

MATHEMATICAL METHODS

Units 3 & 4 – Written examination 2



2018 Trial Examination

SOLUTIONS

SECTION A

Question 1

Answer: B

Explanation:

period = $\frac{2\pi}{n}$ and range = $[1 - 2, 1 + 2]$

Question 2

Answer: A

Explanation:

Define $f(x)=x^2-1$ Done

Define $g(x)=\frac{3-x}{4}$ Done

$g(f(2))$ 0

Question 3

Answer: C

Explanation:

$$RB + BR + RR = \left(\frac{5}{9} \times \frac{4}{8}\right) + \left(\frac{4}{9} \times \frac{5}{8}\right) + \left(\frac{5}{9} \times \frac{4}{8}\right)$$

Question 4

Answer: E

Explanation:

The interval where the tangents to the graph have negative gradients.

Question 5

Answer: D

Explanation:

$$y + 5 = b^{a - \frac{x}{2}} \rightarrow \log_b(y + 5) = a - \frac{x}{2} \rightarrow \frac{x}{2} = a - \log_b(y + 5) \rightarrow x = 2(a - \log_b(y + 5))$$

Question 6

Answer: A

Explanation:

Define $f(x) = 5 \cdot x + x^3$ Done

 solve $\left(\frac{f(a) - f(0)}{a - 0} = 8, a \right)$ $a = -\sqrt{3}$ or $a = \sqrt{3}$

Question 7

Answer: E

Explanation:

$$x' = -x, y' = \frac{1}{4}y \rightarrow x = -x', y = 4y'$$

$$4y' = \sin\left(3\left(-x' - \frac{\pi}{2}\right)\right) \rightarrow y = -\frac{1}{4}\sin\left(3x + \frac{\pi}{2}\right)$$

Question 8

Answer: B

Explanation:

$$\text{solve}\left(\cos(2 \cdot x) = \frac{1}{2}, x\right) | -\pi \leq x \leq \pi$$

$$x = \frac{-5 \cdot \pi}{6} \text{ or } x = \frac{-\pi}{6} \text{ or } x = \frac{\pi}{6} \text{ or } x = \frac{5 \cdot \pi}{6}$$

$$\frac{-5 \cdot \pi}{6} + \frac{-\pi}{6} + \frac{\pi}{6} + \frac{5 \cdot \pi}{6} \quad 0$$

Question 9

Answer: C

Explanation:

$$\text{Define } h(x) = \frac{1}{(x-1)^2} \quad \text{Done}$$

$$h(x) \cdot h(-x) = h(x^2) \quad \text{true}$$

Question 10

Answer: A

Explanation:

$$\text{Var}(X) = E(X^2) - (E(X))^2 = 4 - 5p - (2 - 3p)^2$$

Question 11

Answer: E

Explanation:

$$\text{Solve } 20p = 2\sqrt{20p(1-p)} \text{ on CAS}$$

Question 12

Answer: D

Explanation:

The area is below the x-axis between -2 and $\frac{1}{3}$, hence the negative sign.

Question 13

Answer: A

Explanation:

$$\text{solve} \left(\int_0^{\pi} (k \cdot \sin(x)) dx = 1, k \right) | 0 < k < 2 \quad k = \frac{1}{2}$$

Question 14

Answer: B

Explanation:

Solve $f'(x) = 0$ for each option

Question 15

Answer: E

Explanation:

$$\text{solve} \left((1-p)^4 = \frac{1}{625} \cdot p \right) \quad p = \frac{4}{5} \text{ or } p = \frac{6}{5}$$

$$\text{binomCdf} \left(4, \frac{4}{5}, 3, 4 \right) \quad 0.8192$$

Question 16

Answer: C

Explanation:

$$\text{solve}\left(4+4 \cdot (2 \cdot p-1) \cdot (1-p) \geq 0, p\right) \quad 0 \leq p \leq \frac{3}{2}$$

Question 17

Answer: C

Explanation:

$$\text{invNorm}(0.1488, 16, 2) \quad 13.9168122047$$

Question 18

Answer: B

Explanation:

Since 6 is not included in the domain it cannot yield the maximum value.

Question 19

Answer: E

Explanation:

$$\text{solve}\left(\frac{m}{2} \neq \frac{4}{m-2}, m\right) \quad m \neq -2 \text{ and } m \neq 4$$

Question 20

Answer: A

Explanation:

$$\frac{1}{4-0} \cdot \int_0^4 (x \cdot \sqrt{x}) dx \quad \frac{16}{5}$$

SECTION B

Question 1

a. $\left(\frac{-\sqrt{6}}{6}, \frac{-\sqrt{6}}{9}\right)$ and $\left(\frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{9}\right)$

Define $f(x) = x - 2 \cdot x^3$	Done
solve $\left(\frac{d}{dx}(f(x)) = 0, x\right)$	$x = \frac{-\sqrt{6}}{6}$ or $x = \frac{\sqrt{6}}{6}$
$f\left(\frac{-\sqrt{6}}{6}\right)$	$\frac{-\sqrt{6}}{9}$
$f\left(\frac{\sqrt{6}}{6}\right)$	$\frac{\sqrt{6}}{9}$

3 marks

b.

i. $(-2, 14)$ and $(2, -14)$
 $y = -7x + k \rightarrow 14 = 14 + k \rightarrow k = 0$
 Equation : $7x + y = 0$

2 marks

ii. $20\sqrt{2}$ units

$\sqrt{28^2 + 4^2}$	$20 \cdot \sqrt{2}$
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1 mark

c.

i. The 2 points will be $C(-2, 8k - 2)$ and $D(2, 2 - 8k)$
 $y = (1 - 4k)x + c \rightarrow 8k - 2 = -2(1 - 4k) + c \rightarrow c = 0$
 Equation : $y = (1 - 4k)x$

2 marks

ii. $-1 = 5(1 - 4k) \rightarrow k = \frac{3}{10}$

1 mark

d.

$$\begin{aligned} \text{i. } -a &= a - ka^3 \\ -2 &= -ka^2 \\ a &= \sqrt{\frac{2}{k}} \end{aligned}$$

2 marks

ii.

$$\int_{-\sqrt{\frac{2}{k}}}^0 (-x - x + k \cdot x^3) dx + \int_0^{\sqrt{\frac{2}{k}}} (x - k \cdot x^3 + x) dx$$

$\frac{2}{k}$

2 marks

Question 2

$$\begin{aligned} \text{a. } \int_0^2 \frac{ct}{2} dt + \int_2^6 \frac{c}{4} (10 - t) dt &= 1 \\ \left[\frac{ct^2}{4} \right]_0^2 + \left[\frac{c}{4} \left(10t - \frac{t^2}{2} \right) \right]_2^6 &= 1 \\ c + \frac{c}{4} (60 - 18 - 20 + 2) &= 1 \rightarrow c = \frac{1}{7} \end{aligned}$$

3 marks

$$\begin{aligned} \text{b. } \int_1^2 \frac{t}{14} dt + \int_2^5 \frac{(10-t)}{28} dt \\ = \left[\frac{t^2}{28} \right]_1^2 + \left[\frac{20t - t^2}{56} \right]_2^5 \\ = \frac{3}{28} + \frac{39}{56} = \frac{45}{56} \end{aligned}$$

2 marks

c. This is a conditional probability.

$$\frac{\Pr(T \leq 1 \cap T \leq 5)}{\Pr(T \leq 5)} = \frac{\Pr(T \leq 1)}{\Pr(T \leq 5)} = \frac{\Pr(T \leq 1)}{\Pr(T \leq 1) + \Pr(1 \leq T \leq 5)} = \frac{\frac{1}{28}}{\frac{1}{28} + \frac{45}{56}} = \frac{2}{47}$$

2 marks

d. $\Pr(0 \leq T \leq 2) = \frac{1}{7}$
 $\Pr(2 \leq T \leq a) = \frac{1}{7}$

$$\text{solve} \left(\int_2^a \left(\frac{1}{7} \cdot (10-t) \right) dt = \frac{1}{7}, a \right)$$

$$a = 2.51668522645 \text{ or } a = 17.4833147735$$

$$a = 2.5167$$

3 marks

e.

i. $Bi\left(7, \frac{3}{7}\right)$
 $\Pr(T \geq 4) = \text{binomcdf}\left(7, \frac{3}{7}, 4\right) = 0.8734$

2 marks

ii. $\frac{3}{7} \times \frac{3}{7} \times \frac{3}{7} = \frac{27}{343}$

1 mark

f. $\Pr(T \geq 13) = 0.2312 \rightarrow \Pr\left(Z \geq \frac{2}{b}\right) = 0.2312$
 $\frac{2}{b} = \text{invnorm}(1 - 0.2312, 0, 1) \rightarrow \frac{2}{b} = 0.73490 \rightarrow b = 2.72 \text{ days}$

3 marks

Question 3

a. $a = 3, b = -1, c = -3$

Dilation by a factor of 3 from the x-axis, horizontal translation 1 unit to the left and vertical translation of 3 units down.

1 mark for each

b. $x = 3 \times 2^{y+1} - 3 \rightarrow \frac{x+3}{3} = 2^{y+1} \rightarrow y + 1 = \log_2\left(\frac{x+3}{3}\right) \rightarrow y = \log_2\left(\frac{x+3}{3}\right) - 1$
 $f^{-1}(x) = \log_2\left(\frac{x+3}{3}\right) - 1, \text{ Domain: } (-3, \infty)$

2 marks

c. 2.60 square units

$$\int_{-1}^0 \left(3 \cdot 2^{x+1} - 3 - \log_2 \left(\frac{x+3}{3} \right) + 1 \right) dx$$

2.60085516211

2 marks

d.

- i. Range of $f: (-3, \infty)$, Domain of $g: \mathbb{R} \setminus \{0\}$
 R_f is not part of D_g , hence $g(f(x))$ is not defined

2 marks

- ii. $f(g(x)) = f\left(\frac{1}{x}\right) = 3 \times 2^{\frac{1}{x}+1} - 3$
 Domain of $f(g(x)): \mathbb{R} \setminus \{0\}$

2 marks

e.

- i. $\text{tangentLine}(f(x), x, -3)$
 $y = 0.52x - 0.69$

2 marks

- ii. $\tan(\theta) = 0.52 \rightarrow \theta = 27.5^\circ$

1 mark

- iii. $k = 1.837$

$$\text{tangentLine}\left(3 \cdot 2^{x+1} - 3, x, -3\right)$$

$0.5199 \cdot x - 0.6904$

$$\text{tangentLine}\left(3 \cdot 2^{x+1} - 3, x, k\right)$$

$$4.159 \cdot (2.)^k \cdot x - 4.159 \cdot ((k-1.443) \cdot (2.)^k + 0.7)$$

 $\text{solve}(-0.39 = 4.1588830833597 \cdot (2.)^k \cdot 0.5$
 $k = -2.997 \text{ or } k = 1.837$

Substitute (0.57, -0.39) into the tangent line at $x = k$ obtained on CAS

3 marks

Question 4

a. $\text{Max} = 45 + 30 = 75 \text{ m}$, $\text{Min} = 45 - 30 = 15 \text{ m}$

1 mark for each

b.

i. Period, $T = \frac{2\pi}{\frac{\pi}{3}} = 6 \text{ minutes}$

Ride finishes in 12 minutes

2 marks

ii. After 3 minutes and 9 minutes

Define $h(t) = 45 - 30 \cdot \cos\left(\frac{\pi \cdot t}{3}\right)$ Done

solve $(h(t) = 75, t) | 0 \leq t \leq 12$ $t = 3$ or $t = 9$

2 marks

c.

i.

$\frac{d}{dt}(h(t))$ $10 \cdot \pi \cdot \sin\left(\frac{\pi \cdot t}{3}\right)$

1 mark

ii. $t = 3, 6, 9$

solve $\left(\frac{d}{dt}(h(t)) = 0, t\right) | 0 \leq t \leq 12$

$t = 0$ or $t = 3$ or $t = 6$ or $t = 9$ or $t = 12$

2 marks

d. $\tan^{-1}\left(\frac{75}{350}\right) = 12.09^\circ$

1 mark

e. $CF = \frac{75}{\tan \alpha}$

1 mark

f. $CF = \frac{75}{\tan(26.09)} = 153.16 \text{ m}$

2 marks