

2019 Mathematical Methods Trial Exam 1 Solutions
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Q1a $y = -kx^2 + 3$

Q1b $-kx^2 + 3 = x + 4$, $kx^2 + x + 1 = 0$, $\Delta = 0$, $k = \frac{1}{4}$

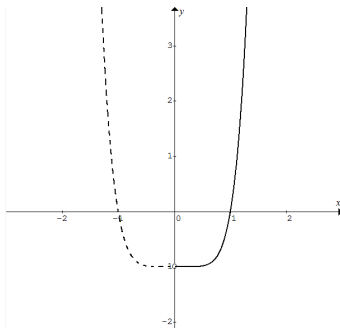
Q1c $-\frac{1}{4}x^2 + 3 = 0$, $x^2 = \pm\sqrt{12} = \pm 2\sqrt{3}$, $(-2\sqrt{3}, 0)$, $(2\sqrt{3}, 0)$

Q2a $(a + \sqrt{a} + 1)(a - \sqrt{a} + 1) = a^2 + a + 1$
 $(a + \sqrt{3a} + 1)(a - \sqrt{3a} + 1) = a^2 - a + 1$

Q2b
 $x^6 - 1 = (x^2 - 1)(x^4 + mx^2 + 1) = x^6 + (m-1)x^4 + (1-m)x^2 - 1$,
 $m = 1$

Q2c
 $P(x) = (x^2 - 1)(x^4 + x^2 + 1) = (x+1)(x-1)(x^2 + x + 1)(x^2 - x + 1)$
 $= (x+1)(x-1)(x + \sqrt{x} + 1)(x - \sqrt{x} + 1)(x + \sqrt{3x} + 1)(x - \sqrt{3x} + 1)$

Q2d



Q3 $e^x + e^{-x} = 7e^x - 7e^{-x}$, $6e^x - 8e^{-x} = 0$, $6 - 8(e^{-x})^2 = 0$,
 $(e^{-x})^2 = \frac{3}{4}$, $e^{-x} = \frac{\sqrt{3}}{2}$

Q4a

$f(-x) = \log_e(a(-x)^2) = \log_e(a^2(-x)^2) = \log_e(a^2x^2) = \log_e(ax)^2 = f(x)$

Q4b $f(x) = \log_e(ax)^2$, $f(y) = \log_e(ay)^2$

$f(x) + f(y) = \log_e(ax)^2 + \log_e(ay)^2 = \log_e(ax)^2(ay)^2 = \log_e(a^2xy)^2$
 $f(xy) = \log_e(axy)^2$, $\therefore f(xy) \neq f(x) + f(y)$

Q4c $f(x) = \log_e(ax)^2$, $f'(x) = \frac{1}{(ax)^2} \times 2a^2x$

$\therefore f'(x) = \frac{2}{x}$ for $x \in \mathbb{R} \setminus \{0\}$

$\therefore f'(-x) = \frac{2}{-x}$ for all $x \in \mathbb{R} \setminus \{0\}$, this statement is the same as

$f'(x) = \frac{2}{x}$ for all $x \in \mathbb{R} \setminus \{0\}$

Also, $f(-x) = f(x) = \log_e(ax)^2$, $\therefore f'(-x) = f'(x)$

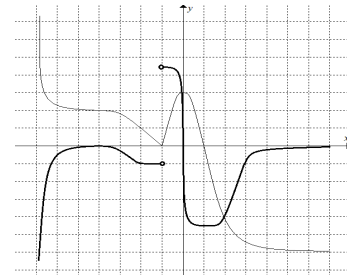
Q5 $f(g(x)) = f\left(x^{\frac{2}{3}}\right) = \left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = x$ for $x \geq 0$

$g(f(x)) = g\left(x^{\frac{2}{3}}\right) = \left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = x$ for all $x \in \mathbb{R}$.

$\therefore f(g(x)) - g(f(x)) = 0$ for $x \geq 0$.

Q6 Translate $\frac{\pi}{10}$ units to the right, dilate in the y-direction by factor 4, dilate in the x-direction by factor n , translate 9 units up.

Q7



Q8a $A = \int_a^b f(x) dx$

Q8b $\int_{\sqrt{2}a}^{\sqrt{2}b} \left(\sqrt{2}f\left(\frac{x}{\sqrt{2}}\right) + \sqrt{2} \right) dx$

Q8c $\sqrt{2} \times \sqrt{2} \times A + \sqrt{2} \times (\sqrt{2}b - \sqrt{2}a) = 2(A + b - a)$

Q9a $p = \frac{10000}{100000} = 0.1$, $\text{sd}(\hat{p}) = \sqrt{\frac{0.1 \times 0.9}{400}} = 0.015$

Q9b $\Pr(\hat{p} > 0.10) = 0.5$

Q9c $\Pr(10 \leq N \leq 70) = \Pr(0.025 \leq \hat{p} \leq 0.175) = \Pr(-5 \leq Z \leq 5) \approx 1$

Q9d (0.07, 0.13)

Q10a $\int_{\frac{\pi}{4}-\alpha}^{\frac{\pi}{4}+\alpha} 2 \sin(2x) dx = 1$, $[-\cos(2x)]_{\frac{\pi}{4}-\alpha}^{\frac{\pi}{4}+\alpha} = 1$

$-\cos\left(\frac{\pi}{2} + 2\alpha\right) + \cos\left(\frac{\pi}{2} - 2\alpha\right) = 1$, $\sin(2\alpha) + \sin(2\alpha) = 1$

$\sin(2\alpha) = \frac{1}{2}$, $2\alpha = \frac{\pi}{6}$, $\alpha = \frac{\pi}{12}$

Q10b $\Pr\left(\frac{5\pi}{24} < X < \frac{7\pi}{24}\right) = \int_{\frac{5\pi}{24}}^{\frac{7\pi}{24}} 2 \sin(2x) dx = [-\cos(2x)]_{\frac{5\pi}{24}}^{\frac{7\pi}{24}}$

$= -\cos\left(\frac{7\pi}{12}\right) + \cos\left(\frac{5\pi}{12}\right) = \cos\left(\frac{5\pi}{12}\right) + \cos\left(\frac{5\pi}{12}\right) = 2\cos\left(\frac{5\pi}{12}\right)$

$= 2\sqrt{\frac{1 + \cos\frac{5\pi}{6}}{2}} = \sqrt{2 - \sqrt{3}}$

Please inform mathline@itute.com re conceptual and/or mathematical errors