YEAR 12 *Trial Exam Paper*

2019

MATHEMATICAL METHODS

Written examination 2

Worked solutions

This book presents:

- worked solutions
- mark allocations
- tips.

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SECTION A – Multiple-choice questions

Question 1

Answer: D

Explanatory notes

The period is $2\pi / \frac{2\pi}{5} = 5$.

The range of $3\cos\left(\frac{2\pi x}{5}\right)$ is $[-3,3]$. Therefore the range of $3\cos\left(\frac{2\pi x}{5}\right)-2$ is $[-5,1]$.

Question 2

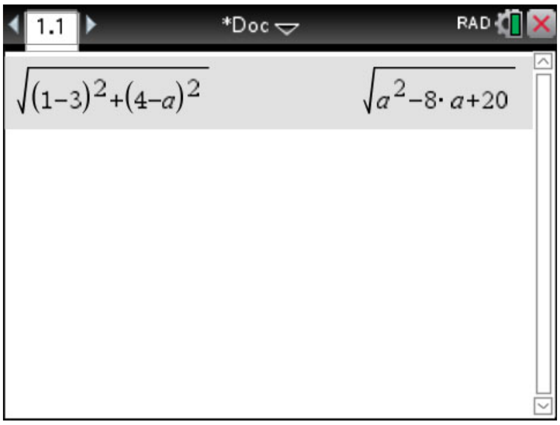
Answer: B

Explanatory notes

Use the formula for the distance between two points:

$$\sqrt{(1-3)^2+(4-a)^2}=\sqrt{a^2-8a+20}$$

Use CAS to do the algebra and avoid common algebraic errors.



Question 3

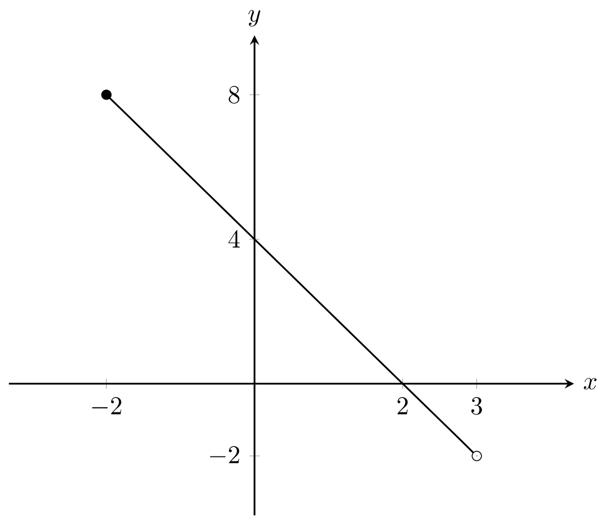
Answer: A

Explanatory notes

Since $f(-2)=8$ is included and $f(3)=-2$ is excluded, the range of f is $(-2,8]$.

Tip

- *It may be helpful for you to draw a sketch of the function on the domain given:*

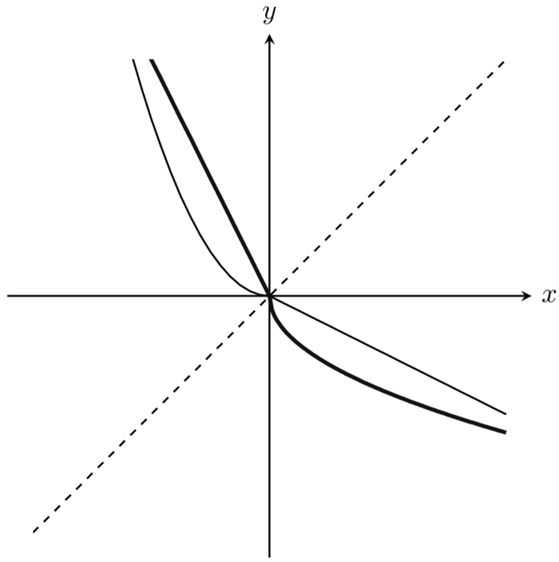


Question 4

Answer: B

Explanatory notes

The inverse may be found by reflecting the graph of function f around the line $y = x$ as shown in the diagram below.



Note: The function with the thickest line is the inverse function.

Tip

- *You could eliminate some options in this question using the fact that points in the 2nd quadrant are always reflected to/from the 4th quadrant, while points in the 1st and 3rd quadrants don't change quadrants.*

Question 5

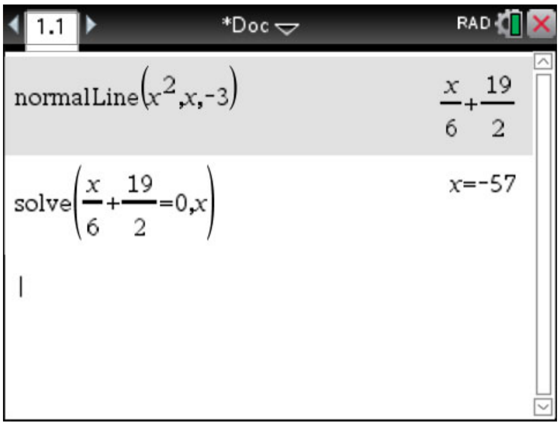
Answer: C

Explanatory notes

$\frac{dy}{dx}=2x$. When $x=-3$, $\frac{dy}{dx}=-6$. So the gradient of the perpendicular line is $\frac{1}{6}$ and the equation of this line is $y-9=\frac{1}{6}(x+3)$.

When $y=0$, $-9=\frac{1}{6}(x+3)$ and so $-54=x+3 \Rightarrow x=-57$.

Alternatively, CAS may be used. The equation of the perpendicular line of $y=x^2$ at the point $(-3,9)$ is $y=\frac{x}{6}+\frac{19}{2}$. This linear function passes through the x-axis when $x=-57$.



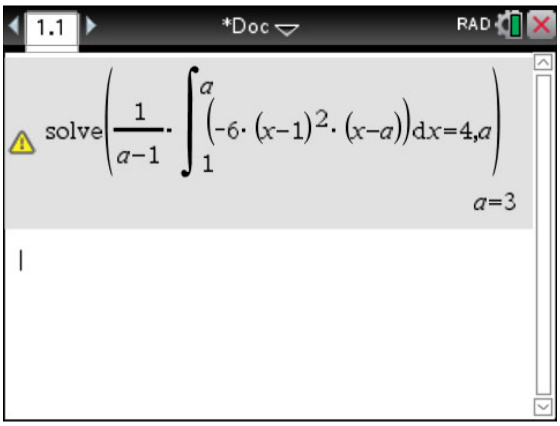
Question 6

Answer: E

Explanatory notes

We need to solve the equation $\frac{1}{a-1}\int_1^a -6(x-1)^2(x-a)dx=4$ for *a*.

Using CAS, we find that $a=3$.



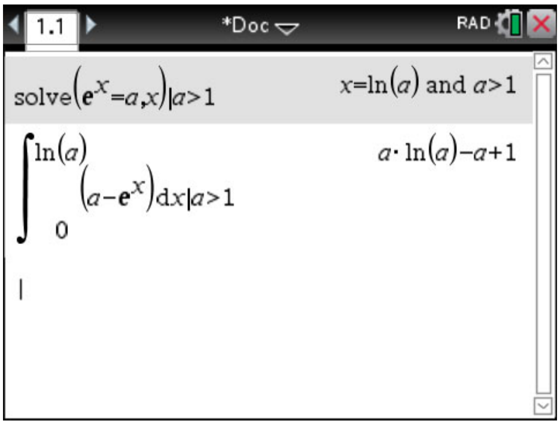
Question 7

Answer: C

Explanatory notes

The graphs of $y = e^x$ and $y = a$ meet when $x = \log_e(a)$.

Use CAS to find the required area (in terms of a).

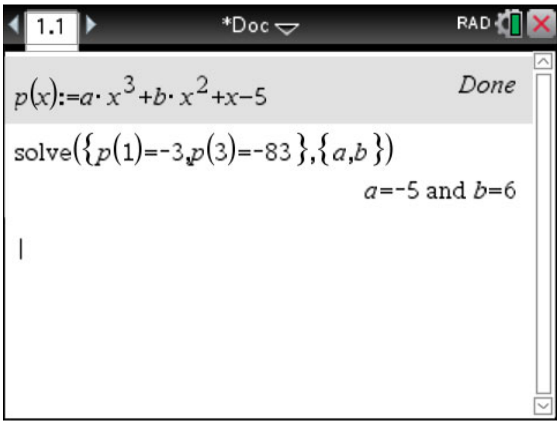


Question 8

Answer: C

Explanatory notes

Use CAS to determine the values of a and b by first defining the function, then substituting in the two points and finally solving for the parameters.



The values of a and b are -5 and 6 respectively. The sum of a and b is 1.

Question 9

Answer: C

Explanatory notes

First the value of a must be found. Since X is a probability distribution, it must be the case that

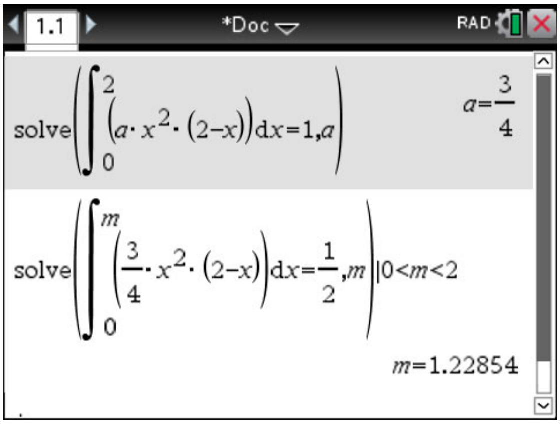
$$\int_{-\infty}^{\infty} f(x)dx = \int_0^2 ax^2(2-x)dx = 1.$$

Use CAS to find that $a = \frac{3}{4}$.

Now find the median by solving

$\int_0^m \frac{3}{4}x^2(2-x)dx = \frac{1}{2}$ for m.

It is found that $m \approx 1.23$.



Question 10

Answer: A

Explanatory notes

The average value of f on the interval $[0,2]$ is $\frac{1}{2}\int_0^2 f(x)dx = \frac{5}{2}$. Therefore, it must be the case that $\frac{1}{4}\int_{-4}^{0} f\left(-\frac{x}{2}\right)dx = \frac{5}{2}$, since dilation from or reflection in the y-axis does not change a function's average value.

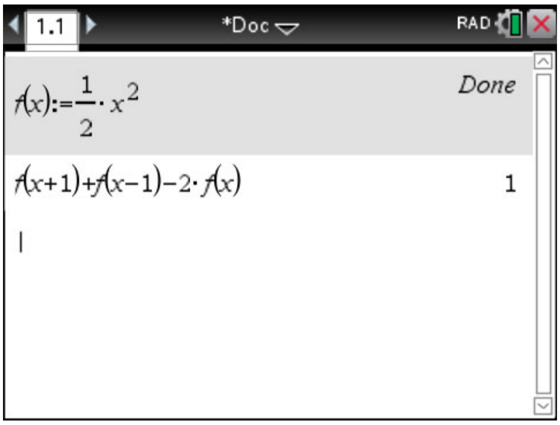
Therefore, $\int_{-4}^{0} 3f\left(-\frac{x}{2}\right)dx = 4 \times 3 \times \frac{5}{2} = 30.$

Question 11

Answer: E

Explanatory notes

If $f(x)=\frac{1}{2}x^2$ then $f(x+1)+f(x-1)-2f(x)=1$. This can be checked using CAS.



Tip

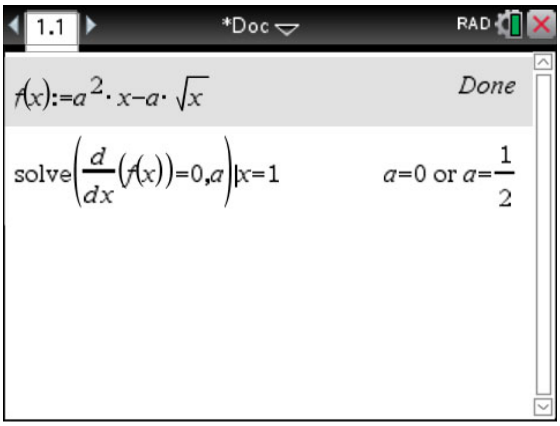
- *In the absence of a better method, you may use trial and error. In this case, starting from the last option and working upwards can be a good strategy.*

Question 12

Answer: B

Explanatory notes

Use CAS to solve $f'(x)=0$ for a when $x=1$.



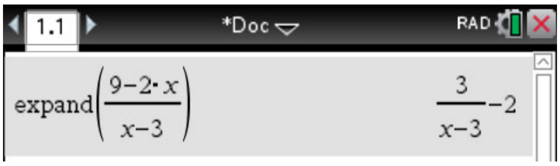
Since $a>0$ the trivial solution is rejected.

Question 13

Answer: D

Explanatory notes

Express f in the form $a+\dfrac{b}{x-3}$ to find the vertical and horizontal asymptotes.



So the graph of f has a vertical asymptote of $x = 3$ and a horizontal asymptote of $y = -2$.

Tip

- *While you could do this by hand, having a good knowledge of the CAS calculator allows the computation to be performed more easily.*

Question 14

Answer: E

Explanatory notes

Equate the functions, rearrange and multiply through by e^x to obtain

$ke^{2x} - 3e^x + 1 = 0$.

This is a quadratic in e^x and the discriminant is $9 - 4k$. From the discriminant there can be two solutions if $9 - 4k > 0 \Rightarrow k < \dfrac{9}{4}$. Solving the equation $ky^2 - 3y + 1 = 0$ gives $y = \dfrac{3 \pm \sqrt{9-4k}}{2k}$. Since $y = e^x$ it is necessary for both these solutions to be positive in order for the original equation to have two solutions. If k is negative or zero, then $\dfrac{3+\sqrt{9-4k}}{2k} \le 0$ so at most one solution is possible, whereas for $0 < k < \dfrac{9}{4}$ both solutions are positive as required.

Tip

- *Use sliders on your CAS graphing screen to quickly visualise the effect when changing the parameters.*

Question 15

Answer: B

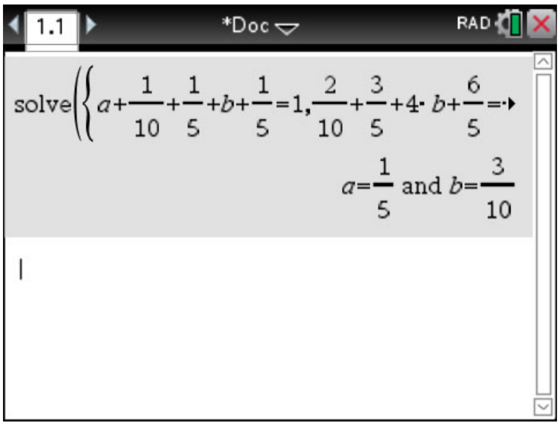
Explanatory notes

From the information provided, we can write down two simultaneous equations:

$$a+\frac{1}{10}+\frac{1}{5}+b+\frac{1}{5}=1$$

$$\frac{2}{10}+\frac{3}{5}+4b+\frac{6}{5}=\frac{32}{10}$$

These are solved to give $a=\frac{1}{5}$ and $b=\frac{3}{10}$.



Tip

- *You should use CAS to solve simultaneous equations in Examination 2.*

Question 16

Answer: E

Explanatory notes

Use the standard normal $Z \sim N(0,1)$:

$$\Pr(Z > 120) = 0.2$$

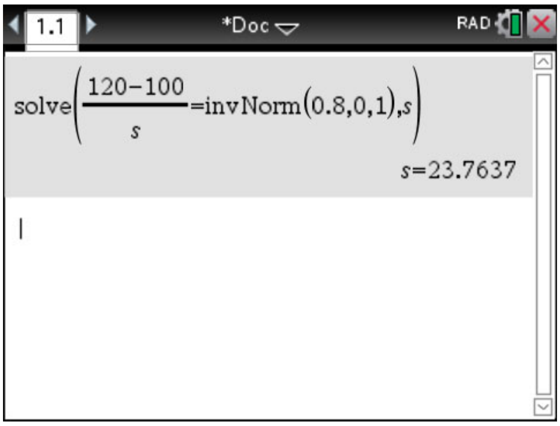
$$\Pr\left(Z > \frac{120-100}{\sigma}\right) = 0.2$$

$$\frac{120-100}{\sigma} = 0.8416$$

$$\sigma = 23.7637$$

So σ is closest to 24.

This can be done in a single line using CAS.



Question 17

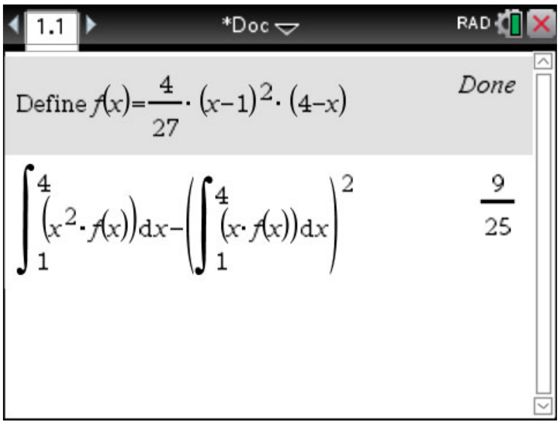
Answer: A

Explanatory notes

Use the formula for the variance of a random variable X:

$\text{Var}(X) = E\left(X^2\right) - \left(E(X)\right)^2$

This is best done using CAS.



So $\text{Var}(X) = \dfrac{9}{25} \Rightarrow \sigma = \dfrac{3}{5}$.

Question 18

Answer: B

Explanatory notes

Since $\Pr(B) = 0.3$ it follows that $\Pr(B') = 1 - 0.3 = 0.7$ and since A and B are independent events, $\Pr(A \cap B') = 0.28$.

So

$$\begin{aligned}\Pr(A \cup B') &= \Pr(A) + \Pr(B') - \Pr(A \cap B') \\ &= 0.4 + 0.7 - 0.28 \\ &= 0.82\end{aligned}$$

Question 19

Answer: A

Explanatory notes

The approximate area is

$$\left(5-\frac{5}{4}\right)+(5-2)+\left(5-\frac{13}{4}\right)=\frac{17}{2}$$

The actual area is

$$\int_1^5 2\sqrt{x-1}\,dx=\frac{32}{3}$$

The error is

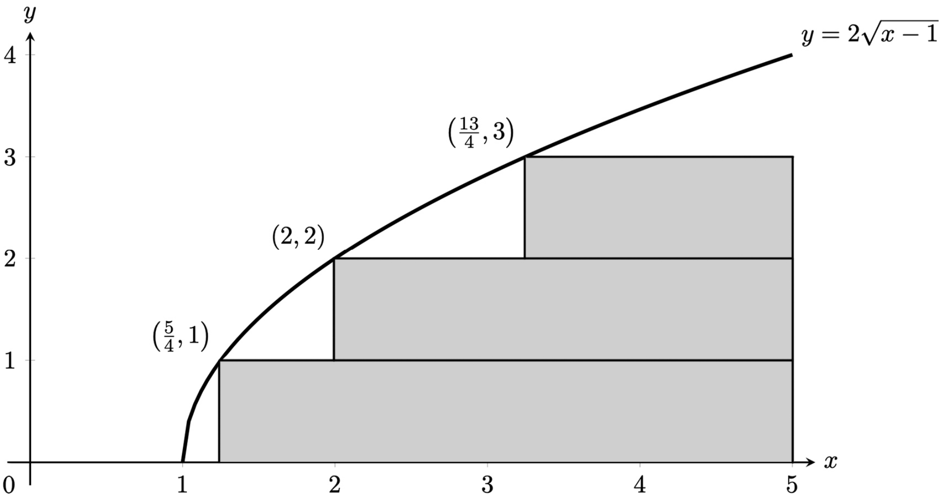
$$\begin{aligned}\text{actual} - \text{approximate} &= \frac{32}{3}-\frac{17}{2}\\ &= \frac{13}{6}\end{aligned}$$

Therefore, the percentage error is

$$\frac{\frac{13}{6}}{\frac{32}{3}}\times 100 \approx 20.3\%$$

Tip

- *The inverse function of* $y=2\sqrt{x-1}$ *is* $y=\frac{x^2}{4}+1$. *This will assist you in finding the length of the rectangles, for when* $x=1$, $y=\frac{5}{4}$, *when* $x=2$, $y=2$ *and when* $x=3$, $y=\frac{13}{4}$. *This is shown below:*



Question 20

Answer: B

Explanatory notes

Let $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)=\begin{bmatrix} x' \\ y' \end{bmatrix}$ and so $x'=-2x+1$, $y'=3y$.

Since the original function needs to be found from the transformed function, substitute x' and y' into the equation $y'=3\sqrt{4-x'}$ to obtain

$$3y=3\sqrt{4-(-2x+1)}$$
$$y=\sqrt{3+2x}$$

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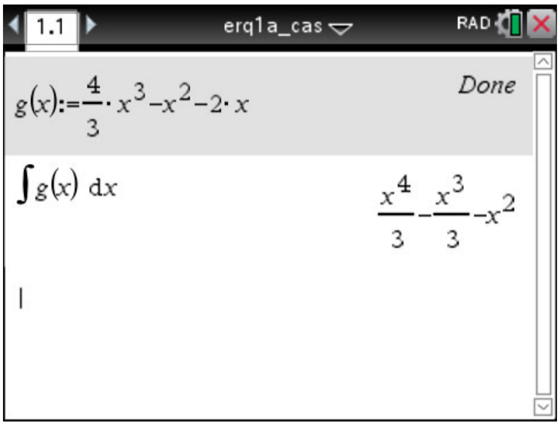
SECTION B

Question 1a.i.

Worked solution

Integrate by hand or use CAS to obtain $f(x)=\frac{1}{3}x^4-\frac{1}{3}x^3-x^2+c$, and apply the condition that $f(0)=-1$ to find the value of c.



Answer: $\frac{1}{3}x^4-\frac{1}{3}x^3-x^2-1$

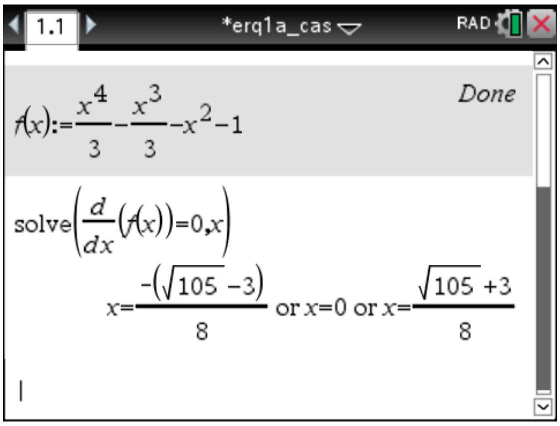
Mark allocation: 1 mark

- 1 mark for finding $f(x)=\frac{1}{3}x^4-\frac{1}{3}x^3-x^2+c$ and applying the condition to give the required answer

Question 1a.ii.

Worked solution

Use CAS to solve $f'(x)=0$ to find the stationary points.



Answer: $x=\dfrac{3\pm\sqrt{105}}{8}$

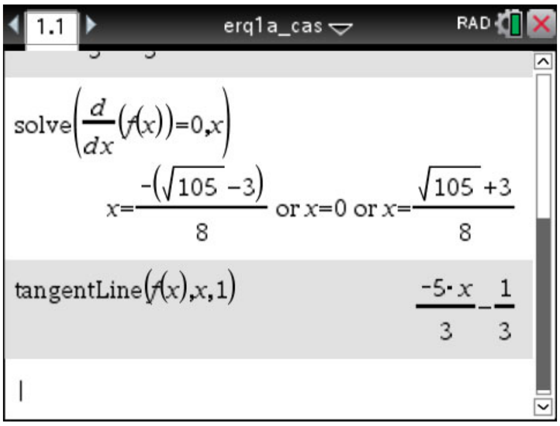
Mark allocation: 2 marks

- 1 mark for solving $f'(x)=g(x)=\dfrac{4}{3}x^3-x^2-2x=0$
- 1 mark for the correct answer

Question 1b.i.

Worked solution

Use CAS to find the equation of the tangent to $f(x)$ at $x=1$.



Answer: $y=-\dfrac{5}{3}x-\dfrac{1}{3}$

Mark allocation: 1 mark

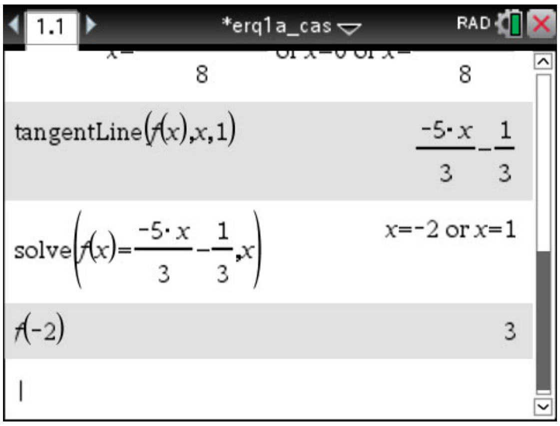
- 1 mark for the correct answer

Question 1b.ii.

Worked solution

Use CAS to solve $f(x)=-\dfrac{5}{3}x-\dfrac{1}{3}$ for x. It is found that $x=-2$ and $x=1$.

Since $f(-2)=3$, the coordinates of A are $(-2,3)$.



Answer: $(-2,3)$.

Mark allocation: 1 mark

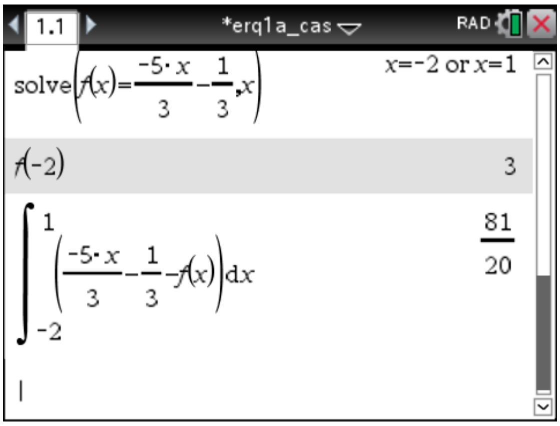
- 1 mark for the correct answer

Question 1b.iii.

Worked solution

The area bounded by the tangent l and the graph of $f(x)$ is equal to

$$\int_{-2}^{1}\left(-\frac{5}{3}x-\frac{1}{3}-f(x)\right)dx=\frac{81}{20}.$$



Answer: $\frac{81}{20}$

Mark allocation: 2 marks

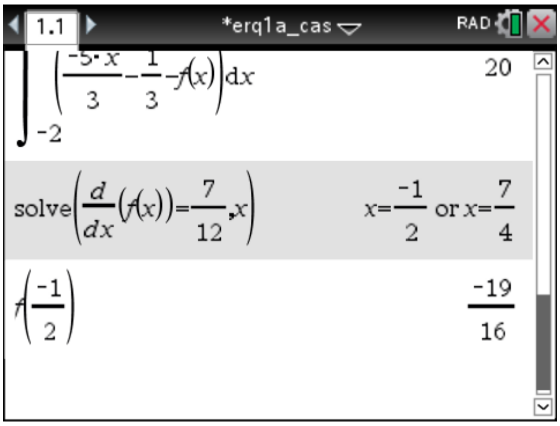
- 1 mark for setting up appropriate integral
- 1 mark for the correct answer

Question 1c.i.

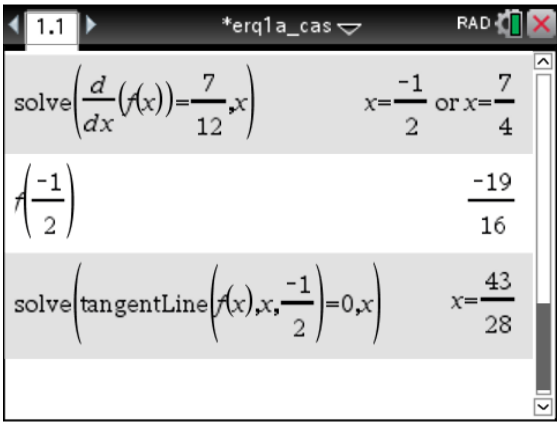
Worked solution

Using CAS to solve $f'(x)=\frac{7}{12}$ gives $x=-\frac{1}{2}$ and $\frac{7}{4}$

$f\left(-\frac{1}{2}\right)=-\frac{19}{16}$ and so the coordinates of B are $\left(-\frac{1}{2},-\frac{19}{16}\right)$.



Now use CAS to determine when the tangent line to the graph of f crosses the x-axis. The coordinates of C are $\left(\frac{43}{28},0\right)$.



Answer: $\left(-\frac{1}{2},-\frac{19}{16}\right)$ and $\left(\frac{43}{28},0\right)$

Mark allocation: 3 marks

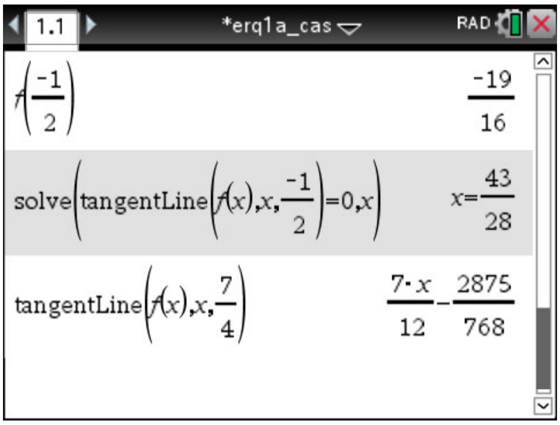
- 1 mark for solving $f'(x)=\frac{7}{12}$ and finding $x=-\frac{1}{2}$
- 1 mark deriving the coordinates of B
- 1 mark deriving the coordinates of C

Question 1c.ii.

Worked solution

The other tangent with gradient $\frac{7}{12}$ meets the graph of f when $x=\frac{7}{4}$.

Use CAS to find the equation of this tangent.



Answer: $y=\frac{7}{12}x-\frac{2875}{768}$

Mark allocation: 1 mark

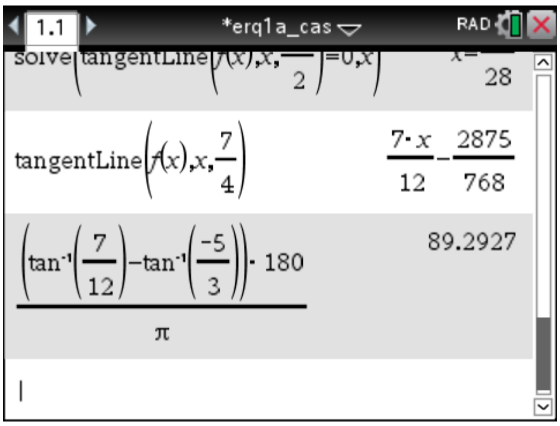
- 1 mark for the correct answer

Question 1d.

Worked solution

The angle θ between the two tangents is

$\theta=\tan^{-1}\left(\frac{7}{12}\right)-\tan^{-1}\left(-\frac{5}{3}\right)=89.29^{\circ}$.



Answer: 89.29°

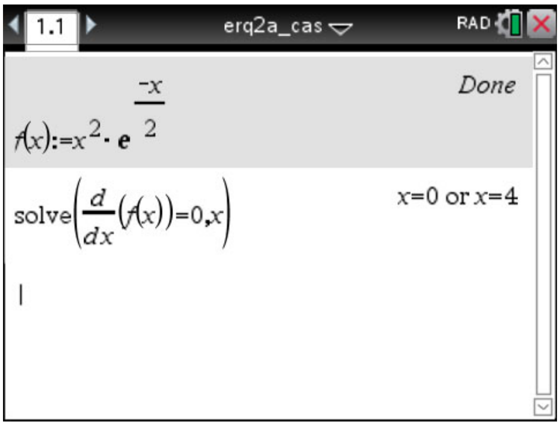
Mark allocation: 2 marks

- 1 mark for correct use of inverse tan formula to find the angle
- 1 mark for the correct answer

Question 2a.

Worked solution

Use CAS to solve $f'(x)=0$ and find that the turning points are $x=0$ and $x=4$.



Answer: $[0,4]$

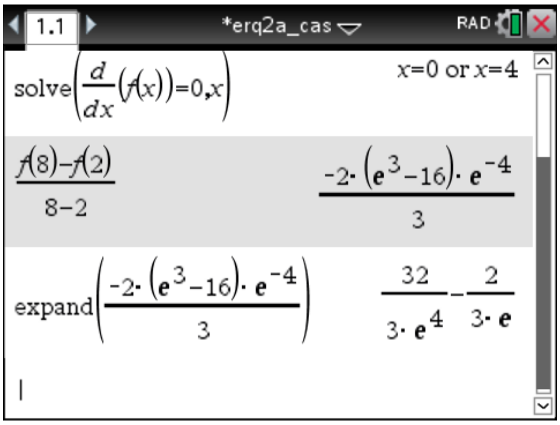
Mark allocation: 1 mark

- 1 mark for the correct answer

Question 2b.

Worked solution

Use CAS to perform computation.



$$\frac{f(8)-f(2)}{8-2}=\frac{32}{3e^4}-\frac{2}{3e}$$

Answer: $\frac{32}{3e^4}-\frac{2}{3e}$

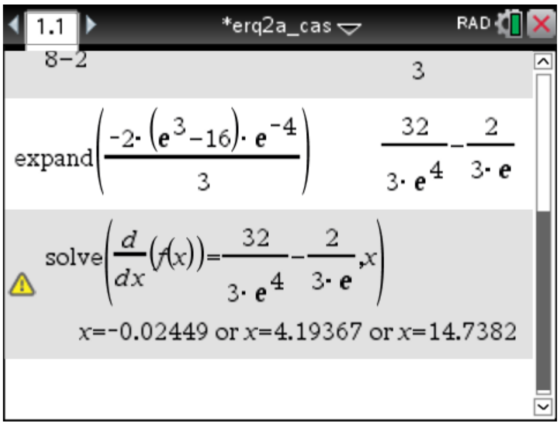
Mark allocation: 2 marks

- 1 mark for using the gradient formula
- 1 mark for the correct answer

Question 2c.

Worked solution

Use CAS to solve $f'(x)=\dfrac{32}{3e^4}-\dfrac{2}{3e}$.



Answer: $x = 4.194$

Mark allocation: 2 marks

- 1 mark for solving $f'(x)=\dfrac{32}{3e^4}-\dfrac{2}{3e}$
- 1 mark for the correct answer

Question 2d.i.

Worked solution

In order for $f(g(x))$ to be defined we require $\text{rang} \subseteq \text{dom}f = [0,10]$. Since $g(x)=x^2$ it must be the case that $D=\left[-\sqrt{10},\sqrt{10}\right]$.

Therefore, the domain of h' is $\left(-\sqrt{10},\sqrt{10}\right)$ as the derivative is not defined at the endpoints.

Answer: $\left(-\sqrt{10},\sqrt{10}\right)$

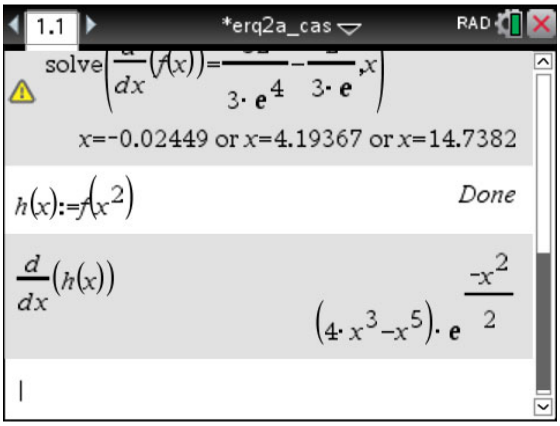
Mark allocation: 2 marks

- 1 mark for finding $D=\left[-\sqrt{10},\sqrt{10}\right]$
- 1 mark for the correct answer

Question 2d.ii.

Worked solution

Use CAS to find $h'(x) = \left(4x^3 - x^5\right)e^{-\frac{x^2}{2}}$.



Answer: $h'(x) = \left(4x^3 - x^5\right)e^{-\frac{x^2}{2}}$

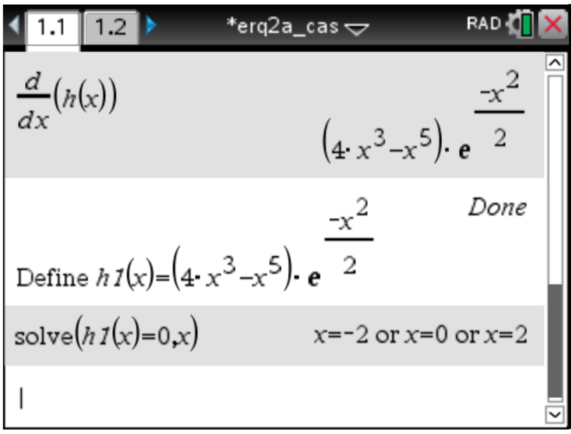
Mark allocation: 1 mark

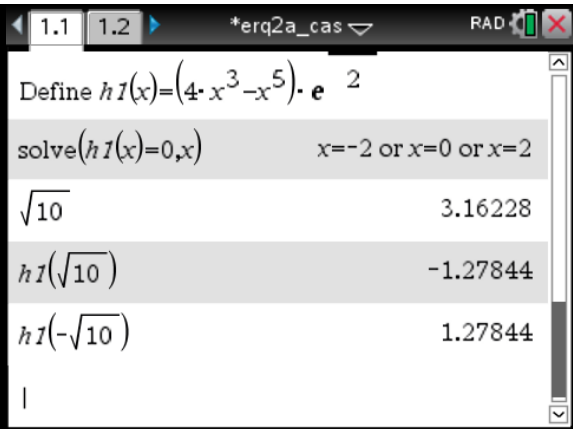
- 1 mark for the correct answer

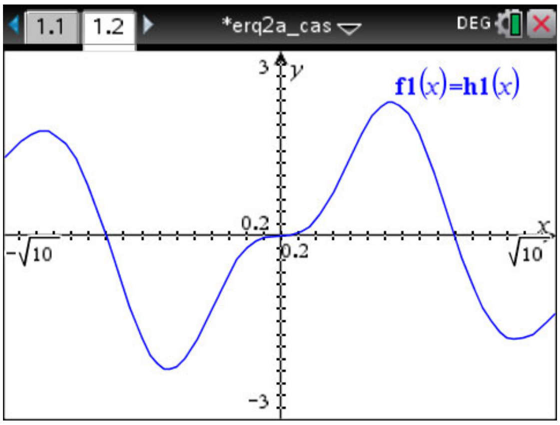
Question 2d.iii.

Worked solution

Use CAS to assist in graphing and to help find the coordinates of the endpoints.



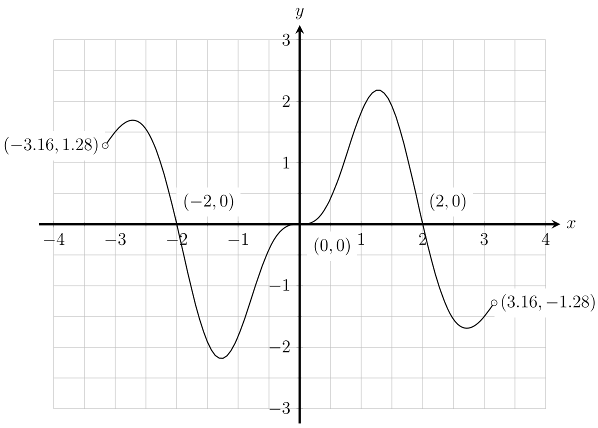




The x-intercepts are at (–2, 0) and (2, 0)

The endpoint coordinates are at (–3.16, 1.28) and (3.16, –1.28)

Answer:



Mark allocation: 3 marks

- 1 mark for a graph with the correct shape
- 1 mark for the endpoints correct
- 1 mark for the correct coordinates of axis intercepts

Question 3a.

Worked solution

The amplitude of f is 2, so the length of OC is 12 cm. The area $OABC$ is $15\times 12 = 180\text{ cm}^2$

Answer: 180 cm^2

Mark allocation: 1 mark

- 1 mark for the correct answer

Question 3b.

Worked solution

Note that the y-intercept of the parabola is $(0,8)$. Therefore

$$g(0) = 225a = 8 \Rightarrow a = \frac{8}{225}.$$

Answer: $\frac{8}{225}$

Mark allocation: 1 mark

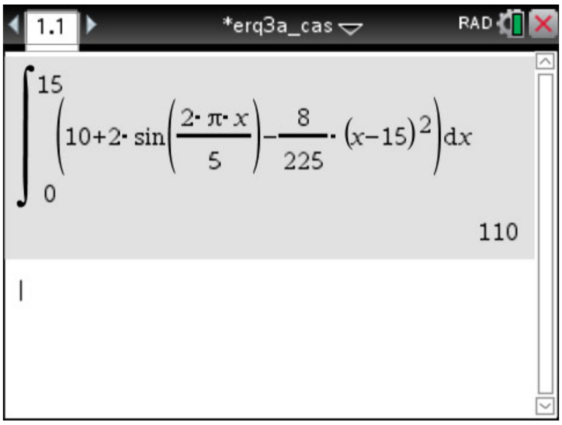
- 1 mark for the correct answer

Question 3c.

Worked solution

The area of the component is given by

$$\int_0^{15}\left(10+2\sin\left(\frac{2\pi x}{5}\right)-\frac{8}{225}(x-15)^2\right)dx = 110\text{ cm}^2$$



Answer: 110 cm^2

Mark allocation: 2 marks

- 1 mark for an appropriate integral
- 1 mark for the correct answer

Question 3d.

Worked solution

The distance between the two functions is $d(x)=10+2\sin\left(\frac{2\pi x}{5}\right)-\frac{8}{225}(x-15)^2$

Answer: $d(x)=10+2\sin\left(\frac{2\pi x}{5}\right)-\frac{8}{225}(x-15)^2=2\sin\left(\frac{2\pi x}{5}\right)-\frac{8}{225}x^2+\frac{16}{15}x+2$

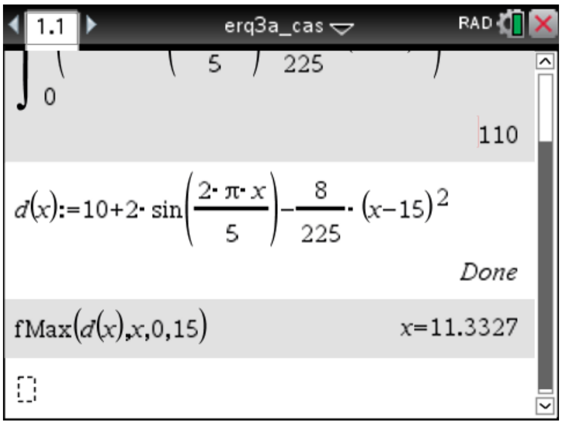
Mark allocation: 1 mark

- 1 mark for the correct answer

Question 3e.

Worked solution

Use CAS to find the maximum value of $d(x)$ on the interval $[0,15]$. It is found that $x=11.333$ to three decimal places.



Answer: $x=11.333$

Mark allocation: 1 mark

- 1 mark for the correct answer

Question 3f.

Worked solution

The average value is $\frac{1}{15}\int_0^{15}\left(10+2\sin\left(\frac{2\pi x}{5}\right)-\frac{8}{225}(x-15)^2\right)dx=\frac{110}{15}=\frac{22}{3}$

Answer: $\frac{22}{3}$

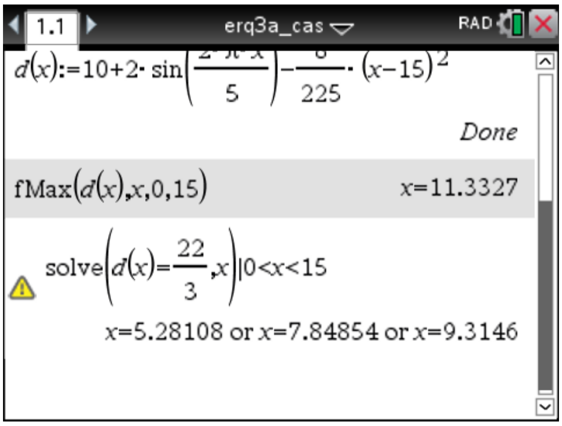
Mark allocation: 1 mark

- 1 mark for the correct answer

Question 3g.

Worked solution

Use CAS to solve the equation $d(x) = \frac{22}{3}$.



Answer: $x = 5.281$, $x = 7.849$ and $x = 9.315$

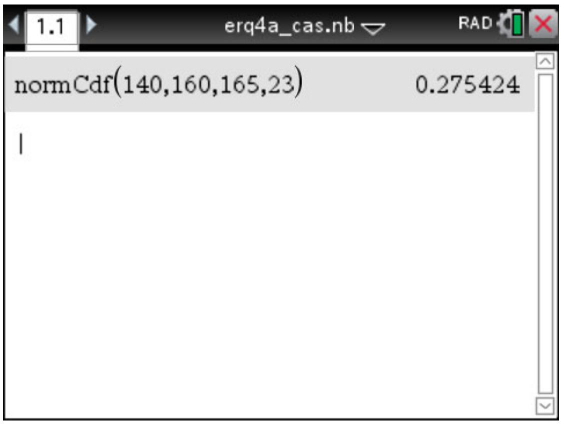
Mark allocation: 3 marks

- 1 mark for solving equation $d(x) = \frac{22}{3}$
- 2 marks for all three correct answers

Question 4a.

Worked solution

Use CAS to find the required probability.



Answer: 0.2754

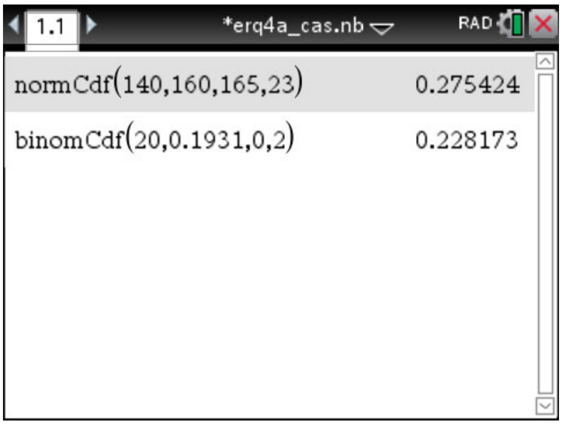
Mark allocation: 1 mark

- 1 mark for the correct answer

Question 4b.i.

Worked solution

Let $Y \sim Bi\left(20, 0.1931\right)$. Then $\Pr(Y \leq 2) = 0.228$.



Answer: 0.228

Mark allocation: 2 marks

- 1 mark for the correct binomial distribution
- 1 mark for the correct answer

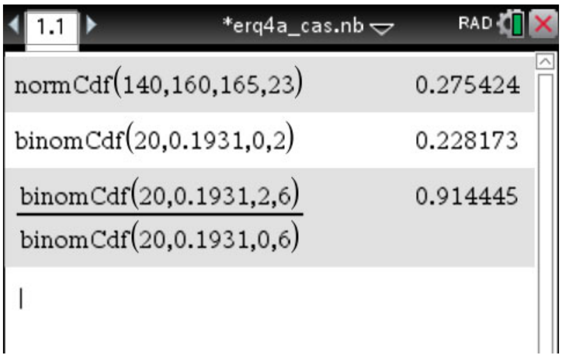
Question 4b.ii

Worked solution

$$\Pr\left(\hat{P} \geq 0.1 \mid \hat{P} \leq 0.3\right) = \frac{\Pr(0.1 \leq \hat{P} \leq 0.3)}{\Pr(\hat{P} \leq 0.3)}$$

$$= \frac{\Pr(2 \leq Y \leq 6)}{\Pr(Y \leq 6)}$$

$$= 0.914$$



Answer: 0.914.

Mark allocation: 3 marks

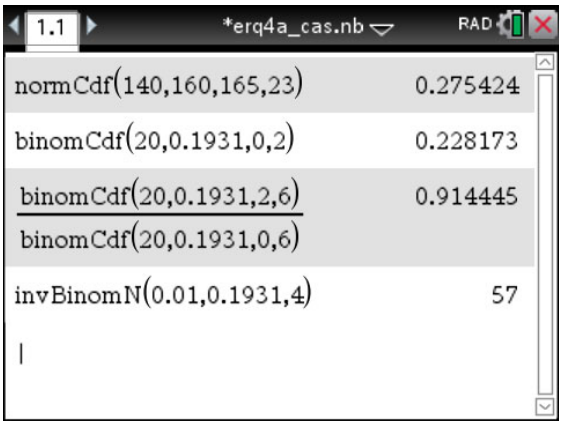
- 1 mark for the correct conditional probability (first line)
- 1 mark for the conversion to Y
- 1 mark for the correct answer

Question 4b.iii.

Worked solution

Let $W \sim Bi(n, 0.1931)$. We want to find n such that $\Pr(W \geq 5) > 0.99$. This is equivalent to $\Pr(W < 5) < 0.01$.

Use the inverse binomial function of the CAS calculator to find that $n = 57$.



Answer: 57

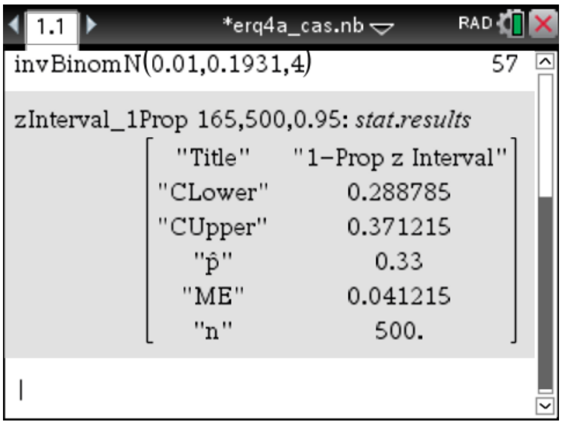
Mark allocation: 2 marks

- 1 mark for the statement equivalent to $W \sim Bi(n, 0.1931)$
- 1 mark for the correct answer

Question 4c.

Worked solution

Use the CAS calculator to determine the confidence interval.



Answer: $(0.289, 0.371)$

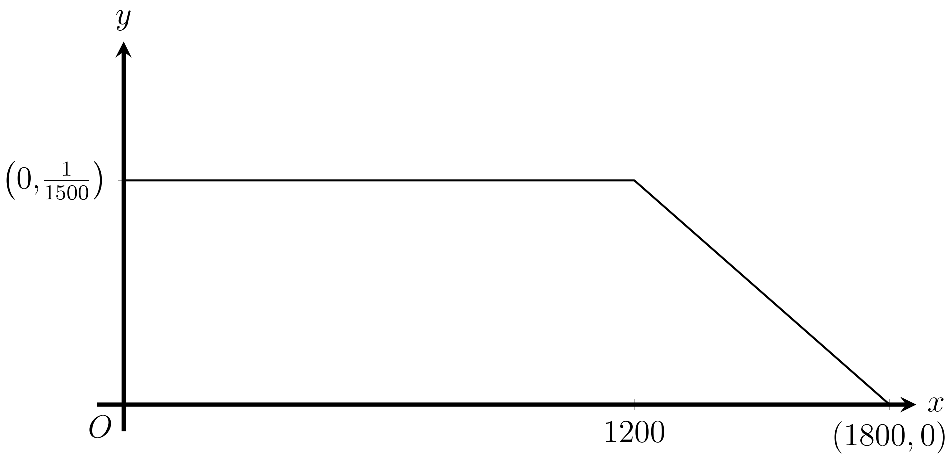
Mark allocation: 1 mark

- 1 mark for correct answer

Question 4d.i

Worked solution

The graph is plotted below.



Mark allocation: 2 marks

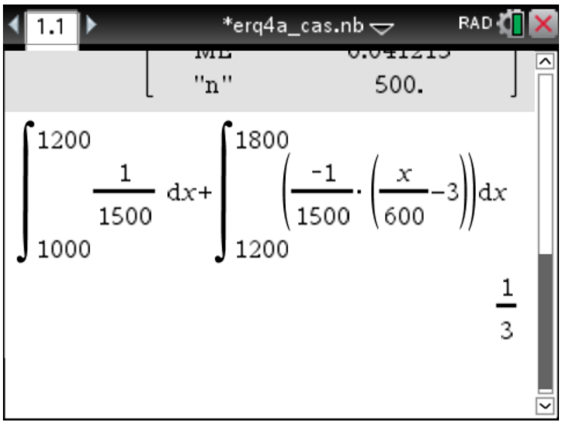
- 1 mark for the correct shape (two straight lines)
- 1 mark for the correct axis coordinates

Question 4d.ii.

Worked solution

Use CAS to evaluate:

$$\Pr(X > 1000) = \int_{1000}^{1200} \frac{1}{1500}\,dx + \int_{1200}^{1800} -\frac{1}{1500}\left(\frac{x}{600} - 3\right)dx = \frac{1}{3}$$



Alternatively,

$$\begin{aligned}\Pr(X > 1000) &= 1 - \Pr(X < 1000)\\ &= 1 - \int_{0}^{1000} \frac{1}{1500}\,dx\\ &= 1 - \frac{1000}{1500}\\ &= 1 - \frac{2}{3}\\ &= \frac{1}{3}\end{aligned}$$

Answer: $\frac{1}{3}$

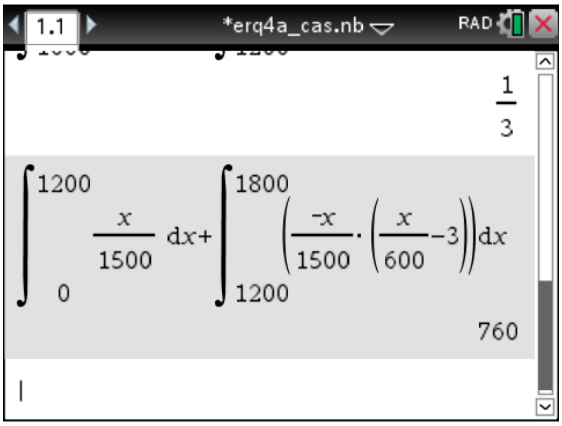
Mark allocation: 1 mark

- 1 mark for the correct answer

Question 4d.iii.

Worked solution

$$\mathrm{E}(X) = \int_0^{1200} \frac{x}{1500}\,dx + \int_{1200}^{1800} -\frac{x}{1500}\left(\frac{x}{600} - 3\right)dx = 760$$



Answer: 760 m

Mark allocation: 2 marks

- 1 mark for the integrals
- 1 mark for the correct answer

Question 4e.

Worked solution

Let H be the event that a randomly selected apple is hybrid and A be the event that a randomly selected apple is grown at an altitude greater than 1000 m.

Then $\Pr(H) = 0.4$ and $\Pr(A) = \frac{1}{3}$ and so $\Pr(A') = 1 - \frac{1}{3} = \frac{2}{3}$.

Now $\Pr(H \mid A) = 0.15 \Rightarrow \Pr(H \cap A) = 0.15 \times \frac{1}{3} = 0.05$.

Therefore, $\Pr(H \cap A') = 0.4 - 0.05 = 0.35$.

Finally, $\Pr(H \mid A') = \dfrac{\Pr(H \cap A')}{\Pr(A')} = \dfrac{0.35}{2/3} = 0.525$.

Answer: 0.525

Mark allocation: 2 marks

- 1 mark for recognising conditional probability and finding $\Pr(H \cap A) = 0.05$
- 1 mark for the correct answer

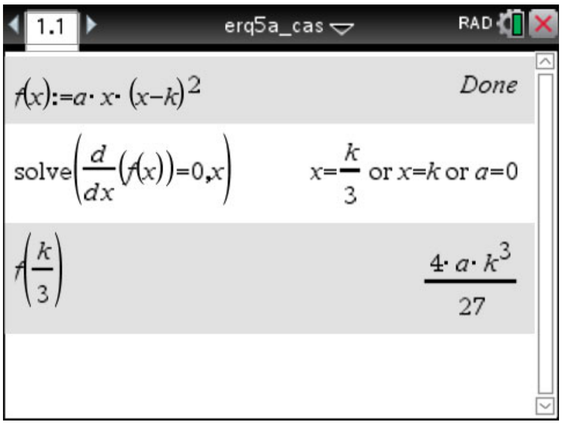
Question 5a.

Worked solution

Use CAS to find that $f'(x) = 0 \Rightarrow x = \dfrac{k}{3}$ or $x = k$.

The turning point at $x = k$ is the local minimum.

Use CAS to evaluate $f\left(\dfrac{k}{3}\right) = \dfrac{4ak^3}{27}$.



Answer: $\left(\dfrac{k}{3}, \dfrac{4ak^3}{27}\right)$

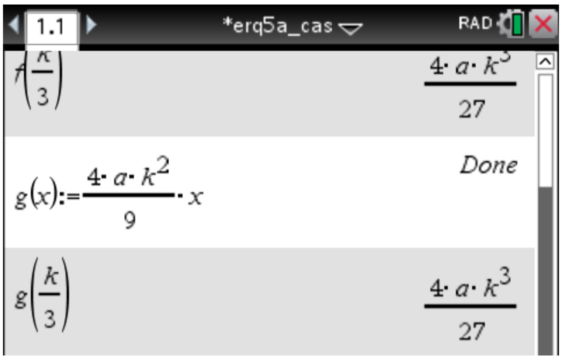
Mark allocation: 2 marks

- 1 mark for stating solving $f'(x) = 0$
- 1 mark for the correct coordinates

Question 5b.

Worked solution

Use CAS to confirm that $g\left(\frac{k}{3}\right)=\frac{4ak^3}{27}$.



Answer: $g\left(\frac{k}{3}\right)=\frac{4ak^3}{27}$

Mark allocation: 1 mark

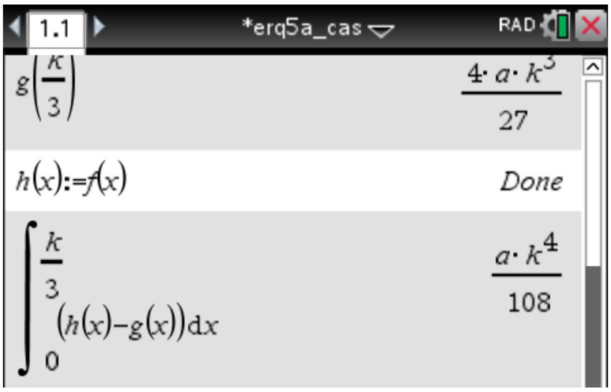
- 1 mark for confirming that $g\left(\frac{k}{3}\right)=\frac{4ak^3}{27}$

Question 5c.

Worked solution

Use CAS to find that the area bounded by the graphs of h and g is

$\int_0^{\frac{k}{3}}\big(h(x)-g(x)\big)dx=\frac{ak^4}{108}$.



Answer: $\frac{ak^4}{108}$

Mark allocation: 2 marks

- 1 mark for the integral $\int_0^{\frac{k}{3}}\big(h(x)-g(x)\big)dx$
- 1 mark for the answer $\frac{ak^4}{108}$

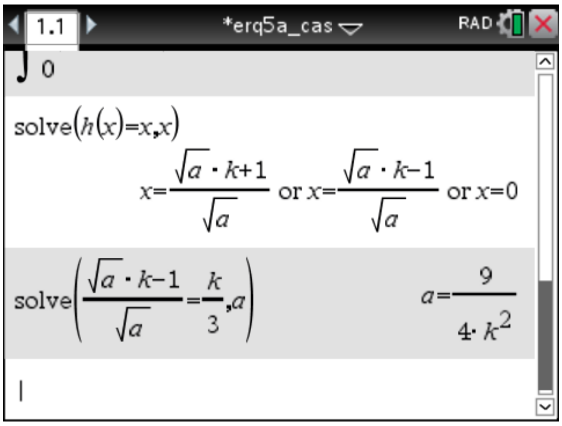
Question 5d.

Worked solution

The graphs of $y = h(x)$ and $y = x$ meet when $x = 0$ and when $x = \dfrac{\sqrt{a}k \pm 1}{\sqrt{a}}$.

For the graphs of $y = h(x)$ and $y = x$ to meet exactly twice on the interval $\left[0, \dfrac{k}{3}\right]$ with the value of a as large as possible, it must be the case that $\dfrac{\sqrt{a}k - 1}{\sqrt{a}} = \dfrac{k}{3}$.

Therefore $a = \dfrac{9}{4k^2}$



Mark allocation: 2 marks

- 1 mark for finding $x = 0$ and $x = \dfrac{\sqrt{a}k \pm 1}{\sqrt{a}}$
- 1 mark for solving $\dfrac{\sqrt{a}k - 1}{\sqrt{a}} = \dfrac{k}{3}$

Question 5e.

Worked solution

If h and h^{-1} meet at O when $x = \frac{k}{3}$, then the area bounded by the graphs is

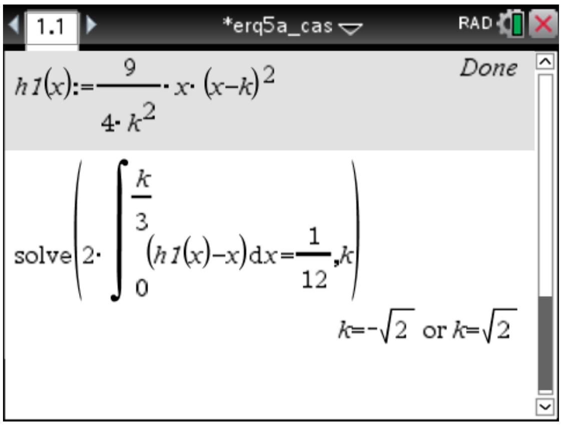
$$\int_0^{\frac{k}{3}} \left(h(x) - h^{-1}(x)\right) dx = 2\int_0^{\frac{k}{3}} \left(h(x) - x\right) dx$$

So solve

$$2\int_0^{\frac{k}{3}} \left(h(x) - x\right) dx = \frac{1}{12}$$

with $a = \frac{9}{4k^2}$.

It is found that $k = \sqrt{2}$.



Answer: $k = \sqrt{2}$

Mark allocation: 3 marks

- 1 mark for recognising that $\int_0^{\frac{k}{3}} \left(h(x) - h^{-1}(x)\right) dx = 2\int_0^{\frac{k}{3}} \left(h(x) - x\right) dx$
- 1 mark for solving $2\int_0^{\frac{k}{3}} \left(h(x) - x\right) dx = \frac{1}{12}$
- 1 mark for the answer $k = \sqrt{2}$

END OF WORKED SOLUTIONS