(\hat{P}) = \sqrt{\frac{p(1-p)}{n}}$$

$$\text{sd}(\hat{P}) = \sqrt{\frac{0.64 \times (1-0.64)}{100}}$$

$$\text{sd}(\hat{P}) = 0.048$$

A1

ii. $\Pr(0.64 - 0.048 \leq \hat{P} \leq 0.64 + 0.048)$

$$= \Pr(0.592 \leq \hat{P} \leq 0.688)$$

for 100 potatoes, we require $\Pr(59.2 \leq P_{100} \leq 68.8)$

M1

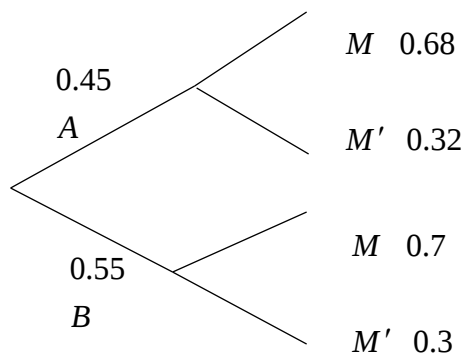
Since it is discrete, $P_{100} \stackrel{d}{=} \text{Bi}(n=100, p=0.64)$

$$\Pr(60 \leq P_{100} \leq 68) = 0.651$$

A1

$\sqrt{\frac{0.64 \cdot 0.36}{100}}$	0.048
$0.64 + 0.048$	0.688
$0.64 - 0.048$	0.592
$\text{binomCdf}(100, 0.64, 59.2, 68.8)$	0.651
$\text{binomCdf}(100, 0.64, 60, 68)$	0.651

e. tree diagram



$$\Pr(M') = \Pr(M' \cap A) + \Pr(M' \cap B) = 0.45 \times 0.32 + 0.55 \times 0.3 = 0.309 \quad \text{M1}$$

$$\Pr(B | M') = \frac{0.55 \times 0.3}{0.309} = 0.534 \quad \text{A1}$$

$\frac{0.55 \cdot 0.3}{0.55 \cdot 0.3 + 0.45 \cdot 0.32} \rightarrow 0.534$

Question 4

a. $f(x) = \frac{1}{x+2} + 3$

crosses the x-axis $y = 0$, $0 = \frac{1}{x+2} + 3 \Rightarrow x = -\frac{7}{3}$

crosses the y-axis $x = 0$, $y = \frac{1}{2} + 3 \Rightarrow y = \frac{7}{2}$, the coordinates are required

$$\left(-\frac{7}{3}, 0\right), \left(0, \frac{7}{2}\right) \quad \text{A1}$$

b. domain $B = \mathbb{R} \setminus \{-2\}$ range $\mathbb{R} \setminus \{3\}$ A1

Define $f1(x) = \frac{1}{x+2} + 3$	Done
⚠ solve($f1(x)=0, x$)	$x = -\frac{7}{3}$
$f1(0)$	$\frac{7}{2}$
domain($f1(x), x$)	$x \neq -2$

- c. $s = d(OP) = \sqrt{p^2 + (f(p))^2} = \sqrt{p^2 + \left(\frac{1}{p+2} + 3\right)^2} = \frac{\sqrt{p^4 + 4p^3 + 13p^2 + 42p + 49}}{p+2}$
- $\frac{ds}{dp} = 0 \Rightarrow p = -2.310$ or $p = 0.530$ M1
- $s(-2.310) = 2.321$ and $s(0.530) = 3.436$
- minimum value of p is $p = -2.310$ and the minimum distance is 2.321 A1

Define $s(p) = \sqrt{p^2 + (f(p))^2}$	Done
$s(p)$	$\frac{\sqrt{p^4 + 4 \cdot p^3 + 13 \cdot p^2 + 42 \cdot p + 49}}{ p+2 }$
$\frac{d}{dp}(s(p))$	$\frac{p^4 + 6 \cdot p^3 + 12 \cdot p^2 + 5 \cdot p - 7}{(p+2) \cdot \sqrt{p^4 + 4 \cdot p^3 + 13 \cdot p^2 + 42 \cdot p + 49} \cdot p+2 }$
solve $\left(\frac{d}{dp}(s(p)) = 0, p\right)$	$p = -2.31032$ or $p = 0.530301$
$s(-2.3103)$	2.32101
$s(0.53)$	3.43637

- d. $f: y = \frac{1}{x+2} + 3$
- $f^{-1}: x = \frac{1}{y+2} + 3, \quad x-3 = \frac{1}{y+2}, \quad y+2 = \frac{1}{x-3}$
- $f^{-1}: \mathbb{R} \setminus \{3\} \rightarrow \mathbb{R}, \quad f^{-1}(x) = -2 + \frac{1}{x-3}, \quad a = -2, b = 1, c = -3$ A1

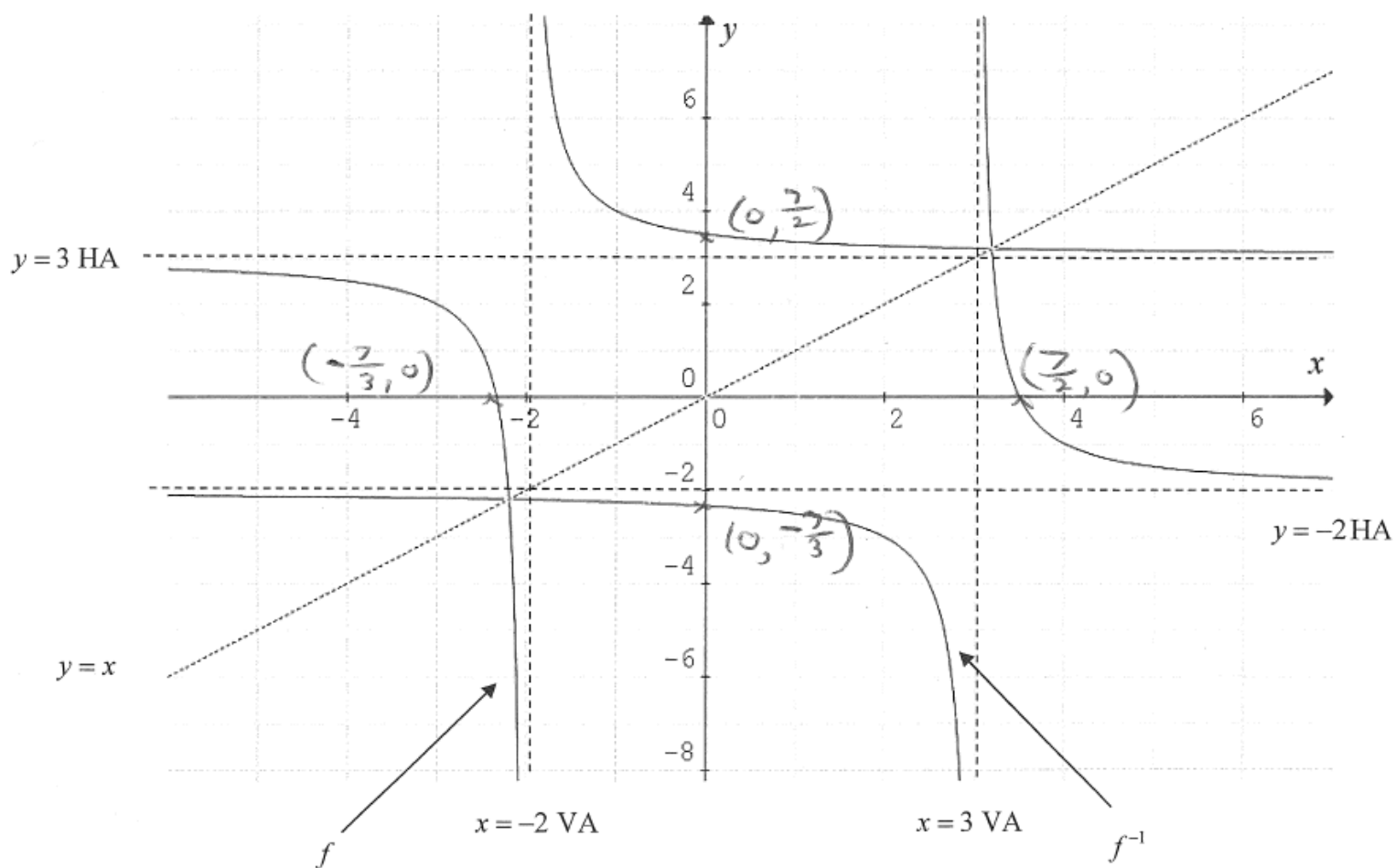
solve $(f(y) = x, y)$	$y = \frac{-(2 \cdot x - 7)}{x - 3}$
$\text{propFrac}\left(y = \frac{-(2 \cdot x - 7)}{x - 3}\right)$	$y = \frac{1}{x - 3} - 2$

- e. solving $f(x) = x$ or $f^{-1}(x) = x$ or $f(x) = f^{-1}(x)$
- $\frac{1}{u+2} + 3 = u$ or $\frac{1}{u-3} - 2 = u$ or $\frac{1}{u+2} + 3 = \frac{1}{u-3} - 2$
- all give $u = \frac{1 \pm \sqrt{29}}{2}$ A1

Define $f_2(x) = \frac{1}{x-3} - 2$	Done
$\triangle$ solve $(f_1(x) = f_2(x), x)$	$x = \frac{-(\sqrt{29}-1)}{2}$ or $x = \frac{\sqrt{29}+1}{2}$
$\triangle$ solve $(f_1(x) = x, x)$	$x = \frac{-(\sqrt{29}-1)}{2}$ or $x = \frac{\sqrt{29}+1}{2}$
$\triangle$ solve $(f_2(x) = x, x)$	$x = \frac{-(\sqrt{29}-1)}{2}$ or $x = \frac{\sqrt{29}+1}{2}$

f.

G2



- g. Note that $u = \frac{1+\sqrt{29}}{2} \approx 3.19 < 3.5$, must give two of the following three possible areas,

$$A = 2 \left[\int_0^{\frac{1+\sqrt{29}}{2}} (f(x) - x) dx \right] = 2 \left[\int_0^{\frac{1+\sqrt{29}}{2}} \left(\frac{1}{x+2} + 3 - x \right) dx \right]$$

$$\text{or } A = \int_0^{\frac{1+\sqrt{29}}{2}} f(x) dx + \int_{\frac{1+\sqrt{29}}{2}}^{\frac{7}{2}} f^{-1}(x) dx = \int_0^{\frac{1+\sqrt{29}}{2}} \left(\frac{1}{x+2} + 3 \right) dx + \int_{\frac{1+\sqrt{29}}{2}}^{\frac{7}{2}} \left(\frac{1}{x-3} - 2 \right) dx$$

A1

$$\text{or } A = 2 \left[\int_0^{\frac{1+\sqrt{29}}{2}} x dx + \int_{\frac{1+\sqrt{29}}{2}}^{\frac{7}{2}} f^{-1}(x) dx \right] = 2 \left[\int_0^{\frac{1+\sqrt{29}}{2}} x dx + \int_{\frac{1+\sqrt{29}}{2}}^{\frac{7}{2}} \left(\frac{1}{x-3} - 2 \right) dx \right]$$

$$A = 10.871$$

A1

$u := \frac{\sqrt{29}+1}{2}$	$\frac{\sqrt{29}+1}{2}$
$2 \cdot \int_0^u (f(x) - x) dx$	10.8711
$\int_0^u f(x) dx + \int_u^{\frac{7}{2}} f^{-1}(x) dx$	10.8711
$2 \cdot \left(\int_0^u x dx + \int_u^{\frac{7}{2}} f^{-1}(x) dx \right)$	10.8711

h. $g(x) = \frac{1}{x+k} + 3$

crosses the x-axis $y = 0$, $0 = \frac{1}{x+k} + 3 \Rightarrow x = -\frac{1}{3} - k$

crosses the y-axis $x = 0$, $y = \frac{1}{k} + 3$

$$\left(-\left(k + \frac{1}{3} \right), 0 \right), \left(0, \frac{1}{k} + 3 \right)$$

A1

i. $g^{-1}(x) = \frac{1}{x-3} - k$

A1

j. solving $g(x) = x$ or $g^{-1}(x) = x$ or $g(x) = g^{-1}(x)$

$$\frac{1}{x+k} + 3 = x$$

$$\frac{1}{x-3} - k = x$$

$$\frac{1}{x+k} = x-3$$

$$\frac{1}{x-3} = k+x$$

both give

$$1 = (k+x)(x-3)$$

$$1 = x^2 + (k-3)x - 3k$$

$$x^2 + (k-3)x - (3k+1) = 0$$

$$\Delta = (k-3)^2 + 4(3k+1)$$

$$\Delta = k^2 - 6k + 9 + 12k + 4 = k^2 + 6k + 13$$

$$x = \frac{-k \pm \sqrt{(k+3)^2 + 4}}{2}$$

M1

$\Delta = (k+3)^2 + 4$ since $\Delta > 0$ for any value of k ,
the graphs always intersect at two distinct points.

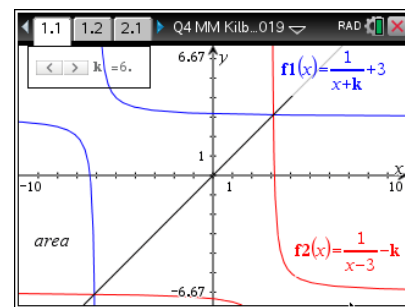
A1

k. The area approaches the area of a rectangle

with side lengths $\frac{1}{k} + 3$

$$A(k) = \left(\frac{1}{k} + 3\right)^2 = \frac{1}{k^2} + \frac{6}{k} + 9$$

$$\lim_{k \rightarrow \infty} A(k) = 9$$



A1

END OF SECTION B SUGGESTED ANSWERS

End of detailed answers for the
2019 Kilbaha VCE Mathematical Methods Trial Examination 2

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