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2019

***Mathematical
Methods***

***Trial Examination 2
(2 hours)***

SECTION A Multiple-choice questions

Instructions for Section A

Answer **all** questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this examination are **not** drawn to scale.

Question 1 Which one of the following is the correct solution set when $\log_a(b-x)^2 = c$ is solved for x given $a, b, c \in R$?

- A. $\{b - a^{\frac{c}{2}}\}$
- B. $\{-b + a^{\frac{c}{2}}, b + a^{-\frac{c}{2}}\}$
- C. $\{b - a^{\frac{c}{2}}, b + a^{\frac{c}{2}}\}$
- D. $\{b - a^{\sqrt{c}}, b - a^{-\sqrt{c}}\}$
- E. $\{-b + a^{\sqrt{c}}, b + a^{-\sqrt{c}}\}$

Question 2 Which one of the following is a correct solution when $\left(\cos\left(x + \frac{\pi}{4}\right)\right)\left(\tan\left(x - \frac{\pi}{4}\right)\right) = 0$ is solved for x ?

- A. $-\frac{9\pi}{4}$
- B. $-\frac{5\pi}{4}$
- C. $-\frac{3\pi}{4}$
- D. $-\frac{\pi}{4}$
- E. $\frac{7\pi}{4}$

Question 3 The domain of the inverse of $\{(3, -3), (-1, -2), (1, -1), (-2, 2), (3, -1), (-2, -3)\}$ is D . Which one of the following statements is true?

- A. D is $\{x: -2 \leq x \leq 3\}$
- B. D is $\{x: -3 \leq x \leq 2\}$
- C. A subset of D is $\{x: -2 \leq x \leq 2\}$
- D. $3 \notin D$
- E. There are 3 positive integers in D

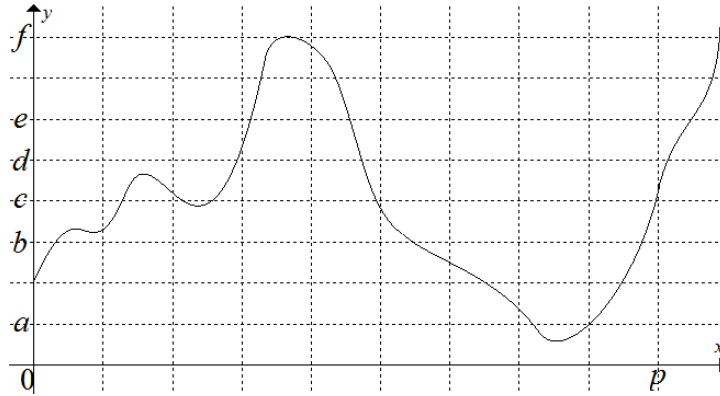
Question 4 Given $a \in \mathbb{R} \setminus \{0\}$, the **inverse** of $y = a\left(1 \pm \frac{1}{x-a}\right)$ is

- A. $y = a\left(1 \pm \frac{1}{x-a}\right)$
- B. $y = a\left(1 \pm \frac{1}{x+a}\right)$
- C. $y = a\left(1 - \frac{1}{x \pm a}\right)$
- D. $y = a\left(1 + \frac{1}{x \pm a}\right)$
- E. $y = \pm a\left(1 - \frac{1}{x-a}\right)$

Question 5 The area of the region(s) bounded by the graphs of $y = x^a$ and $y = x^{\frac{1}{a}}$ for $0 < a < 1$ is A . The value of A is in the interval

- A. $(0, 0.5)$
- B. $(0, 1)$
- C. $(1, 1.5)$
- D. $(1, 2)$
- E. $(0, 2)$

Question 6 The graph of $y = h(x)$ is shown below. The average value of $h(x)$ in the interval $[0, p]$ is H .



The value of H is in the interval

- A. (a, b)
- B. (b, c)
- C. (c, d)
- D. (d, e)
- E. (e, f)

Question 7 Consider $h(x)$ in **Question 6**.

The average rate of change of $h(x)$ with respect to x in the interval $[0, p]$ is closest to

- A. $\frac{c - a + b}{p}$
- B. $\frac{b - a + c}{p}$
- C. $\frac{2c - a - b}{2p}$
- D. $\frac{2c - a + b}{2p}$
- E. $\frac{0.5c - a + b}{p}$

Question 8 If $f(x-a) + f(a-x) = 0$ and $a \in \mathbb{R}$, $f(x)$ cannot be

- A. $\tan x$
- B. $2x^3$
- C. $\sin 3x$
- D. $4x^{-3}$
- E. $\cos 5x$

Question 9

The number of solutions to simultaneous equations $y = \cos(nx)$ and $y = \cos(3nx)$ in the interval $\left[-\frac{\pi}{n}, \frac{\pi}{n}\right]$ for $n > 0$ is

- A. 1
- B. 2
- C. 3
- D. 4
- E. 5

Question 10 $f(x)$ is a cubic polynomial. The graph $y = f(x)$ cuts the x -axis at $x = a, b$ and c from left to right in that order. (m, n) and (p, q) are the local maximum and minimum points respectively. Using two triangles to approximate the regions bounded by $y = f(x)$ and the x -axis, the approximate area is closest to

- A. $\frac{1}{2}(nc - (n+q)b + qa)$
- B. $\frac{1}{2}(nc + (n+q)b + qa)$
- C. $\frac{1}{2}((n-q)b - na - qc)$
- D. $\frac{1}{2}((n-q)b + na - qc)$
- E. $\frac{1}{2}((n-q)b - na + qc)$

Question 11 A fair die is rolled 18 times. The probability of getting 5 or 6 appearing on the uppermost face of the die 5 times or 6 times out of the 18 rolls of the die is closest to

- A. 0.25
- B. 0.33
- C. 0.38
- D. 0.43
- E. 0.51

Question 12 A bag contains 3 green, 4 blue and 5 red marbles. A random sample of three marbles are taken out of the bag. From the following statements choose one which could be a random variable.

- A. All three marbles in the sample are green.
- B. The number of blue marbles is greater than the number of red marbles in the sample.
- C. There are two blue marbles and one red marble in the sample.
- D. The colour of each marble in the sample.
- E. The number of red marbles in the sample.

Question 13 A large number of random samples of size 256 people are taken from a large city population. The distribution of $\hat{P}$, proportion of people wearing glasses in a sample, has a standard deviation of 0.025. The proportion of people wearing glasses in the population is closest to

- A. 0.55
- B. 0.45
- C. 0.40
- D. 0.30
- E. 0.20

Question 14 A large number of random samples of size 300 people are taken from another large city. Many samples have $\hat{p} = 0.25$. A statistician decides to use 0.25 as an estimation of p . Using this value the proportion of random samples with sample proportion greater than 0.2 is closest to

- A. 0.98
- B. 0.78
- C. 0.68
- D. 0.58
- E. 0.57

Question 15 The graph of $y = f(x)$ undergoes the following sequence of transformations. Firstly the graph is translated in the negative y -direction by b units, then the resulting graph is reflected in the x -axis, and lastly dilated from the x -axis by factor a .

The equation of the graph after the sequence of transformations is

- A. $y = ab - af(x)$
- B. $y = b - af(x)$
- C. $y = -b - af(x)$
- D. $y = -b + af(x)$
- E. $y = -ab - af(x)$

Question 16 The probability distribution of random variable X is given by the table below.

X	1	2	3	4
$\Pr(X = x)$	0.50	a^2	0.35	$0.2a$

The standard deviation of X is closest to

- A. 1.0436
- B. 1.0522
- C. 1.0713
- D. 1.0715
- E. 1.9007

Question 17 The probability density function of random variable X is given by

$$f(x) = \begin{cases} \frac{1}{1+a-x} & \text{for } 0 \leq x \leq a \\ 0 & \text{elsewhere} \end{cases}$$

The value of a is

- A. e
- B. e^{-1}
- C. $e - 1$
- D. $1 - e^{-1}$
- E. $1 + e^{-1}$

Question 18 The shortest distance from the origin O to the curve $y = \frac{(x-2)^3}{3}$ is closest to

- A. 1.01
- B. 0.99
- C. 0.97
- D. 0.96
- E. 0.95

Question 19

The graph of $y = 3x^3 + ax^2 + b^2x + c$ for $b, c \in R^+$ has no stationary points.
Which of the following statements is true?

- A. $a < 3b$
- B. $a > 3b$
- C. $a = 3b$
- D. $-b < 2a < 3b$
- E. $a < 3b + c$

Question 20 Given $f(x) = \frac{a}{x-a} + b$ where $a, b \in R$ and $a \neq 0$, the number of intersections of the graphs of $y = f(x)$ and $y = f^{-1}(x)$ **cannot** be

- A. 0
- B. 1
- C. 2
- D. 3
- E. infinitely many

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise stated.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this examination are **not** drawn to scale.

Question 1 A parabola has its vertex at $(2, 0)$ and passes through point $(3, 1)$.

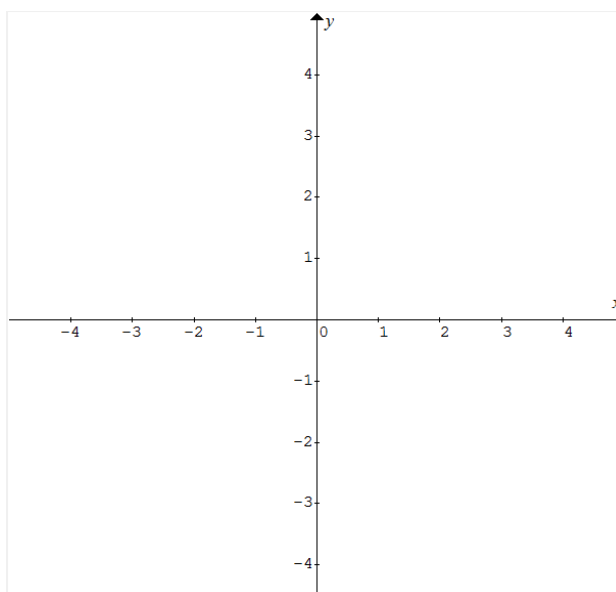
- a. The equation of the parabola is $y = x^2 - 4x + 4$. Show working in finding the equation. 1 mark

- b. The inverse of the parabola in part a is given by $y = f(x)$ and $y = g(x)$ together with point $(0, 2)$.
Given $f(x) > g(x)$, write down the rule for each of functions $f(x)$ and $g(x)$.

1 mark

- c. Show that the parabola and its inverse intersect at $(1, 1)$ and $(4, 4)$. 1 mark

- di. Sketch the parabola and its inverse with $(1, 1)$ and $(4, 4)$ as the endpoints of each curve. 2 marks



dii. Show that the area of the region bounded by the parabola and its inverse is 9.

2 marks

Horizontal line $y = d$ divides the bounded region into two regions of equal area.

e. Determine the value of d . Give the value correct to two decimal places.

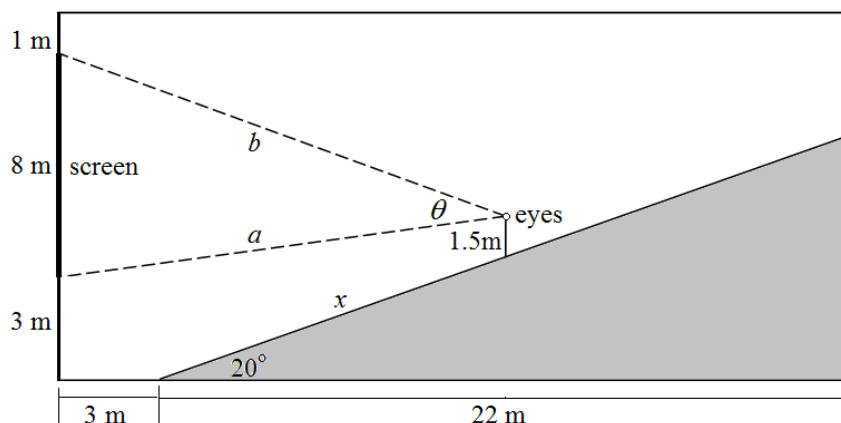
2 marks

A line segment of equation $y = -x + c$ is drawn inside the bounded region.

f. Find the maximum length of such a line segment inside the region and the corresponding value of c .
Give exact values for your answers.

3 marks

Question 2 The following diagram shows the side elevation of a cinema.



There are 23 rows of seats. The first row is 3 m horizontally from the screen. The last row is at the top of the sloping floor inclined at 20° to the horizontal. The rows are 1 m apart horizontally. The diagram shows a viewer sitting in a row x m from the first row. The eyes of the viewer are 1.5 m above the sloping floor. The bottom and the top of the screen are 3 m and 11 m respectively above the ground. The viewing angle is θ . Lengths are measured in metres and angles in degrees.

- a. Determine the possible exact values of x .

1 mark

- b. Show that $a^2 = x^2 + (6\cos 20^\circ - 3\sin 20^\circ)x + 11.25$

2 marks

- c. Show that $b^2 = x^2 + (6\cos 20^\circ - 19\sin 20^\circ)x + 99.25$

1 mark

- di. Using $\cos \theta = \frac{a^2 + b^2 - c^2}{2ab}$ where c is the height of the screen, show that

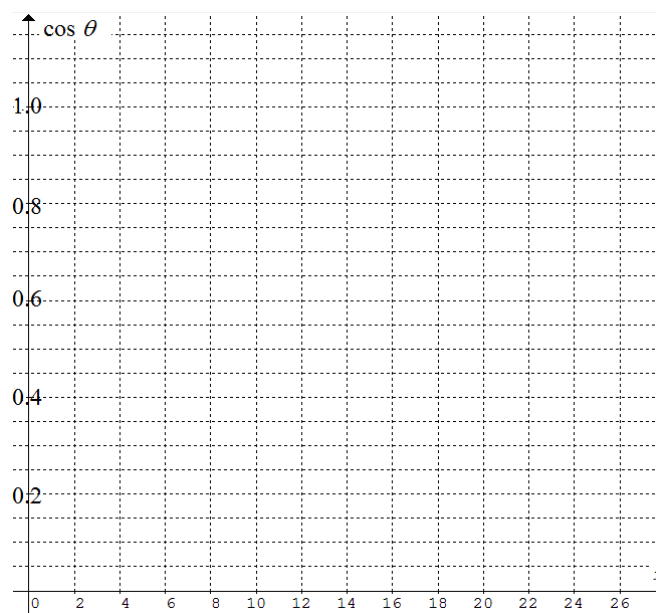
$$\cos \theta = \frac{x^2 + (6\cos 20^\circ - 11\sin 20^\circ)x + 23.25}{\sqrt{(x^2 + (6\cos 20^\circ - 3\sin 20^\circ)x + 11.25)(x^2 + (6\cos 20^\circ - 19\sin 20^\circ)x + 99.25)}}.$$

1 mark

- dii. Given $\cos \theta \approx \frac{x^2 + 1.88x + 23.25}{\sqrt{x^4 + bx^3 + 106.53x^2 + 448.07x + 1116.56}}$, show that $b \approx 3.75$. 1 mark

- e. Calculate the viewing angle θ (correct to the nearest degree) if the viewer is seated in the tenth row. 2 marks

- f. Use the expression in dii to sketch the graph of $\cos \theta$ versus x . 2 marks



- g. Use the graph to determine the greatest viewing angle θ (correct to the nearest degree) and the value of x (correct to the nearest whole number) for maximum θ . 2 marks

- h. State which row gives the viewer the greatest viewing angle. 1 mark

Question 3 Consider $f(x) = e^x - mx$ where $m > 0$.

- a. If $f(x) = 0$ has exactly one solution, find the solution and the exact value of m . 2 marks

- b. Write down the values of m such that $f(x) = 0$ has two distinct solutions. 1 mark

- c. If the graph of $y = f(x)$ has x -intercepts at x_1 and x_2 , where $x_2 > x_1$, write a definite integral for the area of the region bounded by the graph of $y = f(x)$ and the x -axis.

1 mark

- d. Evaluate the definite integral in part c in terms of m , x_1 and x_2 . 2 marks

- e. Hence show that the sum of the two solutions to $f(x) = 0$ is always greater than 2. 3 marks

Now consider $g(x) = \log_e x - nx^2$ where $n > 0$.

- f. If $g(x) = 0$ has exactly one solution $x = a$, find the exact value of a and the exact value of n .

3 marks

- g. Write down the n values such that the equation $g(x) = 0$ has two distinct solutions.

1 mark

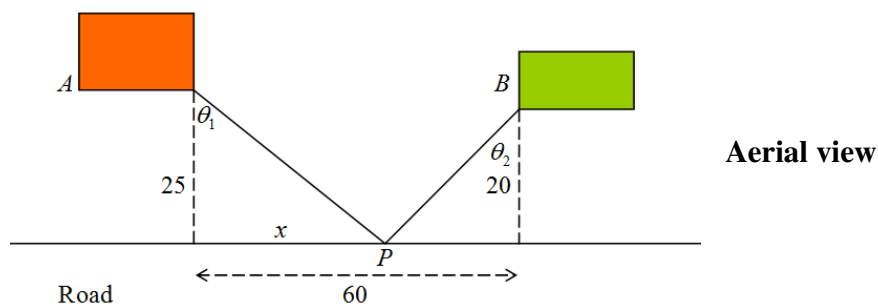
- h. If $g(x) = \log_e x - nx^2$ where $n < 0$, and $x = b$ is the solution to $g(x) = 0$, explain why $0 < b < 1$.

1 mark

Question 4 Two farm houses, A and B , are 60 m apart. Farm House A is 25 m from the road and Farm House B is 20 m from the road.

A telephone pole marked as P is located at the edge of the road x m from A along the edge of the road. The telephone lines make angles θ_1 and θ_2 as shown in the following diagram.

Assume that the telephone lines are straight and horizontal.



- a. Given $x = 30$ and $\tan(\theta_1 + \theta_2) = \frac{\tan \theta_1 + \tan \theta_2}{1 - \tan \theta_1 \tan \theta_2}$, find the exact value of $\theta_1 + \theta_2$. 2 marks

- b. Find the value(s) of x such that $\theta_1 + \theta_2 = 90^\circ$. 2marks

- c. Show that the total length of the two telephone lines is a minimum when $\theta_1 = \theta_2$. 2 marks

- d. Find the exact value of θ_1 when the total length of the two telephone lines is a minimum. 1 mark

- e. Find the minimum total length of the two telephone lines. 1 mark

Question 5 The following piecewise function is a differentiable probability density function for continuous random variable X , where k is a real constant.

$$f(x) = \begin{cases} ke^{-2}x, & 0 \leq x < 1 \\ ke^{-\frac{(x-5)^2}{8}}, & 1 \leq x \leq 9 \\ -ke^{-2}(x-10), & 9 < x \leq 10 \\ 0 & \text{elsewhere} \end{cases}$$

- a. Show that $k \approx 0.203232$.

2 marks

Let random variable X be the length (in cm) of an earth worm in Worm Farm A.
The probability distribution of X is given by $f(x)$ in part a.

- b. Evaluate $\Pr(X < 6 | X > 1)$, correct to 4 decimal places.

1 mark

Let p be the proportion of earth worms in the farm with $X > 6$.
Random samples of 100 worms are taken from the farm.

- c. Determine the value of p , correct to 4 decimal places.

1 mark

- d. Determine the mean and standard deviation of sample proportion of earth worms with $X > 6$.
Correct your answers to 4 decimal places.

2 marks

- e. Find the probability, correct to 4 decimal places, that the proportion of earth worms in a sample with $X > 6$ is less than 0.4

1 mark

For parts f and g, let $p = 0.3$ be the proportion of earth worms in the farm with $X > c$.

- f. Find the value of c , correct to 2 decimal places. 1 mark

- g. Find how large a sample is required so that $\text{sd}(\hat{p}) \leq 0.01$. 1 mark

Let random variable Y be the length (in cm) of an earth worm in Worm Farm B .

Random variable Y has a normal distribution.

A random sample of 400 earth worms is taken from Worm Farm B .

The sample contains 100 worms of length $Y > 8$.

Correct your answers to parts h, i and j to 2 decimal places.

- h. For $Y < 8$ cm, write down a suitable approximate value of p in the expression for estimating the standard deviation of p in Worm Farm B .

1 mark

- i. Calculate the estimated standard deviation of p in Worm Farm B . 1 mark

- j. Find an approximate 70% confidence interval for p in Worm Farm B . 2 marks

End of Examination 2