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Mathematical Methods

2019

Trial Examination 1 (1 hour)

Instructions

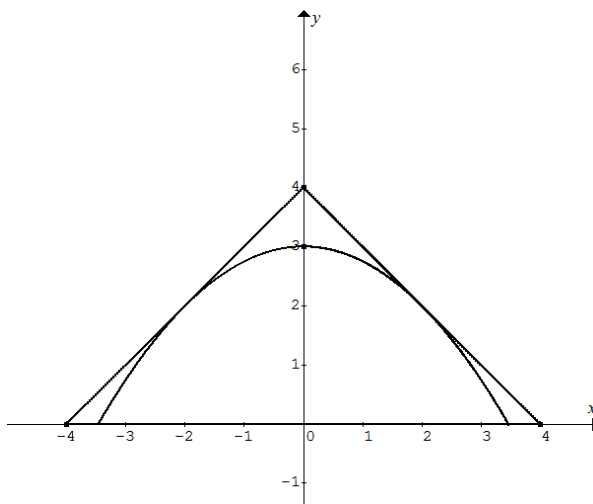
Answer **all** questions.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Question 1 The following diagram shows a triangle with vertices $(-4, 0)$, $(0, 4)$ and $(4, 0)$, and a parabola with vertex $(0, 3)$.



- a. Write down the equation of the parabola in turning point form in terms of dilation factor k in the y -direction.

1 mark

- b. The parabola can just fit inside the triangle. Determine the value of k in part a.

2 marks

- c. Determine the coordinates of the x -intercepts of the parabola.

1 mark

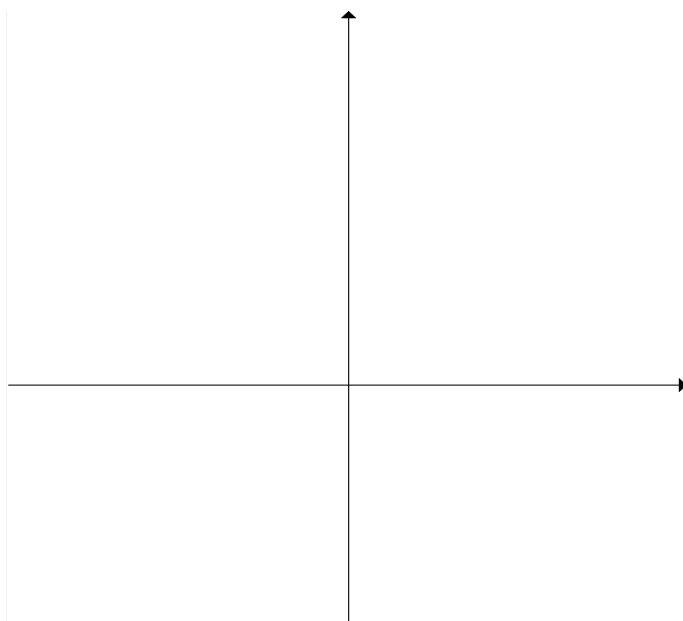
Question 2 Polynomial $P(x) = x^6 - 1$ can be expressed in the form of 6 factors for $x > 0$.

- a. Firstly, expand $(a + \sqrt{a} + 1)(a - \sqrt{a} + 1)$. 1 mark

- b. Given $P(x) = (x^2 - 1)(x^4 + mx^2 + 1)$, find the value of m . 1 mark

- c. Hence or otherwise express $P(x)$ in the form of 6 factors. 2 marks

- d. Sketch the graph of $P(x)$. 1 mark



Question 3 Solve $\frac{e^x + e^{-x}}{e^x - e^{-x}} = 7$ for e^{-x} .

2 marks

Question 4 Consider $f(x) = \log_e(ax)^2$ where $a \in \mathbb{R} \setminus \{-1, 1\}$.

a. Show that $f(-x) = f(x)$.

1 mark

b. Show that $f(xy) \neq f(x) + f(y)$.

2 marks

c. Find $f'(x)$ and show that $f'(-x) = f'(x)$.

2 marks

Question 5 Given $f(x) = x^{\frac{2}{3}}$ and $g(x) = x^{\frac{3}{2}}$, solve $f(g(x)) - g(f(x)) = 0$ for x .

2 marks

Question 6 $y = \frac{1}{2} \cos\left(n\left(x + \frac{\pi}{10}\right)\right) - 2$ is the transformation of $y = 2 \cos(x) + 1$.

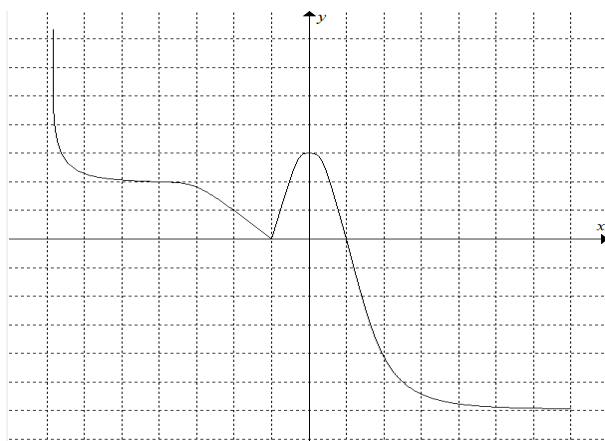
A sequence of transformations can change $y = \frac{1}{2} \cos\left(n\left(x + \frac{\pi}{10}\right)\right) - 2$ back to $y = 2 \cos(x) + 1$.

Write down the required transformations in correct order.

5 marks

Question 7 Sketch on the same axes the graph of the derivative function of the function shown below.

4 marks



Question 8 The graph of function $f(x)$ intersects the x -axis at $x = a$ and $x = b$, where $b > a > 0$ and $f(x) \geq 0$ for $x \in [a, b]$.

Let A be the area of the region bounded by $f(x)$ and the x -axis.

- a. Write a definite integral for A . 1 mark

The graph of $f(x)$ is dilated in both x and y directions by the same factor of $\sqrt{2}$ and then translated in the positive y -direction by $\sqrt{2}$ units.

- b. Write a definite integral for the area of the region after the stated transformations. 2 marks

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- c. Find the area of the region after the stated transformations in terms of A , a and b . 1 mark

Question 9 Country town A has a population of 100 000, and 10 000 are overweight.

A sample of 400 people is selected randomly.

Let $\hat{P}$ be the proportion of overweight people in the sample.

- a. Calculate the standard deviation of $\hat{P}$. 1 mark

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- b. Find $\Pr(\hat{P} > 0.10)$. 1 mark

c. Let N be the number of overweight people in another sample of 400 people from country town A.

Find an approximate value of $\Pr(10 \leq N \leq 70)$ in the sample.

1 mark

d. A random sample of 400 people is taken from country town B. If there are 40 overweight people in the sample, calculate an approximate 95% confidence interval for the population proportion p of country town B.

1 mark

Question 10 $f(x) = 2 \sin(2x)$ over $x \in \left[\frac{\pi}{4} - \alpha, \frac{\pi}{4} + \alpha \right]$ is a probability density function.

a. Calculate the value of α .

3 marks

b. Given $\cos(\theta) = \sqrt{\frac{1 + \cos(2\theta)}{2}}$, find the exact value of $\Pr\left(\frac{5\pi}{24} < X < \frac{7\pi}{24}\right)$.

2 marks

End of Exam