YEAR 12 *Trial Exam Paper*

2019

MATHEMATICAL METHODS

Written examination 1

Reading time: 15 minutes
Writing time: 1 hour

STUDENT NAME:

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials provided

- Question and answer book of 15 pages
- Formula sheet
- Working space is provided throughout this book.

Instructions

- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination

- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination.

Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

a. If $y = (2x+1)\log_e(2x+1)$, find $\frac{dy}{dx}$.

2 marks

b. Let $f(x) = \cos(4x^2)$.

Evaluate $f'\left(\frac{\sqrt{\pi}}{4}\right)$.

2 marks

Question 2 (3 marks)

Let $\frac{dy}{dx} = 2 - e^{-x}$.

Given that $y = 4 - \frac{1}{e^2}$ when $x = 2$, find y in terms of x.

Question 3 (4 marks)

A bag contains two blue blocks and one red block. Two blocks are randomly drawn from the bag without replacement. Each block is equally likely to be drawn.

Let X be the random variable that represents the number of red blocks drawn from the bag.

a. Find $\Pr(X = 1)$.

1 mark

b. Find $\mathrm{E}(X)$.

1 mark

c. Find $\mathrm{Var}(X)$.

2 marks

Question 4 (3 marks)

Let $f:\left(-\frac{1}{2},1\right]\to R,\ f(x)=2(x-1)^2-3$.

a. State the range of f.

1 mark

b. Find the rule for f^{-1}.

2 marks

Question 5 (4 marks)

Let $f(x) = 4\sin^3(x) - 4\sin^2(x) - 3\sin(x) + 3$.

a. Show that $f(x) = \left(4\sin^2(x) - 3\right)(\sin(x) - 1)$.

1 mark

b. Hence, solve $4\sin^3(x) - 4\sin^2(x) - 3\sin(x) + 3 = 0$, for $x \in [0, 2\pi]$.

3 marks

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Question 6 (6 marks)

Let $f(x) = \log_2(x+1) - \log_2(4-x)$.

a. State the domain of *f*.

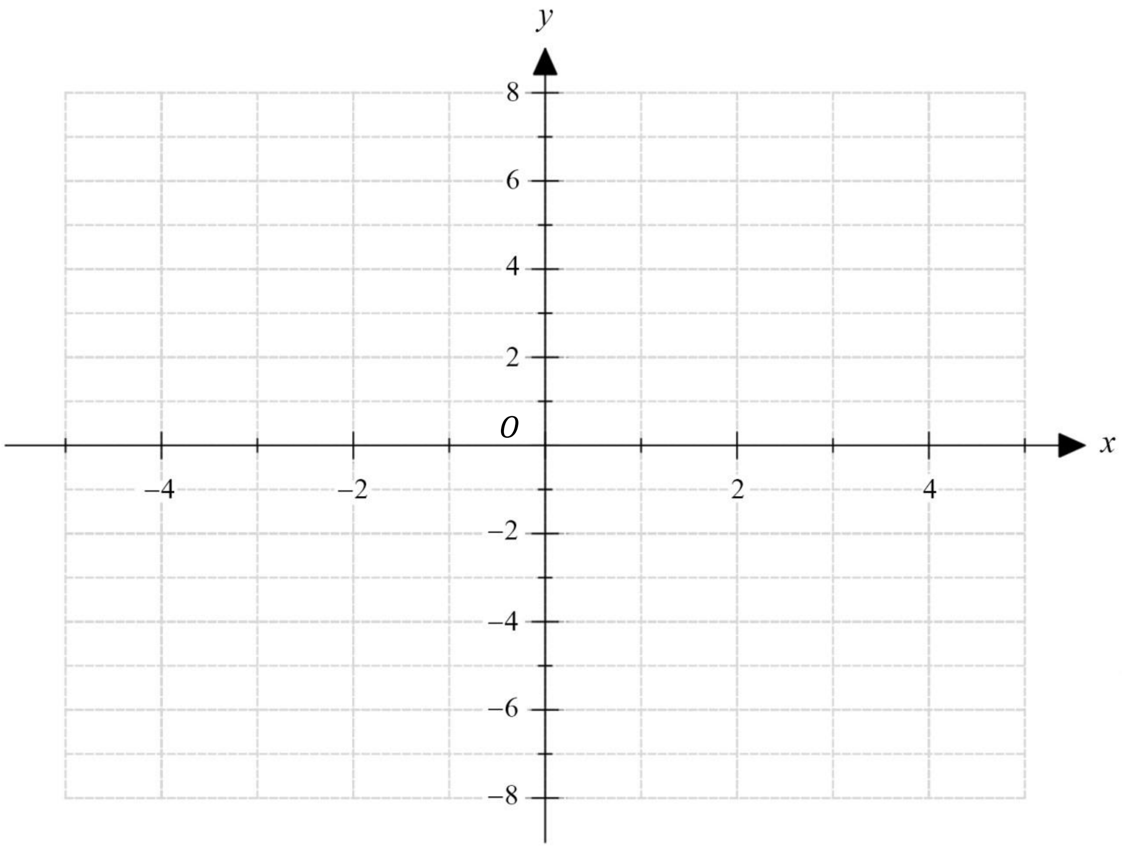
1 mark

b. Solve the equation $\log_2(x+1) - \log_2(4-x) = 0$.

2 marks

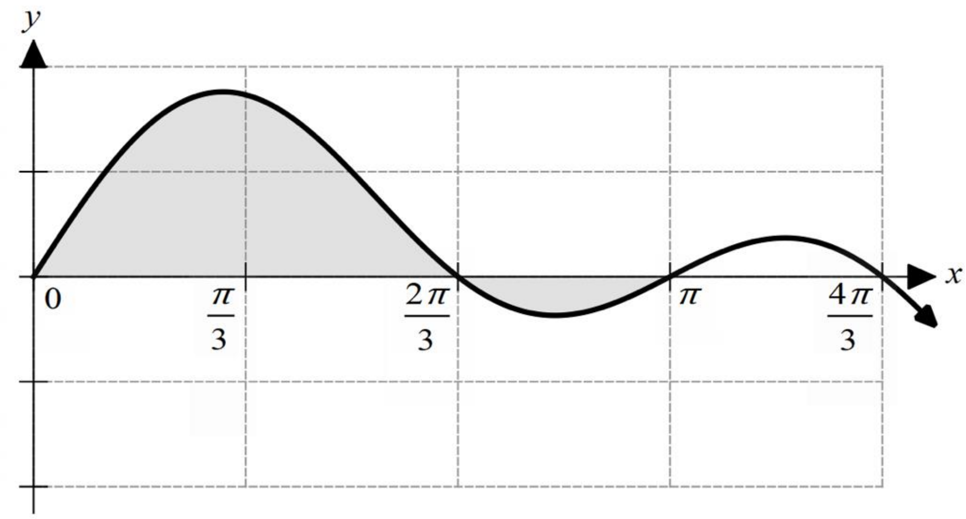
c. Sketch the graph of the function $y = f(x)$ on the axes below. Label any asymptotes with the appropriate equation and label the axis intercepts with their coordinates.

3 marks



Question 7 (3 marks)

The graph of $y = \sin(x) + \sin(2x)$ is shown below.



Calculate the area of the region bounded by the curve and the x-axis between $x = 0$ and $x = \pi$.

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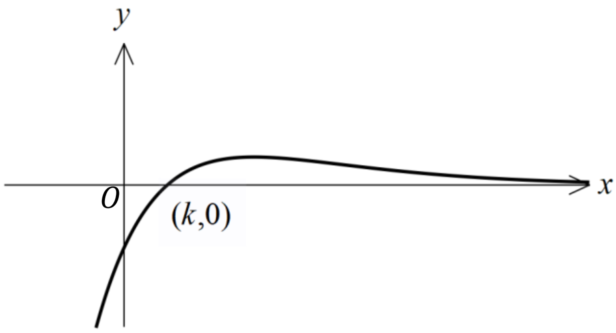
Question 8 (7 marks)

a. Let $f: R \to R,\ f(x) = (x-k)e^{-x}$, where $k \in R$.



i. Show that $f'(x) = -(x-1-k)e^{-x}$.

1 mark

ii. The graph of f has a stationary point at P. Find the coordinates of P in terms of k.

2 marks

Question 8 – continued

b. The continuous random variable X has the probability density function $g:[0,\infty)\to R,\ g(x)=xe^{-x}$, and has an expected value of 2 and variance of 2.

The continuous random variable Y has the probability density function $h:[0,\infty)\to R,\ h(x)=9xe^{-3x}$.

i. The transformation $T:R^2\to R^2$ with rule $T\left(\begin{bmatrix}x\\y\end{bmatrix}\right)=\begin{bmatrix}a&0\\0&b\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$ maps the graph of $y=g(x)$ onto the graph of $y=h(x)$.

Find the values of a and b.

2 marks

ii. Find E(Y).

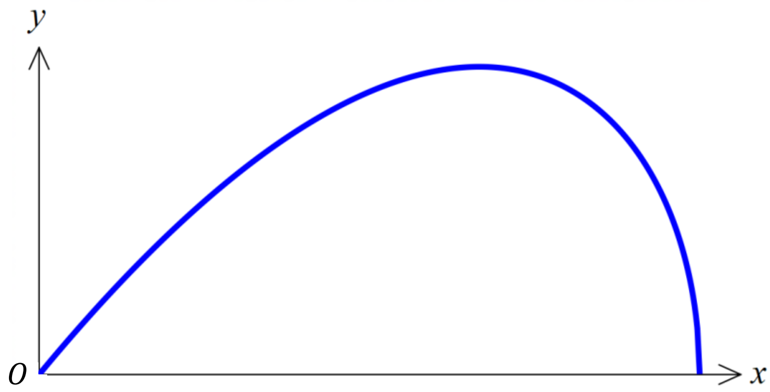
1 mark

iii. Find Var(Y).

1 mark

Question 9 (6 marks)

Let $f: [0, a) \rightarrow R,\ f(x) = \sqrt{-x^3 + kx^2}$, where $k \in R^+$.



a. Find the maximal value of a in terms of k.

2 marks

b. P is a point on the graph $y = f(x)$ with coordinates $(p,\ f(p))$.

i. Show that the length of the chord OP is given by the expression $\sqrt{-p^3 + (k+1)p^2}$.

1 mark

ii. For $0 < k \leq 2$, the chord OP is longest when $p = k$.

Determine the value of p for which the length of the chord OP is longest in terms of k when $k > 2$.

3 marks

END OF QUESTION AND ANSWER BOOK