

THE SCHOOL FOR EXCELLENCE (TSFX)

VCE MATHEMATICAL METHODS UNITS 3 & 4

WRITTEN EXAMINATION 1 – 2018

Reading Time: 15 minutes

Writing Time: 1 hour

QUESTION AND ANSWER BOOK

Student
Number:

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Letter

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Structure of Book

Number of questions	Number of questions to be answered	Number of marks
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are **NOT** permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials Supplied

- Question and answer book of 10 pages.
- A separate sheet of miscellaneous formulas.
- Working space is provided throughout the book.

Instructions

- Write your **name** in the space provided above on this page.
- All written responses must be in English.

Students are **NOT** permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

- Answer **all** questions in the spaces provided.
- In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

QUESTION 1 (4 marks)

$$f(x) = e^{-3x} \cos(2x)$$

- a. Find $f'(x)$. 2 marks

- b. Hence find the values of a and b if $f'\left(\frac{\pi}{3}\right) = (a + b\sqrt{3})e^{-\pi}$ where $a \in R$ and $b \in Z$.

2 marks

QUESTION 2 (4 marks)

- a. Find the anti-derivative of $\frac{1}{(3x+2)^4} - e^{1-2x}$. 2 marks

- b. Evaluate $\int_0^{\frac{\pi}{6}} \left(\sin\left(\frac{3\pi}{2} + x\right) + \cos\left(\frac{\pi}{2} - x\right) \right) dx$. 2 marks

QUESTION 3 (7 marks)

The functions f and g are defined by

$$f : \mathbb{R}^+ \rightarrow \mathbb{R}, f(x) = \log_e x$$

$$g : \mathbb{R}^+ \rightarrow \mathbb{R}, g(x) = x^2 - 1$$

- a. Define $g^{-1}(x)$. Write your answer in similar function notation. 2 marks

- b. Show that $g^{-1}(f(x))$ does not exist. 1 mark

- c. If the domain of $f(x)$ is restricted to $\{x : x > k\}$, find the least value of k for which $g^{-1}(f(x))$ exists. 2 marks

- d. Find the rule and domain of $g^{-1}(f(x))$ for the restricted values of x found in part c. 2 marks

QUESTION 4 (4 marks)

Radioactive substances decay in such a way that the mass (grams) remaining at any time t years is given by $M(t) = M_0 e^{-kt}$ where k represents the decay constant. 500 grams of radioactive substance A, which has a decay constant of 0.05 is added to 100 grams of radioactive substance B, the decay constant which is unknown.

- a. Show that the masses of substance A and substance B will be equal when

$$t = \frac{\log_e 0.5}{0.05 - k} \dots$$

2 marks

Radium-226 decays in such a way that half of its mass disappears every 1590 years (the half-life). The mass of radium at any time t years is given by $M(t) = M_0 e^{-kt}$.

- b. Show that $k = \frac{\log_e 2}{1590}$.

2 marks

QUESTION 5 (3 marks)

Consider the function given by the rule $f(x) = 3\sin\left(\frac{x}{3}\right) + \pi$. Solve the equation

$$3f'(x) + f(x) = \pi \text{ for } 0 \leq x \leq 9\pi.$$

3 marks

QUESTION 6 (7 marks)

A sequence of three transformations takes the graph of $f(x) = 4x - x^2$ to the graph $g(x) = 9 - 6x - 3x^2$. One of these transformations is a reflection in the y -axis.

- a.** Write $f(x)$ and $g(x)$ in the form $a(x-h)^2+k$ and hence describe the three transformations in order. 3 marks

[illegible]

b. Write these transformations in matrix notation.

2 marks

c. Hence find the equation of the image of the graph of the function $y = e^{x+1}$ as the result of these transformations.

2 marks

QUESTION 7 (4 marks)

A bag contains twelve marbles, eight of them blue and four of them red. A sample of four marbles is taken from the bag, one at a time, without replacement.

- a. State the possible values of the sample proportion $\hat{p}$ of red marbles in the sample.

1 mark

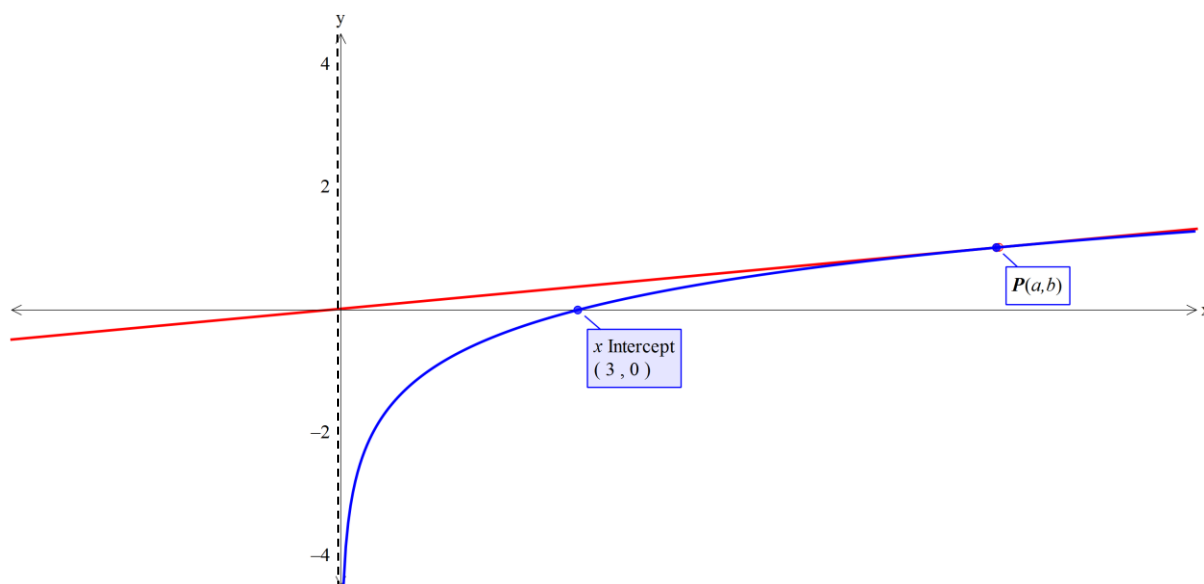
- b. Calculate the probability that the proportion of red marbles in the sample is more than 0.5, giving your answer in simplest form.

3 marks

QUESTION 8 (4 marks)

Part of the graph of the function $f : (0, \infty) \rightarrow \mathbb{R}$, where $f(x) = \log_e \left(\frac{x}{3} \right)$ is shown below.

The tangent to this curve at the point $P(a, b)$ passes through the origin.



- a.** Find a in terms of b where $a, b \in \mathbb{R}$

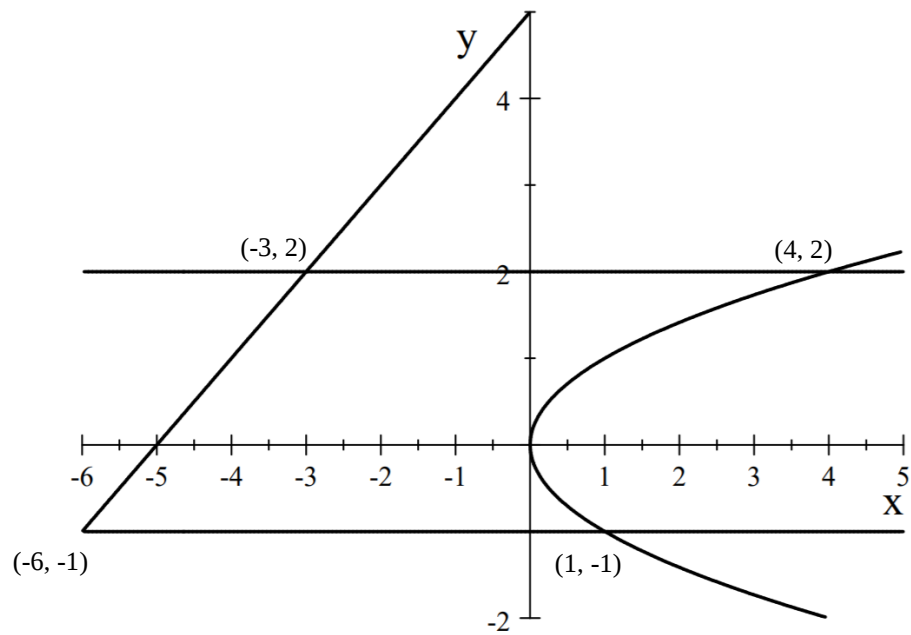
1 mark

- b.** Show that the equation of the tangent at P is given by $x = ay$.

3 marks

QUESTION 9 (3 marks)

Find the area enclosed by the graphs of $y = x + 5$, $y = \pm\sqrt{x}$, $y = 2$ and $y = -1$.



End of Paper