

2019 VCE Mathematical Methods Trial Examination 1 Detailed Answers



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Question 1

a. Let $y = \frac{\sin(2x)}{x^2}$ using the quotient rule

$$u = \sin(2x) \quad v = x^2$$

$$\frac{du}{dx} = 2 \cos(2x) \quad \frac{dv}{dx} = 2x$$

M1

$$\frac{dy}{dx} = \frac{2x^2 \cos(2x) - 2x \sin(2x)}{x^4} = \frac{2x(x \cos(2x) - \sin(2x))}{x^4}$$

$$\frac{dy}{dx} = \frac{2(x \cos(2x) - \sin(2x))}{x^3} = \frac{2f(x)}{x^3}$$

$$f(x) = x \cos(2x) - \sin(2x)$$

A1

b. Let $y = g(x) = \tan\left(\frac{2}{x}\right) = \tan(u) \quad u = \frac{2}{x} = 2x^{-1}$ chain rule

$$\frac{dy}{du} = \frac{1}{\cos^2(u)} \quad \frac{du}{dx} = -2x^{-2} = -\frac{2}{x^2}$$

$$g'(x) = -\frac{2}{x^2 \cos^2\left(\frac{2}{x}\right)}$$

M1

$$g'\left(\frac{6}{\pi}\right) = -\frac{2}{\left(\frac{6}{\pi}\right)^2 \cos^2\left(\frac{\pi}{3}\right)} = -2 \times \frac{\pi^2}{36} \times \frac{1}{\left(\frac{1}{2}\right)^2}$$

$$g'\left(\frac{6}{\pi}\right) = -\frac{2\pi^2}{9}$$

A1

Question 2

$$X \stackrel{d}{=} \text{Bi}(n=7, p=?)$$

$$M = \Pr(X=3) = \binom{7}{3} p^3 (1-p)^4 = 35p^3 (1-p)^4 \text{ differentiating using the product rule}$$

A1

$$\frac{dM}{dp} = 35 \left[\left(\frac{d}{dp}(p^3) \right) \times (1-p)^4 + p^3 \times \frac{d}{dp}((1-p)^4) \right]$$

$$\frac{dM}{dp} = 35 [3p^2 (1-p)^4 - 4p^3 (1-p)^3]$$

M1

$$\frac{dM}{dp} = 35p^2 (1-p)^3 [3(1-p) - 4p]$$

$$\frac{dM}{dp} = 35p^2 (1-p)^3 (3-7p) = 0 \quad \text{since } M \text{ has the largest probability}$$

$$\text{since } 0 < p < 1, \quad p = \frac{3}{7}$$

A1

Question 3

a. $2 - 4\cos\left(\frac{\pi x}{3}\right) = 0$

$$4\cos\left(\frac{\pi x}{3}\right) = 2$$

$$\cos\left(\frac{\pi x}{3}\right) = \frac{1}{2}$$

$$\frac{\pi x}{3} = 2n\pi \pm \cos^{-1}\left(\frac{1}{2}\right) = 2n\pi \pm \frac{\pi}{3} = \frac{\pi}{3}(6n \pm 1)$$

$$x = 6n \pm 1, n \in \mathbb{Z}$$

A1

b. endpoints $f(0) = 2 - 4\cos(0) = -2$, $f(6) = 2 - 4\cos(2\pi) = -2$

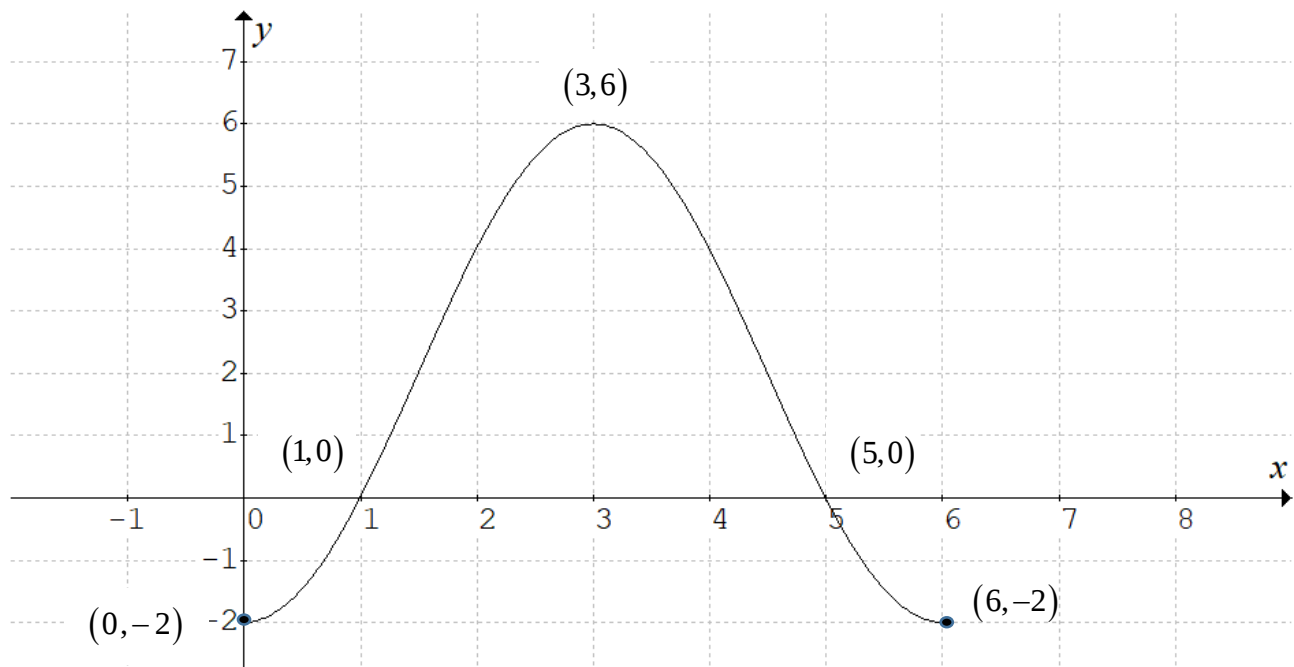
maximum at $y = 6$, when $\cos\left(\frac{\pi x}{3}\right) = -1$, $\frac{\pi x}{3} = \pi$, $x = 3$

crosses x -axis when $2 - 4\cos\left(\frac{\pi x}{3}\right) = 0$ from **a.** $x = 1, 5$

endpoints $(0, -2)$, $(6, -2)$ maximum $(3, 6)$, x -intercepts $(1, 0)$, $(5, 0)$

A1

G1



Question 4

$$y = \sqrt{5+cx} = (5+cx)^{\frac{1}{2}} \quad \text{differentiating using the chain rule}$$

$$\frac{dy}{dx} = \frac{c}{2\sqrt{5+cx}}$$

$$\text{at the point } x=2, \text{ the gradient is } \left. \frac{dy}{dx} \right|_{x=2} = \frac{c}{2\sqrt{5+2c}} \quad \text{M1}$$

this makes an angle of 135° , so the gradient $m = \tan(135^\circ) = -1$

$$\text{so } \frac{c}{2\sqrt{5+2c}} = -1 \text{ and therefore } c < 0 \quad \text{A1}$$

$$c = -2\sqrt{5+2c} \quad \text{squaring both sides}$$

$$c^2 = 4(5+2c) = 20+8c$$

$$c^2 - 8c - 20 = 0 \quad \text{M1}$$

$$(c-10)(c+2) = 0$$

$$c = 10, -2 \quad \text{but } c < 0$$

$$c = -2 \quad \text{only} \quad \text{A1}$$

Question 5

$$\text{a. } \frac{d}{dx} [\log_e(x^2+4)] = \frac{2x}{x^2+4} \quad \text{A1}$$

$$\text{b. } \text{the average value } \bar{f} = \frac{1}{b-0} \int_0^b \frac{x}{x^2+4} dx = \log_e \left(\sqrt[b]{2} \right) = \log_e \left(2^{\frac{1}{b}} \right)$$

$$\text{from a. } \frac{1}{b} \int_0^b \frac{x}{x^2+4} dx = \left[\frac{1}{2b} \log_e(x^2+4) \right]_0^b = \frac{1}{b} \log_e(2) \quad \text{A1}$$

$$= \frac{1}{2} [\log_e(b^2+4) - \log_e(4)] = \frac{1}{2} \log_e(4)$$

$$= \frac{1}{2} \log_e \left(\frac{b^2+4}{4} \right) = \frac{1}{2} \log_e(4)$$

$$\frac{b^2+4}{4} = 4 \quad \text{A1}$$

$$b^2+4=16$$

$$b^2=12$$

$$b = \pm\sqrt{12} \quad \text{but } b > 0$$

$$b = 2\sqrt{3} \quad \text{only} \quad \text{A1}$$

Question 6

- a.** x is the number of broken biscuits, in a total of 12, so $\hat{P} = \frac{x}{12}$

$$\Pr\left(\hat{P} = \frac{1}{12}\right) = \Pr(\text{one broken}) = \Pr(X = 1), X \stackrel{d}{=} Bi\left(n = 12, p = \frac{1}{6}\right)$$

M1

$$\Pr(X = 1) = \binom{12}{1} \left(\frac{1}{6}\right) \left(\frac{5}{6}\right)^{11} = 12 \times \frac{1}{6} \times \left(\frac{5}{6}\right)^{11}$$

$$\Pr(X = 1) = 2 \left(\frac{5}{6}\right)^{11}, \quad c = 2, \quad a = 5, \quad b = 6, \quad n = 11$$

A1

- b.** $n = 144, \quad p = \frac{1}{6}$

$$\text{sd}(\hat{P}) = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{\frac{1}{6} \times \frac{5}{6}}{144}} = \frac{\sqrt{5}}{72}$$

A1

- c.** Packet 1, contains 2 broken and 10 not broken, total of 12.

Let packet 2, contain $12 - b$ broken and b not broken, total of 12.

$$\Pr(\text{both not broken}) = \frac{1}{2} \times \frac{10}{12} \times \frac{9}{11} + \frac{1}{2} \times \frac{b}{12} \times \frac{b-1}{11} = \frac{23}{66}$$

$$90 + b(b-1) = \frac{23}{66} \times 2 \times 132 = 92$$

M1

$$b^2 - b - 2 = 0$$

$$(b-2)(b+1) = 0$$

$$b = 2 \quad \text{since } b > 0$$

there are 10 broken biscuits in the packet that was dropped onto the floor.

A1

- d.** Let W be the weights of a packet, $W \stackrel{d}{=} N(\mu = 250, \sigma^2 = 8^2)$

Z is the standard normal, $Z \stackrel{d}{=} N(\mu = 0, \sigma^2 = 1)$

$$\Pr(244 \leq W \leq 256)$$

$$= \Pr\left(\frac{244 - 250}{8} \leq Z \leq \frac{256 - 250}{8}\right)$$

$$= \Pr\left(-\frac{3}{4} \leq Z \leq \frac{3}{4}\right) = 2 \Pr\left(0 \leq Z \leq \frac{3}{4}\right) = 2 \Pr\left(-\frac{3}{4} \leq Z \leq 0\right)$$

$$w = -\frac{3}{4}$$

A1

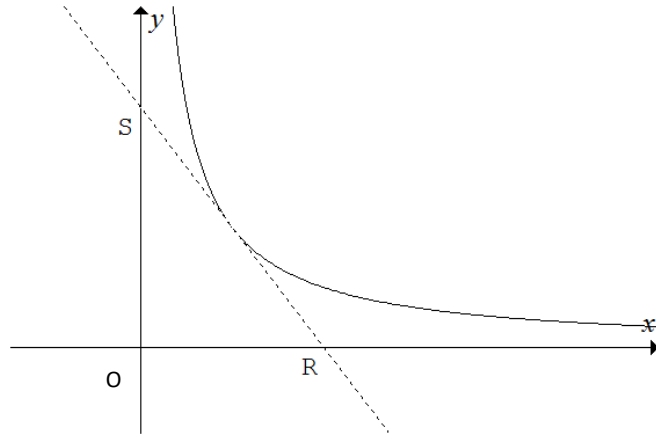
Question 7

a. Let $y = f(x) = \frac{1}{x} = x^{-1}$

$$\frac{dy}{dx} = f'(x) = -x^{-2} = -\frac{1}{x^2}$$

$$m_T = f'(a) = -\frac{1}{a^2}$$

$$T: y - \frac{1}{a} = -\frac{1}{a^2}(x - a)$$



M1

$$T: y = -\frac{x}{a^2} + \frac{2}{a}, \quad m = -\frac{1}{a^2}, \quad c = \frac{2}{a}$$

A1

b. at S $x = 0 \Rightarrow y = \frac{2}{a}$, $S\left(0, \frac{2}{a}\right)$.

at R $y = 0 \Rightarrow \frac{x}{a^2} = \frac{2}{a}$, $x = 2a$, $R(2a, 0)$.

$$\Delta ORS = \frac{1}{2} \times \frac{2}{a} \times 2a = 2$$

A1

Question 8

$$A = \frac{1}{2} \times 2u \times f(u) = uf(u) = ue^{-u^2}$$

using the product rule

$$\frac{dA}{du} = \frac{d}{du}(u)e^{-u^2} + u \frac{d}{du}(e^{-u^2}), \text{ now } \frac{d}{du}(e^{-u^2}) = -2ue^{-u^2} \text{ by the chain rule}$$

A1

$$\frac{dA}{du} = e^{-u^2} - 2u^2e^{-u^2} = e^{-u^2}(1 - 2u^2) = 0 \text{ for maximum}$$

M1

$$2u^2 = 1$$

$$u^2 = \frac{1}{2}$$

$$u = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \text{ since } u > 0$$

A1

Question 9

a. completing the square

$$y = 11 + 2x - x^2 = -(x^2 - 2x + 1) + 1 + 11 = 12 - (x - 1)^2$$

$$y = x^2 \quad y' = 12 - (x' - 1)^2, \quad \frac{y' - 12}{-1} = (x' - 1)^2$$

$$x = x' - 1, \quad y = \frac{y' - 12}{-1} \Rightarrow x' = x + 1, \quad y' = -y + 12$$

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 12 \end{bmatrix}, \quad a = 1, \quad b = -1, \quad h = 1, \quad k = 12 \quad \text{A1}$$

b.

Complete a function domain table

	$f(x)$	$g(x)$
domain	$(-4, \infty)$	R
Range	$R^+ = (0, \infty)$	$(-\infty, 12]$

Note that since range $g = (-\infty, 12] \not\subset$ domain $f = (-4, \infty)$

so that $f(g(x))$ does not exist. A1

c. solving $g(x) = 11 + 2x - x^2 = -4$

$$\Rightarrow x^2 - 2x - 15 = 0$$

$$(x - 5)(x + 3) = 0$$

$$\Rightarrow x = -3, 5 \quad \text{M1}$$

So we can restrict the domain of g , and domain $f(g(x)) = \text{domain } g(x) = D = (-3, 5)$ A1

$$g: (-3, 5) \rightarrow R, \quad g(x) = 11 + 2x - x^2$$

Graph of restricted domain $g(x)$

now domain $g = (-3, 5)$, range $g = (-4, 12]$

range $g \subset$ domain f , so that $f(g(x))$ exists

$$f(g(x)) = f(11 + 2x - x^2)$$

$$= \frac{1}{\sqrt{11 + 2x - x^2 + 4}}$$

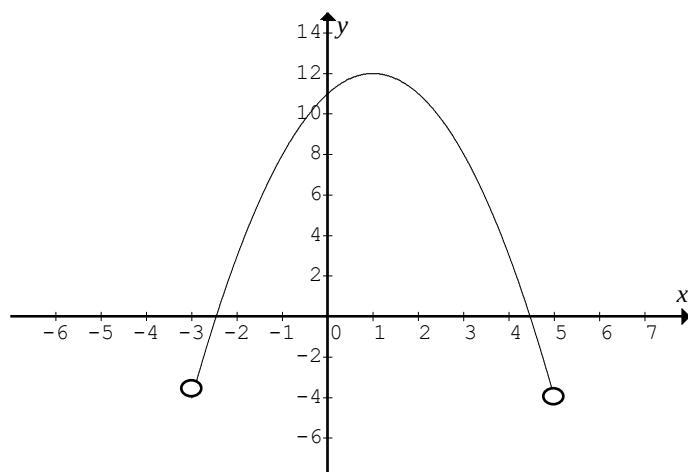
$$= \frac{1}{\sqrt{15 + 2x - x^2}}$$

alternatively $f(g(x))$ now exists when

$$15 + 2x - x^2 > 0$$

$$= -(x^2 - 2x - 15) > 0$$

$$= -(x - 5)(x + 3) > 0 \Rightarrow D = (-3, 5)$$



Question 10

- a. $f(x) = e^{-2x} \sin(2x)$ using the product rule

$$\begin{aligned} f'(x) &= e^{-2x} \frac{d}{dx} [\sin(2x)] + \sin(2x) \frac{d}{dx} (e^{-2x}) \\ &= 2e^{-2x} \cos(2x) - 2e^{-2x} \sin(2x) \\ &= 2e^{-2x} (\cos(2x) - \sin(2x)) \end{aligned} \quad \text{M1}$$

$$\text{for turning points } f'(x) = 0 \Rightarrow 2e^{-2x} (\cos(2x) - \sin(2x)) = 0$$

$$\cos(2x) - \sin(2x) = 0, \quad \cos(2x) = \sin(2x), \quad \tan(2x) = 1$$

$$2x = \frac{\pi}{4}, \frac{5\pi}{4}$$

$$x = \frac{\pi}{8}, \frac{5\pi}{8} \quad \text{since } 0 \leq x \leq \pi \quad \text{A1}$$

- b. Let $y = e^{-2x} (\cos(2x) + \sin(2x))$ using the product rule

$$\begin{aligned} \frac{dy}{dx} &= e^{-2x} \frac{d}{dx} [\cos(2x) + \sin(2x)] + (\cos(2x) + \sin(2x)) \frac{d}{dx} (e^{-2x}) \\ &= e^{-2x} \times (-2\sin(2x) + 2\cos(2x)) - 2e^{-2x} (\cos(2x) + \sin(2x)) \\ &= -4e^{-2x} \sin(2x) \end{aligned} \quad \text{A1}$$

- c. the graph of $f(x) = e^{-2x} \sin(2x)$ crosses the x-axis when $e^{-2x} \sin(2x) = 0$

$$\text{that is when } \sin(2x) = 0, \quad 2x = 0, \pi, 2\pi \quad \text{at } x = 0, \frac{\pi}{2}, \pi$$

$$\text{required area } A = \int_0^{\frac{\pi}{2}} e^{-2x} \sin(2x) dx \quad \text{A1}$$

$$\text{from b. } \frac{d}{dx} [e^{-2x} (\cos(2x) + \sin(2x))] = -4e^{-2x} \sin(2x)$$

$$A = -\frac{1}{4} [e^{-2x} (\cos(2x) + \sin(2x))]_0^{\frac{\pi}{2}} \quad \text{A1}$$

$$A = -\frac{1}{4} (e^{-\pi} (\cos(\pi) + \sin(\pi))) - (\cos(0) + \sin(0))$$

$$A = \frac{1}{4} (e^{-\pi} + 1) \quad \text{A1}$$

**End of detailed answers for the
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